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PFC Design via FRIT Approach and Adaptive PID Control System Design for Discrete-time Systems

Ikuro Mizumoto * **Taro Takagi** *

* *Department of Intelligent Mechanical Systems, Kumamoto University, Kumamoto, Japan (e-mail: ikuro@gpo.kumamoto-u.ac.jp).*

Abstract: This paper deals with a design problem of an adaptive PID control for discrete-time systems with a parallel feedforward compensator (PFC) which is designed for making the augmented controlled system ASPR. A PFC design scheme by a FRIT approach with only using an input/output experimental data set will be proposed for discrete-time systems in order to design an adaptive PID control system. Furthermore, the effectiveness of the proposed PFC design and an adaptive PID control method will be confirmed through numerical simulations for an uncertain discrete-time system.

Keywords: Adaptive PID control, PFC, FRIT, ASPR.

1. INTRODUCTION

PID control is one of the most common control schemes and it has been applied to many industrial process and mechanical systems. However, when there are some changes of system properties, because most PID parameter tuning is done off line, it is difficult to maintain the desired control performance and stability during operation. Therefore, a great deal of attention has been focused on auto-tuning and adaptive PID method (Åström and Hägglund (1995); Chang et al. (2003); Kono et al. (2007)). Recently, an auto-tuning and an adaptive PID control strategies based on the almost strictly positive real (ASPR) property of the controlled system have been proposed (Iwai et al. (2006); Tamura and Ohmori (2007)). These adaptive PID schemes based on the ASPR property of the system can guarantee the asymptotic stability of the resulting PID control system. Unfortunately, the ASPR conditions are very severe restrictions for practical applications of the adaptive PID control. To overcome this problem, an introduction of the parallel feedforward compensator (PFC) has been proposed (Iwai et al. (2006); Mizumoto et al. (2010)). This method fulfills the ASPR conditions of augmented system which consists of the plant and the PFC by designing the PFC accordingly. It has been proposed several methods how to design such a PFC. ,however, most of the methods need *a priori* informations of the controlled plant in order to design the PFC. To obtain *a priori* informations of the plant, we need to derive the system model or do experiment several times. This is time-consuming task and becomes a problem when considering the time and costs. From this reason, recently, PFC design method via fictitious reference iterative tuning (FRIT) approach has been proposed for continuous time system (Mizumoto and Tanaka (2010A); Mizumoto and Tanaka (2010B)). FRIT method can optimizes controller parameters for the uncertain plant from only one shot experimental input/output data without using the plant model (Kaneko et al. (2010)). By applying FRIT method to PFC design, PFC parameters could be optimized without using *a priori* informations.

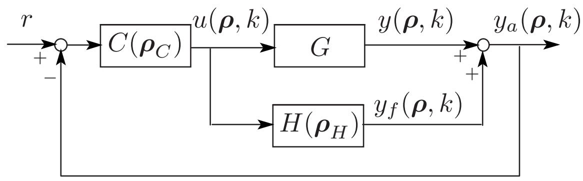


Fig. 1. Closed-loop systems

In this paper, we present a PFC design through FRIT approach for discrete-time systems and propose an adaptive PID control system design for discrete-time systems with a PFC designed via FRIT approach.

2. PFC DESIGN

2.1 Problem Statements

Consider a closed-loop system for a single input/output system G with a controller $C(\boldsymbol{\rho}_C)$ and a PFC $H(\boldsymbol{\rho}_H)$, which are parameterized by $\boldsymbol{\rho} = [\boldsymbol{\rho}_C{}^T \ \boldsymbol{\rho}_H{}^T]^T$, as shown in Figure 1.

The controller and the PFC with the parameter $\boldsymbol{\rho} = [\boldsymbol{\rho}_C{}^T \ \boldsymbol{\rho}_H{}^T]^T$ are satisfies the following assumptions.

Assumption 1. $H(\boldsymbol{\rho}_H) = 0$ with $\boldsymbol{\rho}_H = \mathbf{0}$.

Assumption 2. $C(\boldsymbol{\rho}_C) = \rho_{c1}$(constant) with $\boldsymbol{\rho}_C = [\rho_{C1}, 0, \cdots, 0]^T$.

In this case, the closed-loop system from r to the augmented output $y_a(\boldsymbol{\rho})$ with a controller $C(\boldsymbol{\rho}_C)$ and a PFC $H(\boldsymbol{\rho}_H)$ can be expressed by

$$G_{ac}(\boldsymbol{\rho}) = \frac{(G + H(\boldsymbol{\rho}_H))\, C(\boldsymbol{\rho}_C)}{1 + (G + H(\boldsymbol{\rho}_H))\, C(\boldsymbol{\rho}_C)} \quad (1)$$

Here, we assume that one can obtain an input/output data set $\{u_0(k), y_0(k)\}$ for appropriate controller $C(\boldsymbol{\rho}_{C0})$ and PFC $H(\boldsymbol{\rho}_{H0})$ with parameters $\boldsymbol{\rho}_0 = [\boldsymbol{\rho}_{C0}{}^T\ \boldsymbol{\rho}_{H0}{}^T]^T$. Under this statement, the objective here is to obtain a PFC which renders the augmented system with the PFC ASPR.

To this end, we first consider a desired SPR system:

$$y_{SPR} = G_{SPR} r \tag{2}$$

and then consider to find a parameter $\boldsymbol{\rho} = [\boldsymbol{\rho}_C{}^T\ \boldsymbol{\rho}_H{}^T]^T$ which minimizes the error between SPR model output y_{SPR} and the obtained augmented system's output $y_a(\boldsymbol{\rho}, k)$. That is, to find a parameter $\boldsymbol{\rho} = [\boldsymbol{\rho}_C{}^T\ \boldsymbol{\rho}_H{}^T]^T$ which minimize the following performance function:

$$J(\boldsymbol{\rho}) = \sum_{k=0}^{N} \left(y_a(\boldsymbol{\rho}, k) - y_{SPR}\right)^2 \tag{3}$$

is objective. However, this performance function cannot be obtained directly, because the plant model G is unknown. Therefore we adopt FRIT approach to the parameter tuning.

2.2 PFC Parameter Tuning by FRIT Approach

In order to achieve the objective of PFC design by using an input/output data set $\{u_0(k), y_0(k)\}$, here FRIT approach is considered.

The following relation is satisfied for any parameter vector $\boldsymbol{\rho} = [\boldsymbol{\rho}_C{}^T\ \boldsymbol{\rho}_H{}^T]^T$:

$$C(\boldsymbol{\rho}_C)\left(r^*(\boldsymbol{\rho}, k) - y_{a0}(\boldsymbol{\rho}, k)\right) = u_0(k) \tag{4}$$

where $y_{a0}(\boldsymbol{\rho}, k)$ is the augmented output with the PFC output $H(\boldsymbol{\rho}_H)u_0(k)$:

$$y_{a0}(\boldsymbol{\rho}, k) = y_0(k) + H(\boldsymbol{\rho}_H)u_0(k) \tag{5}$$

This leads

$$\begin{aligned} r^*(\boldsymbol{\rho}, k) &= C(\boldsymbol{\rho}_C)^{-1}u_0(k) + y_{a0}(\boldsymbol{\rho}, k) \\ &= C(\boldsymbol{\rho}_C)^{-1}u_0(k) + y_0(k) + H(\boldsymbol{\rho}_H)u_0(k) \end{aligned} \tag{6}$$

$r^*(\boldsymbol{\rho}, k)$ obtained from (6) is called 'fictitious reference signal'. Taking this signal $r^*(\boldsymbol{\rho}, k)$ as a reference signal, the control system in Figure 1 with controller and PFC of any parameter vector $\boldsymbol{\rho}$ gives the input $u_0(k)$ and the output$y_0(k)$.

Now, impose the following assumption.

Assumption 3. There exists an ideal parameter vector $\boldsymbol{\rho}_d = [\boldsymbol{\rho}_{Cd}^T\ \boldsymbol{\rho}_{Hd}^T]^T$ with $\boldsymbol{\rho}_{Cd} = [K^*, 0, \cdots, 0]$ such that the obtained closed-loop system with the controller $C(\boldsymbol{\rho}_{Cd}) = K^*$ and the PFC $H(\boldsymbol{\rho}_{Hd})$can be expressed by the given SPR model G_{SPR}. That is,

$$G_{SPR} = \frac{(G + H(\boldsymbol{\rho}_{Hd}))K^*}{1 + (G + H(\boldsymbol{\rho}_{Hd}))K^*} \tag{7}$$

Then, considering the fictitious reference signal $r^*(\boldsymbol{\rho}_d, k)$ for the ideal parameter vector $\boldsymbol{\rho}_d$, we have

$$\begin{aligned} y_{a0}(\boldsymbol{\rho}_d, k) &= \frac{(G + H(\boldsymbol{\rho}_{Hd}))K^*}{1 + (G + H(\boldsymbol{\rho}_{Hd}))K^*} r^*(\boldsymbol{\rho}_d, k) \\ &= G_{SPR} r^*(\boldsymbol{\rho}_d, k) \end{aligned} \tag{8}$$

Thus, from (6) and (7), the signal $y_{a0}(\boldsymbol{\rho}_d, k)$ in (8) can be obtained by

$$\begin{aligned} y_{a0}(\boldsymbol{\rho}_d, k) &= G_{SPR}\ r^*(\boldsymbol{\rho}_d, k) \\ &= G_{SPR}(K^{*-1}u_0(k) + y_0(k) + H(\boldsymbol{\rho}_{Hd})u_0(k)) \end{aligned} \tag{9}$$

Furthermore, from (5) and (9), we have that

$$\begin{aligned} y_0(k) &= y_{a0}(\boldsymbol{\rho}_d, k) - H(\boldsymbol{\rho}_{Hd})u_0(k) \\ &= G_{SPR}\left(K^{*-1}u_0(k) + y_0(k) + H(\boldsymbol{\rho}_{Hd})u_0(k)\right) \\ &\quad - H(\boldsymbol{\rho}_{Hd})u_0(k) \end{aligned} \tag{10}$$

From this relation in (10), we define a virtual output $\tilde{y}(\bar{\boldsymbol{\rho}}, k)$ for the system with a controller and a PFC with a parameter $\bar{\boldsymbol{\rho}}$ as follows by using the input/output data set $\{u_0(k),y_0(k)\}$.

$$\begin{aligned} \tilde{y}(\bar{\boldsymbol{\rho}}, k) &= G_{SPR}(C(\bar{\boldsymbol{\rho}}_C)^{-1}u_0(k) + y_0(k) + H(\bar{\boldsymbol{\rho}}_H)u_0(k)) \\ &\quad - H(\boldsymbol{\rho}_H)u_0(k) \end{aligned} \tag{11}$$

$$\bar{\boldsymbol{\rho}}_C = [\bar{\rho}_{c1}, 0, \cdots, 0]^T$$

Then, consider minimizing the following performance function:

$$J_F(\bar{\boldsymbol{\rho}}) = \sum_{k=0}^{N} \left(\tilde{y}(\bar{\boldsymbol{\rho}}, k) - y_0(k)\right)^2 \tag{12}$$

The obtained optimal $\boldsymbol{\rho}_d$ by this FRIT approach can be expected to guarantee the minimization of the performance function given in (3) .

Now, consider a typical PFC:$H(z)$ given as the following nth compensator.

$$H(z) = \frac{b_0 z^n + b_1 z^{n-1} + \cdots + b_n}{z^n + a_1 z^{n-1} + \cdots + a_n} \tag{13}$$

Here, we approximate this PFC with n_mth FIR model as follows:

$$H(z) = f_0 + f_1 z^{-1} + f_2 z^{-2} + \cdots + f_{n_m} z^{-n_m} \tag{14}$$

Then the virtual output $\tilde{y}(\bar{\boldsymbol{\rho}}, k)$ can be represented as

$$\tilde{y}(\bar{\boldsymbol{\rho}}, k) = \boldsymbol{\xi}^T \bar{\boldsymbol{\rho}} + G_{SPR} y_0(k) \tag{15}$$

where $\boldsymbol{\xi} = [\xi_0\ \xi_1\ \xi_2\ \cdots\ \xi_{n_m+1}]^T$, $\xi_0(k) = G_{SPR}u_0(k)$, $\xi_i(k) = (G_{SPR} - 1)u_0(k + 1 - i)$ and $\bar{\boldsymbol{\rho}} = [K^{*-1}\ f_0\ f_1\ \cdots\ f_{n_m}]^T$.

From (15), we obtain

$$\begin{aligned} \begin{bmatrix} \tilde{y}(\bar{\boldsymbol{\rho}}, 0) \\ \tilde{y}(\bar{\boldsymbol{\rho}}, 1) \\ \vdots \\ \tilde{y}(\bar{\boldsymbol{\rho}}, N) \end{bmatrix} &= \begin{bmatrix} \xi_0(0) & \xi_1(0) & \cdots & \xi_{n_m+1}(0) \\ \xi_0(1) & \xi_1(1) & \cdots & \xi_{n_m+1}(1) \\ \vdots & & & \\ \xi_0(N) & \xi_1(N) & \cdots & \xi_{n_m+1}(N) \end{bmatrix} \bar{\boldsymbol{\rho}} \\ &\quad + [\,y_{0SPR}(0)\ y_{0SPR}(1)\ \cdots y_{0SPR}(N)\,]^T \\ &= \boldsymbol{\Phi}\bar{\boldsymbol{\rho}} + \boldsymbol{Y}_{0SPR} \end{aligned} \tag{16}$$

where $y_{0SPR}(k) = G_{SPR} y_0(k)$ and $\boldsymbol{Y}_{0SPR} = [y_{0SPR}(0)\ y_{0SPR}(1)\ \cdots\ y_{0SPR}(N)]^T$.

Moreover, by defining $\boldsymbol{Y}_0 = [y_0(0)\ y_0(1)\ \cdots\ y_0(N)]^T$, the performance function $J_F(\bar{\boldsymbol{\rho}})$ can be represented as

$$\begin{aligned} J_F(\bar{\boldsymbol{\rho}}) &= \sum_{k=0}^{N} \left(\tilde{y}(\bar{\boldsymbol{\rho}}, k) - y_0(k)\right)^2 = \|\boldsymbol{\Phi}\bar{\boldsymbol{\rho}} + \boldsymbol{Y}_{0SPR} - \boldsymbol{Y}_0\|^2 \\ &= \left\|\boldsymbol{\Phi}\bar{\boldsymbol{\rho}} - \bar{\boldsymbol{Y}}\right\|^2 \end{aligned} \tag{17}$$

where $\bar{\boldsymbol{Y}} = \boldsymbol{Y}_0 - \boldsymbol{Y}_{0_{SPR}}$. Then, the optimal $\boldsymbol{\rho}_d$ which minimize the performance function $J_F(\bar{\boldsymbol{\rho}})$ can be obtained by

$$\boldsymbol{\rho}_d = \left(\boldsymbol{\Phi}^T\boldsymbol{\Phi}\right)^{-1}\boldsymbol{\Phi}^T\bar{\boldsymbol{Y}} \tag{18}$$

2.3 The Robustness of The Designed PFC

The designed PFC must be supposed to render the augmented system with the PFC ASPR if the performance function was minimized. Unfortunately however, the obtained closed loop system with the designed parameter vector $\bar{\boldsymbol{\rho}}$ does not perfectly match to the ideal SPR model G_{SPR} in a practical sense. The resulting augmented system G_a with the PFC $H(\bar{\boldsymbol{\rho}}_H)$ can be represented by

$$G_a = G + H(\bar{\boldsymbol{\rho}}_H) = G^*_{ASPR}(1+\Delta) \tag{19}$$

where G^*_{ASPR} is the ideal ASPR model given by $G^*_{ASPR} = G + H(\boldsymbol{\rho}_{Hd})$, and Δ_H and Δ are defined as follows:

$$\Delta_H = 1 - H(\bar{\boldsymbol{\rho}}_H)^{-1}H(\boldsymbol{\rho}_{Hd}), \tag{20}$$

$$\Delta = {G^*_{ASPR}}^{-1}H(\bar{\boldsymbol{\rho}}_H)\Delta_H \tag{21}$$

For the ASPR-ness of the augmented system (19) with a mismatch Δ has been investigated as in the following theorem (Mizumoto et al. (2010)).

Theorem 1. The augmented system (19) is ASPR if

(a) G^*_{ASPR} is ASPR. (b) $\Delta \in RH_\infty$. (c) $\|\Delta\|_\infty < 1.0$.

It is apparent that the conditions (a) and (b) in Theorem 1 are satisfied for the obtained augmented system (19). Thus, if the mismatch Δ between the ideal PFC $H(\boldsymbol{\rho}_{Hd})$ and the obtained PFC $H(\bar{\boldsymbol{\rho}}_H)$ are sufficiently small, then the resulting augmented system is ASPR even if the designed parameter vector $\bar{\boldsymbol{\rho}}$ does not perfectly match to the ideal parameter vector $\boldsymbol{\rho}_d$.

Now, let consider the condition (c) under the Assumption 3. From the Assumption 3, the performance function (12) would be as following.

$$J_F(\bar{\boldsymbol{\rho}}) = \sum_{k=0}^{N}\left[\left\{\left(G_{SPR}-1\right)\left(H(\bar{\boldsymbol{\rho}}_H)-H(\boldsymbol{\rho}_{Hd})\right)\right\}u_0(k)\right]^2 \tag{22}$$

Also, from (19), it can be represented as

$$J_F(\bar{\boldsymbol{\rho}}) = \sum_{k=0}^{N}\left[\left(G_{SPR}-1\right)H(\bar{\boldsymbol{\rho}}_H)\Delta_H u_0(k)\right]^2 \tag{23}$$

Then, by applying the Percival theorem for discrete Fourier transform, (23) can be evaluated as

$$\begin{aligned} &J_F(\bar{\boldsymbol{\rho}}) \\ &= \frac{1}{N+1}\sum_{n=0}^{N}\left\|\left(G_{SPR}(n)-1\right)H(\bar{\boldsymbol{\rho}}_H,n)\Delta_H(n)u_0(n)\right\|^2 \\ &\le \frac{{\delta_H}^2}{N+1}\sum_{n=0}^{N}\left\|\left(G_{SPR}(n)-1\right)H(\bar{\boldsymbol{\rho}}_H,n)\right\|^2\|u_0(n)\|^2 \end{aligned} \tag{24}$$

Here, $\delta_H = \|\Delta_H\|_\infty$ and $u_0(n)$ is discrete Fourier transform of $u_0(k)$, $k = 0, 1, \cdots, N$, that is, $u(n) = \sum_{k=0}^{N} u(k)W^{kn}$ ($W \triangleq e^{-j\frac{2\pi}{N+1}}$). Moreover, by defining

$$\delta_{PH} = \|\left(G_{SPR}(n)-1\right)H(\bar{\boldsymbol{\rho}}_H,n)\|_\infty \tag{25}$$

then, from (25), (24) can be evaluated by

$$J_F(\bar{\boldsymbol{\rho}}) \le {\delta_H}^2{\delta_{PH}}^2\sum_{k=0}^{N}u_0(k)^2 \tag{26}$$

Finally, by defining $\beta_u = \sum_{k=0}^{N}u_0(k)^2$, we have

$${\delta_H}^2 \ge \frac{J_F(\bar{\boldsymbol{\rho}})}{{\delta_{PH}}^2\beta_u} \tag{27}$$

On the other hand, from (21), $\|\Delta\|_\infty$ can be evaluated by

$$\|\Delta\|_\infty = \|{G^*_{ASPR}}^{-1}H(\bar{\boldsymbol{\rho}}_H)\Delta_H\|_\infty \le \delta_{GH}\delta_H \tag{28}$$

where $\delta_{GH} = \|{G^*_{ASPR}}^{-1}H(\bar{\boldsymbol{\rho}}_H)\|_\infty$. This means that $\delta_{GH}\delta_H < 1.0$ is necessary condition to be ASPR. Now, from (28), define the lower limit of δ_H as

$$\bar{\delta}_H = \sqrt{\frac{J_F(\bar{\boldsymbol{\rho}})}{{\delta_{PH}}^2\beta_u}} \tag{29}$$

Then, at least $\delta_{GH}\bar{\delta}_H < 1.0$ have to be fulfilled to be $\delta_{GH}\delta_H < 1.0$. From this, as an one of the standard, when it is $\bar{\delta}_H \ge \frac{1}{\delta_{GH}}$, a PFC have to be redesigned. [1]

3. CONTROL SYSTEM DESIGN

In this section, we propose an adaptive PID controller with an adaptive Neural Network (NN) feedforward control. Consider a SISO discrete-time system G expressed as

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{b}u(k) \\ y(k) &= \boldsymbol{c}^T\boldsymbol{x}(k) \end{aligned} \tag{30}$$

where $\boldsymbol{x}(k) \in \boldsymbol{R}^n$ is a state vector, $u(k) \in \boldsymbol{R}$ and $y(k) \in \boldsymbol{R}$ are the input and the output of the system, respectively.

Suppose that the reference signal $y_r(k)$ which the output $y(k)$ is required to track are generated by the following exosystem:

$$\boldsymbol{\omega}(k+1) = A_\omega\boldsymbol{\omega}(k),\ y_r(k) = \boldsymbol{c}_\omega^T\boldsymbol{\omega}(k) \tag{31}$$

and impose the following assumption.

Assumption 4. There exist an ideal state $\boldsymbol{x}^*(k)$ and an ideal input $v^*(k)$ which attain perfect tracking such that

$$\begin{aligned} \boldsymbol{x}^*(k+1) &= \boldsymbol{A}\boldsymbol{x}^*(k) + \boldsymbol{b}v^*(k) \\ y(k) &= \boldsymbol{c}^T\boldsymbol{x}^*(k) \equiv y_r(k) \end{aligned} \tag{32}$$

and they are given by functions of $\boldsymbol{\omega}(k)$ such as $\boldsymbol{x}^*(k) = \boldsymbol{\pi}(\boldsymbol{\omega}(k))$ and $v^*(k) = c(\boldsymbol{\omega}(k))$.

[1] The condition $\delta_{GH}\bar{\delta}_H < 1.0$ is the necessary condition that to fulfill the condition (c) of Theorem 1. Also, Theorem 1 is the sufficient condition that the control system would be ASPR. Therefore, there exist ASPR control system which does not fulfill the conditions of Theorem 1.

For this system, consider the following PFC of order n_m designed via FRIT:

$$\begin{aligned} \boldsymbol{x}_f(k+1) &= \boldsymbol{A}_f \boldsymbol{x}_f(k) + \boldsymbol{b}_f u(k) \\ y_f(k) &= \boldsymbol{c}_f^T \boldsymbol{x}_f(k) + d_f u(k) \end{aligned} \tag{33}$$

The augmented system with the PFC can be presented by

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{b}u(k) \\ \boldsymbol{x}_f(k+1) &= \boldsymbol{A}_f \boldsymbol{x}_f(k) + \boldsymbol{b}_f u(k) \\ y_a(k) &= y(k) + y_f(k) \end{aligned} \tag{34}$$

Now consider an ideal control input with an ideal PID gain θ_p^*, θ_i^*, θ_d^*, where θ_p^* renders the closed loop system SPR. Such a gain exists from the ASPR-ness of the augmented system. The ideal PID control input with the ideal forward input $v^*(k)$ can be designed as follows:

$$u^*(k) = u_e^*(k) + v^*(k) \tag{35}$$

$$\begin{aligned} u_e^*(k) = &-\theta_p^* \bar{e}_a - \theta_i^* \{\bar{e}_{ai}(k-1) + T\bar{e}_a(k)\} \\ &-\theta_d^* \frac{1}{T}\{\bar{e}_a(k) - \bar{e}_a(k-1)\} \end{aligned} \tag{36}$$

$$\bar{e}_a(k) = \bar{y}_a(k) - y_r(k) \tag{37}$$

$$\bar{e}_{ai}(k) = \bar{e}_{ai}(k-1) + T\bar{e}_a(k) \tag{38}$$

where $\bar{y}_a(k) = y(k) + \bar{y}_f(k)$ and $\bar{y}_f$ is a PFC output with the input $u_e^*(k)$ in (32), thus

$$\begin{aligned} \bar{\boldsymbol{x}}_f(k+1) &= \boldsymbol{A}_f \bar{\boldsymbol{x}}_f(k) + \boldsymbol{b}_f u_e^*(k) \\ \bar{y}_f(k) &= \boldsymbol{c}_f^T \bar{\boldsymbol{x}}_f(k) + d_f u_e^*(k) \end{aligned} \tag{39}$$

Unfortunately, the ideal PID gains and ideal forward input $v^*(k)$ are unknown, and since the augmented ASPR system has a direct feedthrough term of the input, a causality problem will appear for realizing the controller. Then we consider designing the controller adaptively without causality problem.

First, we consider approximation of the ideal forward input $v^*(k)$ by a radial basis function (RBF) NN. We approximate $v^*(k)$ by the form of RBF NN as

$$v_{nn}(k) = W^T S(\boldsymbol{\omega}(k)) \tag{40}$$

where $W = [w_1, \cdots, w_l]^T \in \boldsymbol{R}^l$ is the weight vector, l is the number of NN nodes (weight number) and $S(\boldsymbol{\omega}) = [s_1(\boldsymbol{\omega}), \cdots, s_l(\boldsymbol{\omega})]^T$ is the radial basis function vector. This basis function vector $S(\boldsymbol{\omega})$ is generally designed by the Gaussian functions such as

$$s_i(\boldsymbol{\omega}) = \exp\left[\frac{-(\boldsymbol{\omega} - \boldsymbol{\mu}_i)^T(\boldsymbol{\omega} - \boldsymbol{\mu}_i)}{\eta_i^2}\right] \tag{41}$$
$$i = 1, 2, \cdots, l$$

where $\boldsymbol{\mu}_i = [\mu_{i1}, \cdots, \mu_{iq}]^T$ is the center of the receptive field and η_i is the width of the Gaussian function.

Under Assumption 4, it has been clarified that, for a sufficiently large l and a compact set $\Omega_\omega \subset \boldsymbol{R}^q$, there exists an ideal constant weight vector W^* such that (Zhang et al. (2002))

$$W^* \triangleq \arg \min_{W \in \boldsymbol{R}^l} \{ \sup_{\boldsymbol{\omega} \in \Omega_\omega} |v^* - W^T S(\boldsymbol{\omega})| \} \tag{42}$$

and thus the ideal input $v^*(k)$ can be approximated by

$$v^*(k) = W^{*T} S(\boldsymbol{\omega}) + \epsilon(\boldsymbol{\omega}), \ |\epsilon(\boldsymbol{\omega})| \le \epsilon^* \tag{43}$$

where $\epsilon(\boldsymbol{\omega})$ is an approximation error. Then we impose the following assumption.

Assumption 5. For a given NN nodes l, there exists an ideal weight vector W^* that satisfies (42) for all $\boldsymbol{\omega} \in \Omega_\omega$.

Next, from $u_e^*(k)$ in (36), the following equivalent input $u_{e1}^*(k)$ can be obtained:

$$\begin{aligned} u_e^*(k) = u_{e1}^*(k) = &-\tilde{\theta}_{p1}^* \hat{e}_a(k) - \tilde{\theta}_{i1}^* \bar{e}_{ai}(k-1) \\ &+\tilde{\theta}_{d1}^* \frac{1}{T}\bar{e}_a(k-1) + \mu^*(k) \end{aligned} \tag{44}$$

where

$$\hat{e}_a(k) = y(k) + \boldsymbol{c}^T \bar{\boldsymbol{x}}_f(k) - y_r(k) \tag{45}$$

$$\begin{aligned} &\tilde{\theta}_{p1}^* = \{1 + d_f\theta_p^*\}^{-1}\theta_p^*, \ \tilde{\theta}_{i1}^* = \{1 + d_f\theta_p^*\}^{-1}\theta_i^* \\ &\tilde{\theta}_{d1}^* = \{1 + d_f\theta_p^*\}^{-1}\theta_d^* \end{aligned} \tag{46}$$

$$\mu^*(k) = -\left(T\tilde{\theta}_{i1}^* + \frac{1}{T}\tilde{\theta}_{d1}^*\right)\bar{e}_a(k) \tag{47}$$

Further expanding (36), we have

$$\begin{aligned} u_e^*(k) = &-\left(\theta_p^* + T\theta_i^* + \frac{1}{T}\theta_d^*\right)\bar{e}_a(k) \\ &-\theta_i^* e_{ai}(k-1) + \frac{1}{T}\theta_d^* e_a(k-1) \end{aligned} \tag{48}$$

and then the following equivalent input $u_{e2}^*(k)$ can also be obtained:

$$\begin{aligned} u_e^*(k) &= u_{e2}^*(k) \\ &= -\tilde{\theta}_{p2}^* \hat{e}_a(k) - \tilde{\theta}_{i2}^* e_{ai}(k-1) + \tilde{\theta}_{d2}^* \frac{1}{T} e_a(k-1) \end{aligned} \tag{49}$$

where

$$\begin{aligned} &\tilde{\theta}_{p2}^* = \{1 + d_f\theta_{PID}^*\}^{-1}\theta_{PID}^*, \ \tilde{\theta}_{i2}^* = \{1 + d_f\theta_{PID}^*\}^{-1}\theta_i^* \\ &\tilde{\theta}_{d2}^* = \{1 + d_f\theta_{PID}^*\}^{-1}\theta_d^*, \ \theta_{PID}^* = \theta_p^* + T\theta_i^* + \frac{1}{T}\theta_d^* \end{aligned} \tag{50}$$

It follows that $u_e^*(k) \equiv u_{e1}^*(k) \equiv u_{e2}^*(k)$.

The actual control input is designed by adjusting the equivalent input gains $\tilde{\theta}_{p2}^*$, $\tilde{\theta}_{i2}^*$ and $\tilde{\theta}_{d2}^*$ in $u_2^*(k)$ and W^* in (42) as follows:

$$u(k) = -\tilde{\boldsymbol{\theta}}^T(k)\tilde{\boldsymbol{z}}(k) + \hat{W}^T(k)S(\boldsymbol{\omega}(k)) \tag{51}$$

where

$$\begin{aligned} \tilde{\boldsymbol{\theta}}^T(k) &= \left[\tilde{\theta}_p(k), \tilde{\theta}_i(k), \tilde{\theta}_d(k)\right] \\ \tilde{\boldsymbol{z}}(k) &= \left[\hat{e}_a(k), \bar{\sigma}_i e_{ai}(k-1), -\frac{1}{T}e_a(k-1)\right]^T \end{aligned} \tag{52}$$

The parameter adjusting law is given by

$$\begin{aligned} \tilde{\boldsymbol{\theta}}(k) &= \tilde{\boldsymbol{\theta}}(k-1) + \Gamma\tilde{\boldsymbol{z}}(k)\bar{e}_a(k) - \sigma\tilde{\boldsymbol{\theta}}(k) \\ &= \bar{\sigma}\tilde{\boldsymbol{\theta}}(k-1) + \bar{\sigma}\Gamma\tilde{\boldsymbol{z}}(k)\bar{e}_a(k) \\ \bar{\sigma} &= \frac{1}{1+\sigma}, \ \sigma > 0, \ \Gamma = \Gamma^T > 0 \end{aligned} \tag{53}$$

$$\hat{W}(k) = \bar{\sigma}_n \hat{W}(k-1) - \bar{\sigma}_n \Gamma_n S(\boldsymbol{\omega}(k))\bar{e}_a(k) \quad (54)$$
$$\bar{\sigma}_n = \frac{1}{1+\sigma_n},\ \sigma_n > 0,\ \Gamma_n = \Gamma_n^T > 0$$

In this case, the augmented output error $e_a(t)$ can be obtained from (34), (51) and (53) as

$$\bar{e}_a(k) = \frac{\hat{e}_a(k) - \bar{\sigma} d_f \tilde{\boldsymbol{\theta}}^T(k-1)\tilde{\boldsymbol{z}}(k)}{1+\bar{\sigma} d_f \tilde{\boldsymbol{z}}^T \Gamma \tilde{\boldsymbol{z}}(k)} \quad (55)$$

by using available signals. This means that the proposed adaptive PID controller can be designed without causality problems.

4. STABILITY ANALYSIS

The error system with the control input (51) can be described as

$$\begin{aligned}\boldsymbol{x}_a(k+1) &= A_c \boldsymbol{x}_a(k) + \boldsymbol{b}_a\{\tilde{u}(k) + \Delta u(k) + \mu^*(k)\}\\ &\quad -\bar{\boldsymbol{b}}_f \Delta v(k)\\ e_a(k) &= \boldsymbol{c}_c{}^T \boldsymbol{x}_a(k) + d_f\{\tilde{u}(k) + \Delta u(k) + \mu^*(k)\} \quad (56)\end{aligned}$$

where $A_c = A_a - \tilde{\theta}_{p1}^* \boldsymbol{b}_a \boldsymbol{c}_a^T$, $\boldsymbol{c}_c^T = (1 - d_f \tilde{\theta}_{p1}^*)\boldsymbol{c}_a^T$ and

$$\begin{aligned}\boldsymbol{x}_a(k) &= \bar{\boldsymbol{x}}_a(k) - \bar{\boldsymbol{x}}_a^*(k)\\ \bar{\boldsymbol{x}}_a(k) &= [\boldsymbol{x}(k)^T, \bar{\boldsymbol{x}}_f(k)^T]^T\\ e_a(k) &= \bar{y}_a(k) - \bar{y}_a^*(k) \quad (57)\\ \tilde{u}(k) &= -\theta_i^* e_{ai}(k-1) + \frac{1}{T}\theta_d^* e_a(k-1)\\ \Delta u(k) &= u(k) - u^*(k),\ \Delta v(k) = v(k) - v^*(k)\end{aligned}$$

This error system is SPR, thus there exist positive definite matrices $P = P^T > 0, Q = Q^T > 0$, vector $\boldsymbol{l}$ and w such that the following Kalman-Yakubovich Lemma is satisfied.

$$\begin{aligned}A_c^T P A_c - P &= -Q - \boldsymbol{l}\boldsymbol{l}^T\\ A_c^T P \boldsymbol{b}_a &= \boldsymbol{c}_c + \boldsymbol{l} w\\ \boldsymbol{b}_a^T P \boldsymbol{b}_a &= 2d_f - w^2 \quad (58)\end{aligned}$$

Now, consider a positive definite function V:

$$V(k) = V_1(k) + \rho V_2(k) + \rho V_3(k) + \rho V_4(k) \quad (59)$$

where

$$\begin{aligned}V_1(k) &= \boldsymbol{x}_a(k)^T P \boldsymbol{x}_a(k)\\ V_2(k) &= \frac{1}{T}\bar{\sigma}_i \tilde{\theta}_{i1}^* e_{ai}(k-1)^2 + \frac{1}{T}\tilde{\theta}_{d1}^* e_a(k-1)^2\\ V_3(k) &= \bar{\sigma}\Delta\tilde{\boldsymbol{\theta}}(k-1)^T \Gamma^{-1} \Delta\tilde{\boldsymbol{\theta}}(k-1) \quad (60)\\ V_4(k) &= \bar{\sigma_n}\Delta\hat{W}(k)^T {\Gamma_n}^{-1} \Delta\hat{W}(k)\end{aligned}$$

Here, define $\Delta V(k)$ as

$$\Delta V(k) = V(k+1) - V(k) \quad (61)$$

then $\Delta V_1(k)$, $\Delta V_2(k)$, $\Delta V_3(k)$ and $\Delta V_4(k)$ can be evaluated as

$$\begin{aligned}\Delta V_1(k) &\le -\rho(\lambda_{min}[Q] - \delta\lambda_{max}[P])\|\boldsymbol{x}_a(k)\|^2\\ &\quad +2\rho e_a(k)\{\tilde{u}(k) + \Delta u(k) + \mu^*(k)\}\\ &\quad +\frac{1}{\delta}\lambda_{max}[P]\left\|\bar{\boldsymbol{b}}_f\right\|^2 |\Delta v(k)|^2\\ \Delta V_2(k) &= -\frac{1}{T}\tilde{\theta}_{i1}^*\left(\frac{1}{\bar{\sigma}_i} - \bar{\sigma}_i\right) e_{ai}(k)^2 - 2\tilde{u}(k)e_a(k)\\ &\quad -2\mu^*(k)e_a(k) - T(\bar{\sigma}_i\tilde{\theta}_{i1}^* e_a(k)^2 + \tilde{\theta}_{d1}^* e_{ad}(k)^2)\\ \Delta V_3(k) &\le -\left\{\left(\frac{1}{\bar{\sigma}} - \bar{\sigma}\right) - \delta_1\right\}\lambda_{max}\left[\Gamma^{-1}\right]\left\|\Delta\tilde{\boldsymbol{\theta}}(k)\right\|^2\\ &\quad -2\Delta u_e(k)e_a(k) + \frac{\sigma^2}{\delta_1}\tilde{\boldsymbol{\theta}}_2^{*T}\Gamma^{-1}\tilde{\boldsymbol{\theta}}_2^*\\ \Delta V_4(k) &\le -\left\{\left(\frac{1}{\bar{\sigma}_n} - \bar{\sigma}_n\right) - \delta_2\right\}\lambda_{max}\left[\Gamma_n^{-1}\right]\left\|\Delta\hat{W}(k)\right\|^2\\ &\quad -2\Delta v(k)e_a(k) + \frac{{\sigma_n}^2}{\delta_2}W^{*T}{\Gamma_n}^{-1}W^* \quad (62)\end{aligned}$$

with $0 < \delta < 1$ and positive constants δ_1, δ_2. Thus, ΔV is evaluated by

$$\begin{aligned}\Delta V(k) &\le -\rho(\lambda_{min}[Q] - \delta\lambda_{max}[P])\|\boldsymbol{x}_a(k)\|^2\\ &\quad -\frac{\rho}{T}\tilde{\theta}_{i1}^*\left(\frac{1}{\bar{\sigma}_i} - \bar{\sigma}_i\right) e_{ai}(k)^2\\ &\quad -\rho T\left(\bar{\sigma}_i\tilde{\theta}_{i1}^* e_a(k)^2 + \tilde{\theta}_{d1}^* e_{ad}(k)^2\right)\\ &\quad -\rho\left\{\left(\frac{1}{\bar{\sigma}} - \bar{\sigma}\right) - \delta_1\right\}\lambda_{max}\left[\Gamma^{-1}\right]\left\|\Delta\tilde{\boldsymbol{\theta}}(k)\right\|^2\\ &\quad -\rho\left[\left\{\left(\frac{1}{\bar{\sigma}_n} - \bar{\sigma}_n\right) - \delta_2\right\}\lambda_{max}\left[\Gamma_n^{-1}\right]\right.\\ &\quad \left.-\frac{1}{\rho\delta}\lambda_{max}[P]\|\bar{\boldsymbol{b}}_f\|^2 S_{max}^2 - \delta_3 S_{max}^2\right]\left\|\Delta\hat{W}(k)\right\|^2\\ &\quad -\rho\delta_3\left\{\left\|\Delta\hat{W}(k)\right\| S_{max} - \frac{1}{\rho\delta\delta_3}\lambda_{max}[P]\left\|\bar{\boldsymbol{b}}_f\right\|^2\epsilon^*\right\}^2\\ &\quad +\rho\frac{\sigma^2}{\delta_1}\tilde{\boldsymbol{\theta}}_2^{*T}\Gamma^{-1}\delta_1\tilde{\boldsymbol{\theta}}_2^* + \rho\frac{\sigma_n^2}{\delta_2}W^{*T}\Gamma^{-1}\delta_1 W^*\\ &\quad +\frac{1}{\rho\delta_3}\left(\frac{1}{\delta}\lambda_{max}[P]\left\|\bar{\boldsymbol{b}}_f\right\|^2\epsilon^*\right)^2\\ &\quad +\frac{1}{\delta}\lambda_{max}[P]\left\|\bar{\boldsymbol{b}}_f\right\|^2\epsilon^{*2} \quad (63)\end{aligned}$$

where $\|S(\boldsymbol{\omega}(k))\| \le S_{max}$ and with positive constant δ_3.

Consequently, for appropriate design parameters, $\Delta V(k)$ can be evaluated as

$$\Delta V(k) \le -\alpha V(k) + R,\ \alpha > 0 \quad (64)$$

thus we can conclude that all the signals in the control system are bounded.

5. SIMULATION

To confirm the effectiveness of the proposed method, this section shows a numerical simulation results.

Let's consider a tracking control of the following SISO discrete-time system with the sampling period of 1.0 [s].

$$G(z) = \frac{0.027z^4 + 0.0065z^3 - 0.018z^2 - 2.5\times10^{-4}z - 9.7\times10^{-7}}{z^5 - 2.41z^4 + 1.83z^3 - 0.416z^2 - 0.0074z - 3.3\times10^{-5}} \quad (65)$$

To obtain an input/output data set $\{u_0(k), y_0(k)\}$, we set the feedback gain:$\theta_{p0} = 2.0$, PFC:$H(z) = 0$, and reference signal:$y_r = 1.0$. Here, we considered that white noise is added to the output signal of the plant, and the power spectral density of white noise is 5.0×10^{-3}. The obtained input/output data set is shown in Fig.2. Then, from (18), we designed a second order PFC:$H(\bar{\boldsymbol{\rho}})$ by using SPR model:

$$G_{SPR} = \frac{0.99(15z - 13)}{16z - 14} \tag{66}$$

and obtained

$$H(\bar{\boldsymbol{\rho}}) = 0.2067 + 0.0217z^{-1} + 0.1278z^{-2} \tag{67}$$

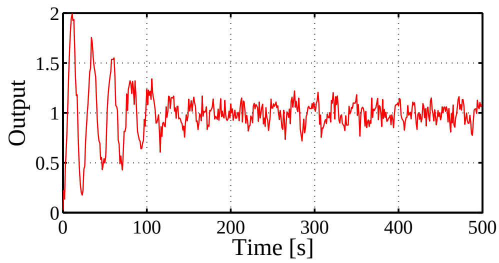
For the system with (70), the design parameters in the adaptive controller are set by $\boldsymbol{\omega} =, 1$ $\sigma = 1.0 \times 10^{-5}$, $\sigma_i = 1.0 \times 10^{-3}$, $\sigma_n = 1.0 \times 10^{-5}$, $\Gamma = \text{diag}[\gamma_p, \gamma_i, \gamma_d]$, $\gamma_p = 600$, $\gamma_i = 200$, $\gamma_d = 100$, and $\Gamma_n = 0.5$. Also, the reference signal $y_r(k)$ is given by

$$y_r(k) = \frac{1/17}{z - (1 - 1/17)} r(k), \ \ r(k) = 10 \tag{68}$$

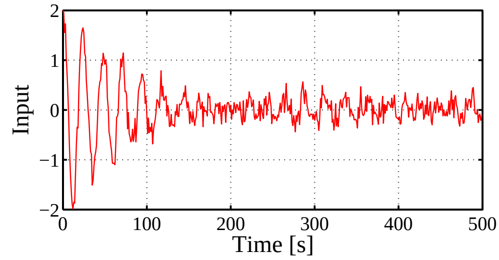
The simulation results are shown in Fig.3.

6. CONCLUSIONS

In this paper, we proposed an adaptive PID control system design for discrete-time systems with PFC designed through FRIT approach. The proposed method ensures the stability of the resulting control system by an adaptive PID control based on ASPR properties of the controlled system and achieves the output tracking by an adaptive NN feedforward control.

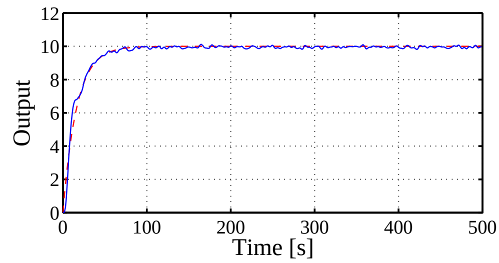


(a)Output of the plant

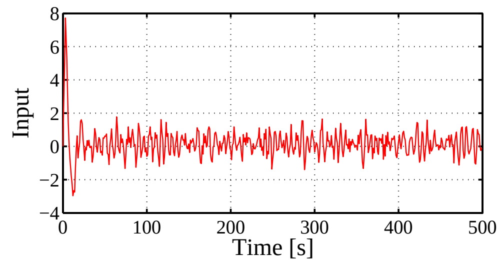


(b)Control input

Fig. 2. The input/output signals for the PFC design.



(a)Output of the plant



(b)Control input

Fig. 3. Simulation results with the designed PFC and adaptive PID controller with adaptive NN.

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