Improvements on the Filtered Smith Predictor using the Clegg Integrator

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Abstract: This paper presents a non-linear control scheme to deal with dead-time (DT) processes where small rise times are required. The control scheme is based on the combination of two strategies appeared in the literature to deal with DT processes, the Filtered Smith Predictor (FSP), which is a Smith Predictor (SP) including a filter to improve the robustness, and the PI+CI, a PI with a partial reset action on the control signal when the process output is equal to the reference input. In the proposed strategy, the reset action allows to achieve very small rise times with small overshoots, improving the results from FSP. On the other hand, the use of the robust predictor allows to deal with the dead time in a better and systematic way than the PI+CI with variable reset ratio and variable reset band does.

Keywords: Dead time, Smith Predictor, Reset Control, Clegg Integrator, Process control, Robustness

1. INTRODUCTION

Most industrial processes involve time delays. These dead times may be intrinsic to the process (such as in chemical and biological processes, distillation columns, etc.) or caused by the processing time of sensors, control algorithms requiring high computational burden, remote control tasks, or communication networks (Normey-Rico and Camacho, 2007). From a control perspective, time delays increase the system phase lag, thereby decreasing the phase and gain margins and limiting the response speed of the system, and thus posing a fundamental limitation over the bandwidth of a well-designed linear and time-invariant (LTI) control system.

Many dead-time compensators (DTC) have been widely studied in literature (Normey-Rico and Camacho, 2007). The Smith predictor(SP) (Smith, 1957) is the best known and most used algorithm for dead-time compensation in industry. The main advantage of the Smith predictor method is that the dead time is eliminated from the characteristic equation of the closed-loop system. Nevertheless, the main drawback of this algorithm is that modeling errors can drive the system to instability. A robust solution is the FSP, in which a filter is included to attenuate the oscillations caused by modeling mismatches (Normey-Rico and Camacho, 2007). The filtered Smith predictor (FSP) is a dead-time compensation structure which can be used to control stable, integrative and unstable processes with a dead-time, providing a unified approach to deal with typical drawbacks of the SP. The structure is based on a simple modification of the SP and both the design and tuning of the controller are simple (Normey-Rico and Camacho, 2009).

However, as well-known, the FSP is a linear control scheme and thus the control specifications are limited to its linear nature. For instance, when fast rise times and small overshoots are required at the same time, it is not possible to find a linear solution. This issue can be solved providing non-linear behavior to the FSP, such as described in this paper where FSP is combined with a non-linear PI controller.

Specifically, the PI+CI reset compensator presented in (Baños and Vidal, 2007a) has been used here as non-linear PI controller. This compensator consists of a PI controller plus a Clegg integrator (CI). Reset control systems were one of the first attempts to overcome fundamental limitations of LTI control systems. Its development was initiated fifty years ago with the work of Clegg (Clegg, 1958), that introduced a nonlinear integrator based on a reset action. Basically, since the integrator output is set to zero when its input is zero, a faster system response without excessive overshooting may be expected, thus avoiding limitation of its LTI counterpart. The implementation of reset control is very simple, it consists of resetting the state (or part of it) of a feedback LTI compensator (referred to as the base LTI compensator or simply base compensator) at every instant

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in which its input is zero (reset times). Usually the design of the reset control is strongly dependent on a proper election of the base control system. A common approach is to design the base system to be stable and to fulfill some performance specifications, and then including reset over some compensator states to improve performance and robustness. In (Baños and Vidal, 2007b), (Baños and Vidal, 2007a) tuning rules of the PI+CI compensator for first and second order systems with time delay were developed. However, in (Vidal, 2009), it is shown that these proposed tuning rules are not working properly for deadtime dominant systems (having also bandwidth limitation problems), and some ad-hoc solutions are provided to face these problems, including the use of a fixed or variable reset band, and also a variable reset percentage.

This work investigates how reset control, and in particular PI+CI compensation, may increase the performance and robustness of FSP, achieving nonlinear specifications. On the other hand, also the potentials of FSP to improve PI+CI compensation, specifically when dominant delays are present, as alternative to the use of a variable reset band and a variable reset percentage.

The paper is organized as follows: section 2 briefly describes the FSP control scheme. The PI+CI reset controller is introduced in section 3. The new combined control algorithm is presented in section 4 discussing the design rules. Section 5 deals with a simulated example to compare the proposed control scheme with FSP controller and PI+CI compensator. Finally, the paper ends with some conclusions and future works.

2. THE FILTERED SMITH PREDICTOR

The FSP control scheme is shown in Figure 1. As can be seen, the structure is the same as in the SP with two additional filters. F_r is a traditional reference filter to improve the set-point response and F is a predictor filter used to improve the predictor properties. This structure was firstly proposed in (Normey-Rico et al., 1997) for FOPDT stable processes to improve the robustness of the traditional SP. Because of its characteristics, the FSP can be used to compute a controller taking into account the robustness, coping with unstable plants, improving the disturbance rejection properties, and decoupling the set-point and disturbance responses. Therefore, all the drawbacks of the SP are considered in the design, using only one structure and a unified design procedure (Normey-Rico and Camacho, 2009).



Fig. 1. The Filtered Smith Predictor control scheme

In the structure, $P_n = G_n e^{-sh_n}$ is a model of the process, G_n is the dead-time-free model, h_n is the model delay and C is the primary controller (for this paper, a PI controller is considered where k_p and τ_i are the proportional gain and the integral time constant, respectively). In this structure that is used for analysis, the nominal closed-loop transfer functions for the reference and load disturbances responses (when the model of the plant is perfect, $P = P_n$) are:

$$H_r = \frac{Y}{R} = \frac{F_r C P_n}{1 + C G_n} \tag{1}$$

$$H_d = \frac{Y}{D} = P_n \left[1 - \frac{CP_n F}{1 + CG_n} \right] \tag{2}$$

To analyze the robustness, consider a family of plants P such that $P = P_n[1 + \delta P]$ and:

$$|\delta P(j\omega)| \le \overline{\delta P}(\omega) \quad \forall \omega > 0$$

Thus, the robust stability condition for the FSP is:

$$\overline{\delta P}(\omega) < dP(\omega) = \frac{|1 + C(j\omega)G_n(j\omega)|}{|C(j\omega)G_n(j\omega)F(j\omega)|} \forall \omega > 0$$
(3)

Note that only $\frac{Y}{D}$ and $dP(\omega)$ are modified by the inclusion of the filter. That is, the filter F can be used to improve the robustness or the disturbance rejection capabilities of the system without affecting the nominal set-point response. Furthermore, F can be tuned to obtain an internal stable system when controlling unstable plants (Normey-Rico and Camacho, 2009). Therefore, this controller has enough degrees of freedom to obtain compromise between robustness and a desired set-point and disturbance rejection responses.

However, such as commented above, there are some situations where the FSP controller is not able to reach the desired closed-loop specifications because of the linear nature of the control scheme. This is the case when small rise times with small overshoots are required simultaneously. Notice that these specifications are conflicting and it is not always possible to reach both of them at the same time using a linear control algorithm. Thus, the nonlinear controller described in the following section will be combined with the FSP scheme in order to face these situations, such as presented later on.

3. THE PI+CI RESET CONTROLLER

This kind of compensator can be interpreted as a PI controller with reset action on its integral term (Baños and Vidal (2007b,a)). Specifically, a PI+CI controller is defined simply by adding a Clegg integrator (CI) (Clegg (1958)) (integrator with reset output to 0 when its input is 0) to a PI controller. From this perspective, the PI+CI controller will have three terms as shown in Figure 2.

In this compensator, k_p and τ_i are the proportional gain and the integral time constant of its counterpart PI compensator. The reset ratio p_{reset} represents the part of the integral term over which the reset action is applied, and it is used to set the partial reset on the integral term. In (Baños and Vidal (2007b,a)), it has been shown that this



Fig. 2. The PI+CI reset control scheme

partial reset of the integral term results in an improvement on the transitory closed-loop response, reducing the overshoot and settling time of the corresponding design without reset. This parameter is adjusted heuristically.

In the frequency domain, the stability of the reset closedloop system can be analyzed by using the describing function method. This method, in spite of being an approximation, has the advantage that delays can be taken into account in a quite natural way. The describing function of the PI+CI compensator can be simply obtained by adding the describing function of a CI (Clegg (1958)) to the frequency response of a PI controller. In that way, the describing function of a PI+CI controller is given by

$$D_{C_{nl}}(j\omega) = k_p \left(1 + \frac{1 - p_{reset}}{\tau_i} \frac{1}{j\omega} + \frac{p_{reset}}{\tau_i} \frac{1.62}{\omega} e^{-j38.1^\circ} \right) (4)$$

As it can be seen (Fig. 3), the difference between a PI+CI controller and its base PI (obtained for $p_{reset} = 0$) is that the reset action give an extra phase lead up to 50 degrees, significantly for frequencies $\omega < 1/\tau_i$, without much increasing the loop gain. This is clearly impossible to achieve by a LTI compensator, and has been one of the main reasons to use PI+CI compensation in order to overcome fundamental limitations of LTI compensation.



Fig. 3. Phase of $D_{C_{nl}}(j\omega)$ vs $\omega \tau_i$

Some simple PI+CI tuning rules for several types of plants has been derived in (Vidal (2009)): the first step consists in tuning the base PI compensator. The tuning rule proposed by Skogestad (2003)) is used to deal with a tradeoff between fast responses and good disturbance rejection. The controller parameters will depend on the system parameters and on the time constant of the desired closed-loop response, λ . Specifically, when the process is modeled as a first-order dead-time (FOPDT) system, with gain k_n , time constant τ_n and dead-time h_n , the controller takes the structure of a PI controller with the following parameters

$$k_p = \frac{\tau_n}{k_n(\lambda + h_n)} \tag{5}$$

$$\tau_i = \tau_n \tag{6}$$

The value of the desired closed-loop time constant λ can be chosen freely, but from equation (5) it has to be within $-h < \lambda < \infty$, in order to get a positive and nonzero controller gain. The optimal value of λ is determined by a tradeoff between fast response and good disturbance rejection (small value of λ), and stability and robustness (large value of λ). From (Skogestad (2003)), a choice with a reasonably fast response with good robustness margins is $\lambda = h_n$, where for lag dominant systems, $\tau_i = 8h_n$ is chosen in order to improve the load disturbance response. In (Vidal (2009)), this rule (5) is used to cope with lag dominant systems and it is shown that is not valid for dead-time dominant systems in the PI+CI context.

In order to overcome these problems for PI+CI with deadtime dominant systems (Vidal (2009)), the reset condition is modified so that the CI is reset when the *e* signal in Figure 2 satisfies the following equation:

$$h\dot{e}(t) + e(t) = 0 \tag{7}$$

As a result, small rise times with small overshoots are achieved. However, harmful effects appear in the undershoot of the output, due to the reset action. Furthermore, another improvement is suggested, the application of a variable reset ratio p_{reset} given by (Vidal (2009); Vidal et al. (2008))

$$p_{reset} = p_{reset}^0 + \tau_d \frac{de_f(t)}{dt} sign(e_f(t))$$
(8)

where $e_f(t)$ is the filtered error and τ_d is tuned as a function of the reference signal change Δr :

$$\tau_d = \kappa \frac{t_r p_{reset}^0}{\Delta r}$$

with $\kappa \in (0,3]$ and t_r the rise time for the closed-loop system. In (Vidal (2009)) $\lambda = 2h_n/3$ is chosen, resulting only there in two free tuning parameters, κ and p_{reset}^0 . When $\Delta r = 0$, τ_d is set to 0, and thus $p_{reset} = p_{reset}^0$ from Eq. (8).

In the following, the potentials of PI+CI compensation to improve the performance of the FSP structure are investigated. Conversely, the PI+CI structure may benefit from FSP for plants with dominant delays.

4. IMPROVING THE FSP AND THE PI+CI: PI+CI-FSP

In this section, a FSP with PI+CI reset is used to cope with the last two problems indicated in the previous sections, related with the control of the dead-time dominant processes and non-linear specifications. Here, the control structures in Figures 1 and 2 are combined in order to achieve a control scheme with the advantages of both schemes (see Figure 4): the fast rise time with small overshoot of the PI+CI and the robust predictive feature of the FSP.



Fig. 4. Combined FSP and PI+CI reset control scheme

The new structure is obtained substituting the C linear controller in Figure 1 with the C_{nl} nonlinear controller in Figure 2. Using the describing function of PI+CI with fixed p_{reset} reset ratio, given by equation (4), the F filter in Figure 1 is designed in order to assure the stability of the new structure in spite of the plant uncertainty. Furthemore, in the design of F, other two important factors are taken into account: the load disturbance response and the noise filtering (Santos et al. (2010)).

Thus, the following algorithm is proposed to design the control system:

- (1) Choose a small rise time for the closed-loop system.
- (2) Assuming null dead-time, design the base PI controller to achieve the previous rise time assuming a second-order closed-loop system and determining the k_p and τ_i constants, for instance, by the pole placement method.
- (3) Choose a p_{reset} reset ratio so that the system output achieves a determined overshoot.
- (4) Design the F filter in Figure 1 to improve the robustness of the new control structure using the describing function of PI+CI given by equation (4).

As was previously indicated in the algorithm, the objective is to achieve a second-order closed-loop system. Thus, the filter F has to be computed using a variation of the structures proposed in (Normey-Rico and Camacho (2009)), shown in equation (9) as follows

$$F = \frac{((1/\omega_n^2)s^2 + 2(\delta/\omega_n)s + 1)(\beta s + 1)}{(T_0s + 1)^3} \tag{9}$$

where ω_n and δ are the undamped natural frequency and the damping factor of the desired closed-loop secondorder system dynamics, respectively, T_o is the free tuning parameter, and

$$\beta = \tau_n (1 - ((1 - T_o/\tau_n)(1 - T_o/\tau_n))e^{-h_n/\tau_n})$$

Notice that after designing the filter F for robustness properties, an extra pole should be added to cope with the noise filtering.

5. ILLUSTRATIVE EXAMPLE

This section presents a simulated example to show the main advantages of the proposed control algorithm. Lets consider the first-order system with dead-time (Vidal (2009)) described by

$$P(s) = \frac{1}{2s+1}e^{-s}$$
(10)

Notice that, although this process is not a dead-time dominant system, it has been included for comparison with the results presented in (Vidal, 2009). Furthermore, this is a representative process showing a intermediate case between lag and dead-time dominant systems.

Such as pointed out in the previous section, the firs step for the proposed combined algorithm consists in setting the rise time specification. For this example, it is selected to $t_r = 1.39$ seconds. The values computed for the PI controller are $k_p = 5$ and $\tau_i = 0.5$. A $p_{reset} = 0.5$ is fixed in order to achieve an overshoot of 3.2% (Vidal, 2009). Finally, the filter F must be derived from equation (9) in order to improve the robustness, the disturbance response and the noise filtering. Figure 5 shows the bound considering 30% uncertainty in the dead time for the new combined structure (PI+CI-FSP) in dash-dot line. Notice that the describing function (4) was used for this case. The tuning filter parameter was set to $T_o = 0.706$ and an extra pole at $\omega = 40 rad/s$ was added to deal with noise filtering effect.

For the FSP (PI-FSP) control scheme, the same PI controller parameters were used ($k_p = 5$ and $\tau_i = 0.5$). Then, as observed from Figure 5, the corresponding robustness bound was calculated (solid line in the figure), where the same filter was used to cope with both control algorithms. From the figure, it can be observed how the resulting filter satisfies the uncertainty bound for both control schemes.



Fig. 5. Robustness analysis for PI+CI-FSP and FSP.

In the case of the PI+CI algorithm, it was not possible to use the same PI controller parameters as in the other algorithms since this would leave to an unstable behavior (because of dead-time limitations). Hence, in this case, the values of $k_p = 1.2$ and $\tau_i = 2$ ($t_r = 1.6$), $p_{reset}^0 = 0.3$, and $\kappa = 1$ are used (Vidal (2009)). Furthermore, a low-pass filter with the same bandwidth than F, $\omega = 34$ rad/s, has been used to filter the error signal as described in (8).



Fig. 6. A comparison among PI+CI-FSP with $p_{reset} = 0.5$, PI+CI with variable reset band and variable p_{reset} , and FSP for nominal case.

Figure 6 shows a comparison among the three structures for the nominal case (when $h = h_n$ in Figure 1). In the simulation, a unit step in the reference input is included at time instant t = 3 seconds and a unit step disturbance at the plant input at time instant t = 35 seconds. A supervisory mechanism is implemented over the new proposed structure, so that the p_{reset} is set to zero when the system is in steady state after a set-point change. Thus, p_{reset} will be different to zero only for reference changes. This fact avoids the problems with the load disturbances when the system output crosses with zero due to the reset action (Vidal (2009)).

Table 1 summarizes the results using several performance indexes for the three structures, where OS is the overshoot, IAE_r is the Integrated Absolute Error for the reference tracking, $ITAE_r$ is the Integrated Time weighted Absolute Error for the reference tracking, t_r is the rise time for the reference tracking, and IAE_d and $ITAE_d$ are equivalent measurements to IAE_r and $ITAE_r$ for the load disturbance response. Notice that for all the results presented in this section, the IAE_r and $ITAE_r$ measurements are calculated from t = 4 seconds for a better comparison.

The smallest values for overshoot, IAE_r , $ITAE_r$ and t_r are achieved for the PI+CI-FSP scheme. Obviously, the rise time for PI-FSP and PI+CI-FSP is the same but a much smaller overshoot is obtained for the new scheme. This new structure improves the PI+CI-PV-BV (section 3) and PI-FSP (section 2) behaviors. It is possible to obtain a faster response in comparison with PI+CI-PV-BV, and also avoiding undershoot; and on the other hand to achieve a small rise time with a small overshoot, improving the PI-FSP.

The overshoot in the PI-FSP controller can be reduced by using the reference filter, F_r , but at the cost of making a slower response. Figure 7 shows a comparison with a PI-FSP where the following reference filter

$$F_r = \frac{1}{0.61s + 1} \tag{11}$$

Structure	OS	IAE_r	$ITAE_r$	t_r	IAE_d	$ITAE_d$
PI+CI-FSP	3.2	0.3	1.5	1.6	1.2	3.6
PI+CI-PV-BV	4.5	1.1	6.5	3.1	1.7	6
PI-FSP	17	0.45	2.1	1.6	1.2	3.6

Table 1. A comparison among PI+CI-FSP with $p_{reset} = 0.5$, PI+CI with variable reset band and variable p_{reset} , and FSP for nominal case in terms of performance indexes.



Fig. 7. A comparison among PI+CI-FSP with $p_{reset} = 0.5$, PI+CI with variable reset band and variable p_{reset} , and FSP with reference filter, for nominal case.

Structure	OS	IAE_r	$ITAE_r$	t_r	IAE_d	$ITAE_d$
PI+CI-FSP	3.2	0.3	1.5	1.6	1.2	3.6
PI+CI-PV-BV	4.5	1.1	6.5	3.1	1.7	6
PI-FSP	3.2	0.8	3.5	2.6	1.2	3.6

Table 2. A comparison among PI+CI-FSP with $p_{reset} = 0.5$, PI+CI with variable reset band and variable p_{reset} , and FSP with reference filter, for nominal case in terms of performance indexes.

is introduced to achieve the same overshoot as the PI+CI-FSP scheme, without variation with respect to the load disturbance response in Fig. 6 (Normey-Rico and Camacho (2007)). As shown from the figure, both schemes have the same overshoot but the rise time has been decreased for the PI+FSP, being much smaller than for the PI+CI-FSP structure. Table 2 summarizes the performance indexes of this simulation for the three structures.

Figure 8 shows the same example presented in Figure 6 but including measurement noise. As can be observed, the three control schemes provide similar responses. Notice that the best results are obtained for the PI+CI-PV-BV scheme. The reason is that, such as commented above, this control structure has a lower bandwidth because its PI base controller was detuned to deal with dead-time limitations. Notice how the PI+CI-FSP scheme allows reaching the desirable bandwidth and provides an acceptable response against noise measurement.



Fig. 8. A comparison among PI+CI-FSP with $p_{reset} = 0.5$, PI+CI with variable reset band and variable p_{reset} , and FSP with reference filter, for noisy case.



Fig. 9. A comparison among PI+CI-FSP with $p_{reset} = 0.5$, PI+CI with variable reset band and variable p_{reset} and FSP, for h = 1.3.

Structure	OS	IAE_r	$ITAE_r$	t_r	IAE_d	$ITAE_d$	
PI+CI-FSP	37	1.2	6.8	1.9	1.3	4.6	
PI+CI-PV-BV	20	1.8	11	3.1	2.4	14	
PI-FSP	46	1.4	8.6	1.9	1.3	4.6	
Table 3. A comparison among PI+CI-FSP							
with $p_{reset} = 0.5$, PI+CI with variable reset							

band and variable p_{reset} and FSP, for h = 1.3in terms of performance indexes.

Finally, the robustness of the three structures is analyzed. Figure 9 shows an example where the three structures are simulated for an uncertainty of 30% in the dead-time $(h = 1.3 \text{ and } h_n = 1)$. For the PI+CI-FSP and PI-FSP, the robustness is guaranteed such as observed from Figure 5. The PI+CI-FSP and the PI-FSP structures have similar responses, although as shown in Table 3, the PI+CI-FSP structure is slightly better. The PI+CI-PV-BV seems to be robust (a similar analysis is missed since its describing function is not reported in the literature) but it has the worst performance indexes.

6. CONCLUSIONS

In this paper, a new control structure based on the combination of two strategies appeared in the literature to deal with DT processes is proposed. This new non-linear control structure, based on the Filtered Smith Predictor and in the PI+CI, referenced as PI+CI-FSP, achieves small rise times with small overshoots with good dead-time compensation. The proposed scheme improves the performance of the FSP keeping its robustness properties, and it also improves the behavior of the PI+CI with variable reset ratio and variable reset band (PI+CI-PV-BV). Furthermore, the PI+CI-FSP is much easier to tune than PI+CI-PV-BV and the robustness is assured.

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