In assignment 1, we will solve the Laplace equation in 2 dimensions by the method of separation of variables; this is a variation of Example 3.4 in Griffiths. The problem is to determine the potential in a long, square, hollow tube, where the four walls have different potentials. The boundary conditions are as follows:

\[ V(x = 0, y) = 0, \]
\[ V(x = L, y) = 0, \]
\[ V(x, y = 0) = 0, \]
\[ \text{and } V(x, y = L) = V_0(x). \]  

Your assignment is as follows:

1. Set up the equation you need to solve, and make it dimensionless (e.g. use \( \xi = x/L \) as a variable instead of \( x \)).

2. Formulate a solution in terms of Fourier coefficients, in a way suitable for numerical calculation.

3. Write a computer program that takes any function \( V_0(x) \), and calculates \( V(x, y) \) inside the square tube. It should be possible to choose to what order your Fourier series should be summed up to, in order to investigate convergence issues.

4. Plot \( V_0(x) \), \( V(x, y) \), and \( V(x, y) \) on the boundaries (e.g. \( V(x, y = L) \)) to verify fulfillment of the boundary conditions.

5. Check how fast the calculation converges towards the correct result. In detail, you should compare how many Fourier orders you need for different \( V_0 \) functions.

6. Calculate and plot \( \vec{E}(x, y) \) using your method of choice.
To plot the potential, $V(x, y)$, there are several possibilities. Possibilities include using a contour plot (pyplot.contour) or a filled contour plot (pyplot.contourf). To plot the electric field, there are two possible approaches. One is to simply plot the vector arrows (pyplot.quiver) with their length being proportional to the electric field. Another possibility is to plot the vector length $|\vec{E}(x, y)|$ as a contour plot, and separately (or on top) plot the field direction by using vector arrows of length 1 (unit vectors). Check out the Matplotlib gallery for inspiration: [http://matplotlib.sourceforge.net/gallery.html](http://matplotlib.sourceforge.net/gallery.html)

Some example functions you can play with are:

\[
V_0(x) = \begin{cases}
V_c \sin\left(\frac{m\pi x}{L}\right), & m = 1, 2, 3, \ldots \\
V_c \left[1 - \left(\frac{x}{L} - \frac{1}{2}\right)^4\right] \\
V_c \theta(x - L/2)\theta(3L/4 - x)
\end{cases}
\]

Here, $V_c$ is a constant that you can choose freely\(^1\), and $\theta$ is the Heaviside step function. Play around with some of your own functions as well, and have fun!

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\(^1\)You should “eliminate” $V_c$ in problem \[1\]

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Figure 1: Sketch of the problem geometry.