Problem 1.
Do the following examples from Griffiths: Ex. 3.2, 3.4, 3.5, and 3.8.
For example 3.2, derive in particular the appropriate position of the image charge inside the sphere.

Problem 2.

(a) Image problem with one interface.

(b) Image problem with two interfaces.

Figure 1: The two image problems. To the left is the “classic” image problem for a point charge outside an infinite conducting half-space. To the right, we have introduced an additional infinite conducting half-space.

In this problem, we are going to investigate two configurations of the image problem. In both problems, we have a point charge \( q \) “stuck” at the origin of our coordinate system, as depicted in Figure 1. Furthermore, we assume rotational symmetry around the \( x_3 \) axis. In all the subproblems, you can assume that \( a \) is a positive constant with dimension length.

a) First, explain briefly the method of images. Second, assume that the conducting half-space at \( x_3 = -a \) in Figure 1(a) is grounded. State explicitly the two boundary conditions for the scalar potential, \( V \), in this case.

b) Solve the classic image problem where we consider a point-like particle with charge \( q \) held at the origin, “hovering” over an infinite conducting half-space at \( x_3 = -a \), as
depicted in Figure 1(a). Sketch the equivalent image problem, give your solution for $V$, and show that it fulfills one of the boundary conditions found in Problem a). You are reminded that the potential from a point charge reads

$$V = \frac{q}{4\pi \varepsilon_0 r},$$

where, in this context, $r$ is the distance from the point charge to the observation point.

c) Now, introduce a second infinite conducting half-space at $x_3 = a$, as shown in Figure 1(b). Sketch and describe your setup of image charges in this case. Note that to solve this problem, you need to take account of both the real charge and the image charge introduced in b). (Hint: you need a lot of image charges, so be systematic!)

d) How would you proceed to determine the induced charge density on the two grounded planes in c)? You do not need to perform the actual calculation.