You are out walking at night with your mother during your Christmas holiday. Suddenly, you are awed by the appearance of a beautiful display of aurora borealis above your heads. Your mother asks you what the aurora is—after all, you should know, being a physicist. You wave your hands and tell her about charged particles. When she asks you how these charged particles actually move around, you are embarrassed: you have no idea! However, you quickly come up with a plan:

(1) Model the Earth as a sphere with a magnetic dipole embedded at its centre (remember that the direction of the magnetic dipole is tilted with regard to the ecliptic). Plot the magnetic field in at least 2 of the 3 planes through the origin: the $xy$ and $xz$ planes. Choose the $x$ axis from the Sun toward the Earth and the $z$ axis perpendicular to the ecliptic.

(2) Model the solar wind as charged particles (e.g. protons) coming in with (initially) constant velocity along the $x$ axis. Experiment with different velocities to get a feeling for the effect of the magnetic field on the particles. Use the simple Euler’s method\(^1\) to solve the equation of motion in 3 dimensions.

(3) Find a test condition to estimate the accuracy of your numerical solution.\(^2\)

(4) To save your results for later analysis, write the final particle trajectories for different initial conditions to one or several text files.\(^3\)

(5) Load some of those files\(^4\) and make some beautiful curve plots showing a projection of the particles’ trajectory into your choice of the $xy$, $xz$, and $yz$ planes (or any other convenient plane). (It is also possible to make 3D plots, but this is not a requirement. Often, such plots are very difficult to read on paper.)

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\(^1\)Euler’s method is described in some detail here: \url{http://en.wikipedia.org/wiki/Euler_method}

\(^2\)Hint: magnetic fields do no work! See Griffiths, Eq. 5.11.

\(^3\)Hint: \url{http://tinyurl.com/284gwez}

\(^4\)Hint: \url{http://tinyurl.com/2dq9spn}