A New Predictive PI Controller with Additonal Filtering

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Abstract: A new predictive PI controller is proposed and applied to the benchmark MIMO PID 2012. Estimate of disturbance, obtained from the Disturbance Observer (DO), is introduced in the loop to obtain the offset free control. First-Order Plus Dead-time (FOPDT) model of stable, integrating and unstable plants is used to design DO, by applying inverse modeling technique. Tuning rules are proposed, and analyzed by simulation of stable, integrating and unstable processes. The high performance of the proposed predictive PI controller with additional filtering is demonstrated by simulation and on the benchmark MIMO PID 2012 plant, with the open-loop dynamics approximated by FOPDT models for the pressure and the water level.

Keywords: PI controllers, Delay compensation, Disturbance rejection, Measurement noise, Filtering, Experimental tuning, Boiler control, Benchmark example.

1. INTRODUCTION

Rejection of the load step disturbance is of primary importance to evaluate controller performance under constraints on the robustness, see Shinskey (1990). The offset free control can be obtained by introducing in the loop an integral action or by introducing in the loop an estimate \hat{d} of the load step disturbance d. In the Modified Smith Predictor (MSP) proposed by Mataušek and Micić (1996, 1999), the estimate \hat{d} is obtained as the output of a Proportional-Derivative controller with the input defined by the difference between the controlled output y(t) and model output $y_m(t)$. The estimate \hat{d} can be obtained from a Disturbance Observer (DO) designed by applying the inverse modeling technique, as in Endo et al. (1996); Kempf and Kobayashi (1999). The same idea is used to design PDDO controller proposed by Mataušek and Ribić (2009), which is a generalization of the series PID controller. This controller can be configured as a predictive PI controller, see Hägglund (1992), or as a PI_{τ}/PID_{τ} controller, see Shinskey (1990, 2001), with a time delay inserted into the integral feedback circuit.

Recently, Mataušek and Ribić (2012) demonstrated that MSP is a parallel PID controller in series with a filter defined by dead-time. For a large class of stable, integrating and unstable processes, faster load disturbance rejection is obtained by the MSP-PID controller, compared with PID controller and recently proposed Dead-time Compensators (DTC), if all controllers are tuned under the same constraints on the sensitivity to measurement noise and the same robustness. However, when the level of measurement noise is high, fast rejection of the load disturbance must be supported by additional filtering to avoid excessive control signal activity, see Larsson and Hägglund (2011). Control signal activity can be measured in the frequency domain as proposed by Larsson and Hägglund (2011), or in time domain by Total Variation (TV), proposed by Skogestad (2003) and used in the benchmark MIMO PID 2012, see Morilla (2011).

According to the above discussion, and since PI controllers are mostly used in industry applications, see Yamamoto and Hashimoto (1991), it is of interest to design a PI like controller in order to obtain fast load disturbance rejection with acceptable control signal activity in the presence of the measurement noise. Tuning of the proposed controller is based on the First-Order Plus Dead-Time (FOPDT) model given by

$$G_m(s) = \frac{e^{-Ls}}{b_1 s + b_0}.$$
 (1)

Model (1) is a representation of process dynamics which can be obtained from a simple tests on the plant. Besides, it can represent a wide class of stable, integrating and unstable processes. For stable processes $|b_0| > 0$ and $sign(b_1) = sign(b_0)$, for integrating processes $b_0 = 0$, while for unstable processes $|b_0| > 0$ and $sign(b_1) \neq sign(b_0)$.

The structure of the new predictive PI controller with additional filtering is presented in Section 2. Tuning and the performance/robustness tradeoff, obtained by the proposed controller, is analyzed in Section 3 by simulation of stable, integrating and unstable processes. Then, in Section 4 results obtained on the benchmark MIMO PID 2012 plant are presented.

2. CONTROLLER STRUCTURE

The block diagram used to derive the proposed controller is presented in Fig. 1. By assuming that the model (1) is

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Fig. 1. Process G_p with: actuator AC, additional filter F_n , observer DO, stabilizing controller C_S and reference prefilter C_{ff} . Signals r, u, w, d, \hat{d}, n and y denotes set-point, controller output, actuator output, load disturbance, estimated value of the load disturbance, measurement noise and process output, respectively.

a perfect representation of the process dynamics, $G_p(s) \equiv G_m(s)$, from Fig. 1 one obtains

$$y = G_m(d+w) + n.$$
(2)

By multiplying left and right side of (2) with $(b_1s + b_0)/(T_Is + 1)$ it follows from (1) that

$$\frac{e^{-Ls}}{T_{I}s+1}d = \frac{b_{1}s+b_{0}}{T_{I}s+1}y - \frac{e^{-Ls}}{T_{I}s+1}w - \frac{b_{1}s+b_{0}}{T_{I}s+1}n.$$
(3)

Since the measurement noise n is unknown, only \hat{d} defined by

$$\hat{d} = \frac{b_1 s + b_0}{T_I s + 1} y - \frac{e^{-Ls}}{T_I s + 1} w, \tag{4}$$

can be used to design DO in Fig. 1.

 F_n is an additional filter used to reduce the amplification of the measurement noise n:

$$F_n(s) = \frac{1}{T_F s + 1}, \ T_F \ge 0.$$
 (5)

The stabilizing controller C_S is defined by

$$C_S = K_C \frac{T_a s + 1}{T_I s + 1}.\tag{6}$$

The gain K_C and time constants T_I , T_a , and T_F are adjustable parameters. From Fig. 1, (4) and (6) one obtains that the control signal u, generated by the proposed predictive PI with additional filtering (PPIF) controller is defined by

$$u = C_{ff}(s)r - \frac{a_1s + a_0}{B_F(s)}y + \frac{e^{-Ls}}{B_F(s)}w,$$
(7)

$$C_{ff}(s) = a_0/B_F(s), B_F(s) = (T_I s + 1)(T_F s + 1),$$
 (8)

where

$$a_0 = b_0 + K_C, \ a_1 = b_1 + K_C T_a. \tag{9}$$

Implementation of the proposed PPIF controller is presented in Fig. 2. For $T_F = 0$, one obtains a predictive PI (PPI) controller, Ribić and Mataušek (2011).

According to (3) and (4) the estimate \hat{d} is related to the real unmeasurable load disturbance d as given by

$$\hat{d} = \frac{e^{-Ls}}{T_I s + 1} d + \frac{b_1 s + b_0}{T_I s + 1} n.$$
(10)



Fig. 2. Implementation of the proposed PPIF controller.

From (10) it follows that faster convergence of estimate \hat{d} to the real load disturbance is obtained for smaller values of time constant T_I . However, excessive control signal activity can be obtained for $T_F = 0$ and for smaller values of T_I , since in this case the high-frequency gain from n to u is equal to a_1/T_I . Thus, in the presence of the high level of measurement noise, it is proposed to include additional filter (5), with $T_F > 0$.

Both cases $T_F = 0$ and $T_F > 0$ are discussed and analyzed by simulations in Section 3, where tuning formulae for the adjustable parameters a_0 , a_1 , T_I and T_F are derived for given values of b_1 , b_0 and L.

In the linear region of the actuator, where $w \equiv u$, the PPIF controller is equal to a two-degree-of-freedom control system given by

$$u = \frac{a_0}{B_F(s) - e^{-Ls}}r - C_{PPIF}(s)y,$$

$$C_{PPIF}(s) = \frac{a_1s + a_0}{B_F(s) - e^{-Ls}}.$$
(11)

If delay term e^{-Ls} is approximated by 1 - Ls from (11) one obtains the filtered PI controller defined by

$$C_{PIF}(s) = \frac{1}{\frac{T_I T_F}{T_I + T_F + L}s + 1} \frac{a_1 s + a_0}{(T_I + T_F + L)s}.$$
 (12)

3. CONTROLLER TUNING AND SIMULATION ANALYSIS

For $T_F = 0$, parameters a_0 and a_1 are determined by relating T_a to dead-time L as $T_a = \beta L$, $\beta > 0$. Then, the gain K_C is obtained from the characteristic equation

$$1 + C_S(s_0)G_m(s_0) = 0, s_0 = -\frac{1}{\lambda L}, \lambda > 0.$$
 (13)

Table 1. Parameters of the PPI/PPIF controllers for processes $G_{p1}(s)$ and $G_{p2}(s)$, and performance/robustness indexes.

Process/Controller	\mathbf{L}	λ	β	a_0	a_1	γ	T_I	T_F	IAE_d	M_S	M_T
G_{p1}/PPI	10	2	0.5	3.53	76.67	-	5	0	4.249	1.72	1.01
G_{p1}/PPIF	10	2	0.5	3.53	76.67	0.5	5γ	5γ	4.249	1.83	1.03
G_{p2}/PPI	8	4	2	2.288	130.6	-	16	0	10.02	1.54	1.29
G_{p2}/PPIF	8	4	2	2.288	130.6	0.5	16γ	16γ	10.52	1.9	1.43

Table 2. Parameters of the PPI/PPIF controllers for processes $G_{p3}(s)$, and performance/robustness indexes compared to the FSP controller. The noise free case.

Controller	\mathbf{L}	λ	β	a_0	a_1	γ	T_I	T_F	IAE_d	M_S	M_T
PPI	20	2	0.5	0.3408	36.35	-	10	0	88.03	2.19	1.86
FSP	-	-	-	-	-	-	-	-	72.62	2.26	1.87
PPIF	20	2	0.5	0.3408	36.35	0.5	10γ	10γ	88.03	2.41	1.94



Fig. 3. Open-loop 10% step responses of pressure/fuel flow (left) and water level/water flow (right) of the benchmark MIMO PID 2012 plant (solid) and step responses of the corresponding models $G_{p1}(s)$ and $G_{p2}(s)$ (dashed).



Fig. 4. Load disturbance unite step responses of process $G_{p3}(s)$ in the loop with the simple PPIF controller and FSP complex DTC controller, in the presence of the Band-Limited White Noise with Noise power=0.001 and sample time=0.3s.

Table 3. The benchmark MIMO PID 2012 plant: relative performance of the PPIF controller(evaluated case in Figs. 5-7) compared to the reference case in Figs. 5-7.

Experiment	$RIAE_1$	$RIAE_2$	$RIAE_3$	$RITAE_1$	$RITAE_3$	$RIAVU_1$	$RIAVU_2$	$J_M(0.25)$
Step on pressure SP, Fig. 5	0.2509	1.0002	0.2531	0	0	1.3463	1.5065	0.6336
Variable load disturbance, Fig. 6	0.2484	0.9990	0.2510	0	0	1.2109	1.1691	0.5981
Step load disturbance, Fig. 7	0.4817	1.0036	0.2814	0.3290	0	1.3996	1.5314	0.6286

59 L



Fig. 5. Benchmark: step change of the pressure set-point as in Morilla (2011).



Fig. 6. Benchmark: variable load disturbance as in Morilla (2011).



Fig. 7. Benchmark: step load disturbance as in Morilla (2011).

For $T_F = 0$, by using $T_I = T_a$, as proposed by Ribić and Mataušek (2011) one obtains tuning formulae

$$K_C = e^{-\frac{1}{\lambda}} \left(\frac{b_1}{\lambda L} - b_0 \right), \tag{14}$$

$$a_0 = b_0 + K_C, (15)$$

$$a_1 = b_1 + \beta L K_C, \tag{16}$$

with adjustable dimensionless parameters $\lambda > 0$ and $\beta > 0$. For stable, integrating and unstable processes it is proposed by Ribić and Mataušek (2011) to use values $\lambda = 2$ and $\beta = 0.5$ as a general rule . For higher values of λ and β , lower values of the maximum sensitivity M_S and maximum complementary sensitivity M_T are obtained, resulting into greater values of Integrated Absolute Error (IAE_d) . Finally, for $T_F = 0$ and $T_I = T_a = \beta L$ one obtains $B_F(s) = (1 + \beta Ls)$. The unit step response of the filter $1/(1 + \beta Ls)$ arrives to the value of 0.632 at time $t = \beta L$.

For the case $T_F > 0$, considered in this paper, it is proposed to use tuning formulae (14)-(16) for obtaining the gains a_0 and a_1 . In this case, additional filtering affects only the term $B_F(s)$ in (7)-(8). It can be easily shown that the unit step response of the filter $1/(1+\gamma\beta Ls)^2$ arrives to the value 0.632 at time $t = \beta L$ for $\gamma \approx 0.465$. Accordingly, for $T_F > 0$ it is proposed to use

$$T_a = \beta L, \ T_I = T_F = \gamma T_a, \ \gamma = 0.5, \tag{17}$$

$$B_F(s) = (1 + \gamma\beta Ls)^2. \tag{18}$$

Tuning defined by (14)-(18), confirmed by simulations, is applied to the benchmark MIMO PID 2012 plant in Section 4.



In simulation analysis the following test batch of stable, integrating and unstable processes:

$$G_{p1} = \frac{e^{-10s}}{75s+3.2}, \ G_{p2} = \frac{e^{-8s}}{94s}, \ G_{p3} = \frac{3.433e^{-20s}}{103.1s-1}, \ (19)$$

is used. Models $G_{p1}(s)$ and $G_{p2}(s)$ are obtained fitting-byeye the open-loop step responses of steam pressure and water level of the benchmark MIMO PID 2012 plant, presented in Fig. 3. Process $G_{p3}(s)$ is used by Normey-Rico and Camacho (2009) to demonstrate the performance/robustness trade-off obtained by the FSP complex DTC controller, and used by Albertos and Garcia (2009) to compare their complex DTC controller with the FSP controller. The similar results are obtained in both papers.

In Table 1, parameters of the PPI and PPIF controllers applied to processes $G_{pj}(s)$, j = 1, 2, are presented with the obtained performance index IAE_d and robustness indexes M_S and M_T . For $T_F=0$ and $T_F > 0$, the same performance index IAE_d is obtained for slightly changed robustness indexes M_S and M_T . The simple PPI and PPIF controllers are compared with the FSP complex DTC controller in Table 2 and Fig. 4. In Table 1 and Table 2, indexes IAE_d , M_S , and M_T are defined by

$$IAE_d = \int_{0}^{\infty} |y_d(t)| dt,$$

$$M_S = \|S(i\omega)\|_{\infty}, M_T = \|1 - S(i\omega)\|_{\infty},$$
(20)

$$S(s) = \frac{1}{1 + C_{PPIF}(s)G_p(s)}, \ y_d = \frac{1}{s}G_p(s)S(s).$$
(21)

4. PPIF CONTROL OF THE BENCHMARK MIMO PID 2012 PLANT

Decentralized Single-Input Single-Output (SISO) design is used in the present paper to define Multiple-Input Multiple-Output (MIMO) controller structure for the benchmark MIMO PID 2012 plant. Two SISO PPIF controllers are applied: one, relating steam pressure with fuel flow, and the other, relating water level with water flow. Both controllers are defined by (7)-(8), with parameters presented in Table 1, cases $G_{p1}/PPIF$ and $G_{p2}/PPIF$, respectively.

Feedforward signal from the load is not included in the evaluated controller. In both loops, saturation limits $l_{low} = 0\%$ and $l_{high} = 100\%$, rate limit of 1%/s, and a small deadband of 0.3% are applied in actuators.

Results of this analysis are presented for: the step change of the pressure set-point in Fig. 5, for the variable load disturbance in Fig. 6 and for the step load disturbance in Fig. 7. In these figures, high performance of the PPIF controller is not demonstrated for the Oxygen level. It is presented only in Table 3. The relative performance of the PPIF controller (evaluated case in Figs. 5-7) is compared to the reference case defined in Morilla (2011).

In Table 3, performance indexes are presented as calculated in the benchmark MIMO PID 2012 plant, see Morilla (2011). Indexes $RIAE_j$, j = 1, 2, 3, are normalized (evaluated/reference) IAE indexes for steam pressure, oxygen level and water level. Indexes $RITAE_j$, j = 1, 3 are the normalized ITAE indexes for steam pressure and water level. Indexes for fuel flow and water flow, while J_M is the total normalized performance index, as in Morilla (2011):

$$J_M(0.25) = \frac{\sum_{j=1}^{3} RIAE_j + \sum_{j=1}^{3} RITAE_j + 0.25 \sum_{j=1}^{2} RIAVU_j}{3 + \sum_{j=1}^{3} typechange_j + 2 \cdot 0.25}$$
(22)

In (22), when $RITAE_j$ is calculated then $typechange_j = 1$, otherwise $typechange_j = 0$. Results presented in Figs. 5-7 and Table 3 confirm the advantages of the PPIF controller compared to the controller used in the benchmark MIMO PID 2012 plant as reference.

5. CONCLUSION

Experiments on the benchmark MIMO PID 2012 plant, confirms that fast set-point response and fast rejection of the load disturbance can be obtained for small values of TV if PPIF controller is applied. Tuning is based on the FOPDT models obtained by fitting-by-eye the open-loop step responses of steam pressure and water level of the plant. These facts recommend the PPIF controller for industry applications where fast load disturbance rejection, obtained by small values of TV and simple experimental tuning, is of the primary importance.

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