Design of a Weigh Feeder Control System for Reduced Energy Consumption

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Abstract: In the present paper, we discuss a design method for controlling a weigh feeder that has been widely used in industry. Since a control system is designed using a performance-adaptive method, the control parameters are adaptively updated based on user-specified control performance. In conventional performance-adaptive methods, control systems are designed such that the variance of the control error is less than or equal to a specified value and the variance of the differences in the control input is minimized without changing the acceptable variance value of the control error. On the other hand, since the design objective of the present study is to reduce energy consumption, the variance of the differences in the control input is first set, and then the variance of the control error is minimized without changing the acceptable variance value of the differences in the control input. Consequently, the variation of the control input can be substantially reduced. In the proposed method, a proportional-integral controller is designed based on generalized minimum variance control (GMVC) with steady-state predictive output (GMVCS). One of the design parameters in GMVCS is automatically decided such that a desired control performance can be attained, and the PI parameters of a PI control law are calculated based on a GMVCS law.

Keywords: PID control, adaptive control, performance drives, mechanical systems, minimum variance control, model approximation, parameter estimation, discrete-time systems.

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1. INTRODUCTION

In the present study, a new design method is proposed for controlling a weigh feeder. Weigh feeders have been used to dispense powder and granular material at a specified rate and have been widely used in industry because of their usefulness (Hopkins, 2006). Most weigh feeder systems used in industry are designed using proportional and integral compensations, and the control parameters are fixed. These control parameters are designed using predesigned values, which are based on both the type of feeder and the characteristics of the material to be dispensed. Therefore, the control systems must be designed such that a desired control performance can be achieved, and a control method that can adapt to changing circumstance is desired.

To this end, a weigh feeder should be adaptively controlled even if the dynamic characteristics are not known beforehand and are changed. Therefore, design methods using a self-tuning method (Clarke, 1984; Omatu and Yamamoto, 1996) have been proposed (Sato and Kameoka, 2007; Sato, 2010). Furthermore, using a performance-adaptive method based on a user-desired control performance (Yamamoto, 2007), a weigh feeder was controlled (Sato et al., 2011). In this method the variance of the control error must be equal to or less than a user-specified value, and the energy consumption is minimized under this condition (Kitano et al., 2011). Therefore, the energy consumption cannot be sufficiently reduced. However, in the present study, in order to further improve energy efficiency, the variance of the differences in the control input is first specified, and the variance of the control error is then minimized under this condition. Consequently, the variation of the control input can be reduced considerably without increasing the control error.

The proposed controller is implemented as a PI controller, and the PI parameters are based on generalized minimum variance control (GMVC) (Clarke, 1984; Omatu and Yamamoto, 1996) with steady-state predictive output (Sato et al., 2010). Hence, the proposed method can be easily used in industry. Finally, numerical examples demonstrated the effectiveness of the proposed method.

2. PROBLEM STATEMENT

A weigh feeder that is the control objective in the proposed study is shown in Fig. 1. This weigh feeder dispenses material charged in the hopper by controlling the rotation velocity of the motor that actuates a discharge mechanism. In this system, the weight of the discharged mass is not directly measured, but rather is obtained by the loss-in-weight method, in which the total weight of the weigh feeder is measured.

The dynamic characteristics from the input voltage to the measured discharged mass are generally of high order, but precise characteristics cannot be obtained. Therefore, control systems were designed using a dominant low-order model (Sato, 2010; Sato et al., 2011). The present study also discusses a design method for a weigh feeder control system based on the following first-order plus integrator system.

$$G(s) = \frac{1}{s}\frac{K}{Ts+1} \tag{1}$$

where K and T correspond to the gain and the time constant in the first-order system, respectively. Generally, these values are unknown because they change due to the environment and the quality of material to be discharged.

The proposed method is designed based on a discrete-time system that is transformed from the continuous-time system using a sampling interval.

$$\begin{aligned} \Delta A(z^{-1})y[k] &= B(z^{-1})u[k-1] + \xi[k] \qquad (2)\\ A(z^{-1}) &= 1 + a_1 z^{-1} \\ B(z^{-1}) &= b_0 + b_1 z^{-1} \end{aligned}$$

where $y[k]$, $u[k]$, and $\xi[k]$ are the discharged mass, the input voltage, and white Gaussian noise, respectively. Moreover, $A(z^{-1})$ is assumed to be stable.

The control objective of the present study is to make the flow-rate of the discharged material follow a desired constant value, and a control system is designed using the following PI control law:

$$\Delta u[k] = C_1(z^{-1})e[k] - C_2(z^{-1})u[k-1] \tag{3}$$

$$C_1(z^{-1}) = k_{c1}(\Delta + \frac{T_s}{T_{I1}}) \tag{4}$$

$$C_2(z^{-1}) = k_{c2}(\Delta + \frac{T_s}{T_{I2}}) \tag{5}$$

$$\begin{aligned} e[k] &= w[k] - y_\Delta[k] \\ y_\Delta[k] &\overset{\triangle}{=} \Delta y[k] \\ \Delta &\overset{\triangle}{=} 1 - z^{-1} \end{aligned}$$

where $w[k]$ is the reference input to be followed by flow-rate $y_\Delta[k]$ at each sampling interval, and k_{ci} and T_{Ii} are the proportional gain and integral time, respectively, where $i = 1, 2$. Finally, T_s denotes the sampling interval.

The PI parameters k_{ci} and T_{Ii}, which are the design parameters of the PI control law, are decided based on generalized minimum variance control using steady-state predictive output (GMVCS) (Sato et al., 2010).

3. CONTROL SYSTEM DESIGN

The design objective of the present study is to obtain a performance-adaptive PI controller such that desired specifications are automatically satisfied. To this end, the derivation of a GMVCS law (Sato et al., 2010) is first described. Next, the design parameters of GMVCS are decided based on performance evaluation. Finally, the design method for a GMVCS-based PI control law is summarized. Consequently, the PI control law can be adaptively updated based on performance evaluation.

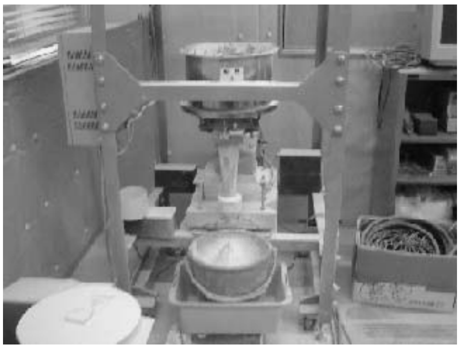

Fig. 1. Weigh feeder

3.1 GMVCS

The control objective of GMVCS is to minimize the variance of a generalized output, and a control law that minimizes the following cost function is derived (Sato et al., 2010):

$$J = E[\Phi_s[k]^2] \tag{6}$$

$$\Phi_s[k] = P(z^{-1})y_\Delta[s|k] + \lambda\Delta u[k] - R(z^{-1})w[k] \tag{7}$$

where $P(z^{-1})$ and $R(z^{-1})$ are design polynomials, and λ is a design parameter.

Steady-state predictive output $y_\Delta[s|k]$ is defined as (Kwok and Shah, 1994):

$$y_\Delta[s|k] = \lim_{j\to\infty} y_\Delta[k+j|k], \tag{8}$$

and is obtained as follows (Kwok and Shah, 1994):

$$\begin{aligned} y_\Delta[s|k] &= g_s\Delta u[k] + F_s(z^{-1})\frac{y_\Delta[k]}{P(z^{-1})} \\ &\quad + G_s(z^{-1})\frac{u[k-1]}{P(z^{-1})} \qquad (9)\\ G_s(z^{-1}) &= g_s P(z^{-1}) - z^{-k_m} e_s B(z^{-1}) \\ F_s(z^{-1}) &= e_s A(z^{-1}) \\ g_s &= \frac{B(1)}{A(1)} \\ e_s &= \frac{P(1)}{A(1)} \end{aligned}$$

The control law minimizing (6) is obtained as follows:

$$\begin{aligned} G_1(z^{-1})\Delta u[k] &= R(z^{-1})w[k] - F_s(z^{-1})y_\Delta[k] \\ &\quad - G_s(z^{-1})u[k-1] \qquad (10)\\ G_1(z^{-1}) &= g_s P(z^{-1}) + \lambda \end{aligned}$$

From the obtained control law, the closed-loop system from the reference input to the plant output is calculated as follows:

$$\begin{aligned} y[k] &= \frac{z^{-(k_m+1)}B(z^{-1})R(z^{-1})}{T(z^{-1})A(z^{-1})}w[k] \\ &\quad + \frac{g_s P(z^{-1}) + \lambda\Delta - z^{-(k_m+1)}e_s B(z^{-1})}{T(z^{-1})A(z^{-1})}\xi[k] \qquad (11) \end{aligned}$$

$$T(z^{-1}) = g_s P(z^{-1}) + \lambda\Delta \tag{12}$$

where k_m is generally positive. However, in the present study, $k_m = 0$ because model (2) has no dead time.

3.2 Decision of Design Parameters based on Performance Evaluation

In this section, the design methods for deciding $P(z^{-1})$ and λ are described, and $R(z^{-1})$ is decided in Section 3.3.

Polynomial $P(z^{-1})$ is designed as follows (Yamamoto and Kaneda, 1998):

$$\begin{aligned} P(z^{-1}) &= p_0 + p_1 z^{-1} + p_2 z^{-2} \qquad (13)\\ p_0 &= 1\\ p_1 &= -2e^{-\frac{\rho}{2\mu}}\cos\left(\frac{\sqrt{4\mu-1}}{2\mu}\rho\right)\\ p_2 &= e^{-\frac{\rho}{\mu}}\\ \rho &= \frac{T_s}{\sigma}\\ \mu &= 0.25(1-\delta)+0.51\delta \end{aligned}$$

where σ is a parameter that corresponds to the rise time, and μ is the damping index and is adjusted using δ. Yamamoto (2007) proposed that σ is set between 1/3 and 1/2 of the sum of the time constant and the dead time using prior information and is desired that δ is designed in $0 \leq \delta \leq 2.0$ by taking practicality into account.

λ is automatically designed based on performance evaluation such that the user-specified performance is achieved. In the conventional method (Kitano et al., 2011), the control error, which corresponds to the product quality is more important than the amount of energy consumed. On the other hand, the design objective of the present study is to reduce the consumption energy more than the conventional method by suppressing the differences in the control input, even if the control error is mildly increased. To this end, the maximum value to be satisfied by the variance of the differences in the control input is set beforehand, and λ is then decided so as to minimize the variance of the control error without exceeding the preliminarily set maximum variance value of the differences in the control input.

3.3 Performance-adaptive GMVCS-PI Controller

The PI parameters are decided such that the GMVCS law (10) is approximated by the PI control law (3). For comparison with the PI control law, (10) is rearranged as follows:

$$\begin{aligned} \Delta u[k] &= \frac{1}{g_{1,0}}\Big(R(z^{-1})w[k] - F_s(z^{-1})y_\Delta[k]\\ &\quad - G_2(z^{-1})u[k-1]\Big) \qquad (14)\\ G_1(z^{-1}) &= g_{1,0} + z^{-1}G_1'(z^{-1})\\ G_2(z^{-1}) &= \Delta G_1'(z^{-1}) + g_s P(z^{-1}) - e_s B(z^{-1})\\ &= g_s(p_0 + P'(z^{-1})) - e_s B(z^{-1})\\ P(z^{-1}) &= p_0 + z^{-1}P'(z^{-1}) \end{aligned}$$

Comparing (3) with (14), the following relations are obtained:

$$R(z^{-1}) = F(z^{-1}) \qquad (15)$$

$$C_1(z^{-1}) = \frac{1}{g_{1,0}}F_s(z^{-1}) \qquad (16)$$

$$C_2(z^{-1}) = \frac{1}{g_{1,0}}G_2(z^{-1}) \qquad (17)$$

Based on these equations, the PI parameters are decided as follows:

$$k_{c1} = -\frac{1}{g_{1,0}}f_{s,1} \qquad (18)$$

$$T_{I1} = -\frac{f_{s,1}}{f_{s,0}+f_{s,1}}T_s \qquad (19)$$

$$k_{c2} = -\frac{1}{g_{1,0}}g_{2,1} \qquad (20)$$

$$T_{I2} = -\frac{g_{2,1}}{g_{2,0}+g_{2,1}}T_s \qquad (21)$$

$$F_s(z^{-1}) = f_{s,0} + f_{s,1}z^{-1}$$

$$G_2(z^{-1}) = g_{2,0} + g_{2,1}z^{-1}$$

The PI parameters are set using these equations, the GMVCS-based PI controller is obtained.

The proposed algorithm is as follows:

(1) Using past input and output dates, the variances of the control error and the differences in the control input are obtained.

(2) The plant parameters are estimated using a recursive least squares method, as follows:

$$\begin{aligned} \hat{\theta}[j] &= \hat{\theta}[j-1]\\ &\quad + \frac{\Gamma[j-1]\psi[j-1]}{1+\psi^T[j-1]\Gamma[j-1]\psi[j-1]}\varepsilon[j] \qquad (22)\\ \Gamma[j] &= \Gamma[j-1]\\ &\quad - \frac{\Gamma[j-1]\psi[j-1]\psi^T[j-1]\Gamma[j-1]}{1+\psi^T[j-1]\Gamma[j-1]\psi[j-1]} \qquad (23)\\ \varepsilon[j] &= y_\Delta[j] - \hat{\theta}^T[j-1]\psi[j-1] \qquad (24)\\ \hat{\theta}[j] &= \left[\, \hat{a}_1[j]\ \hat{b}_0[j]\ \hat{b}_1[j] \,\right]^T\\ \psi[j-1] &= [\, -y_\Delta[j-1]\ u[j-1]\ u[j-2] \,]^T\\ j &= k-N+1, k-N+2, \cdots, k-1, k \end{aligned}$$

Based on the estimated parameters, estimation error $\varepsilon[k]$ and standard deviation σ_ε are calculated.

(3) In order to select the PI parameters such that a specified control performance is achieved, $E[e^2[k]]$ and $E[(\Delta u[k])^2]$ are calculated by changing λ using the H_2 norm (Yamamoto, 2007).

$$E[e^2[k]] = || - \frac{T(z^{-1}) - z^{-1}e_s B(z^{-1})}{T(z^{-1})A(z^{-1})}||_2^2 \sigma_\xi^2 \qquad (25)$$

$$E[(\Delta u[k])^2] = || - \frac{\Delta e_s(z^{-1})}{T(z^{-1})}||_2^2 \sigma_\xi^2 \qquad (26)$$

where σ_ε is used instead of σ_ξ.

(4) λ is decided such that the variance of the control error is minimized, where $\sigma_{\Delta u}^2$ must be less than or equal to the specified value. Using the selected λ, the PI parameters are decided from (18)~(21).

(5) $k = k + 1$

(6) Using the estimated parameters in Step 2, $\eta[k]$ is calculated as:

$$\eta[k] = y_{\Delta}[k] - \hat{\theta}^T[k]\psi[k-1] \tag{27}$$

(7) If (28) is satisfied, return to Step 2. Otherwise, return to Step 5.

$$\mid \eta[k] \mid \geq \gamma\sigma_{\varepsilon} \tag{28}$$

where $3.0 \leq \gamma \leq 5.0$.

4. NUMERICAL EXAMPLE

The effectiveness of the proposed method is demonstrated through numerical examples. For comparison with the conventional method (Kitano et al., 2011), the control result of the conventional method is the plant output and the differences of the control input, respectively, where the design parameters of $P(z^{-1})$ are set as $\delta = 1$ and $\sigma = 1$. Here, λ is searched in the range from 10^{-3} to 10^{1} in 10^{-3} increments, and γ is set to 3. In the conventional method, the specified variance value of the control error must first be satisfied. Then, the variance of the differences in the control input is minimized without changing the preliminarily set maximum variance value of the control error. The variance of the control error to be satisfied is set to 8.8×10^{-6}. From the start of the simulation to step 150, the plant is controlled using fixed control parameters: $k_{c1} = 100$, $T_{I1} = 0.1$, $k_{c2} = 0$, and $T_{I2} = \infty$. After that the conventional performance-adaptive method was applied. Fig. 4 shows that the trade-off curve at step 150 and the selected value of λ is 0.283. The estimated plant parameters are calculated using the recursive least squares method with the input-output data of the past 100 steps. The control performance is shown to improve after step 150.

The control result obtained using the proposed method is shown in Fig. 5∼Fig. 7, where the variance value of the differences in the control input that must not be exceeded was set to 5.0×10^{-4}, and the other conditions are the same as in the previous method. In this method, λ was decided to be 6.0×10^{-3}. The control performance can be improved using the proposed method. Compared with the control results for the conventional method, the variation of the control input is considerably reduced, although the control error of the proposed method is slightly inferior to that of the conventional method. Consequently, the proposed method is effective for reducing energy consumption.

5. CONCLUSION

We have herein proposed a new design method for a weigh feeder control system. In the proposed method, control parameters are adaptively updated based on control performance, and a user-specified control performance can be automatically attained. In the conventional method for controlling a weigh feeder, the primary objective is to improve the control error. On the other hand, the objective of the proposed method is to reduce energy consumption, although the control error might be increased compared with the conventional method. However, the simulation results demonstrated that the control error was slightly increased and the variation of the control input can be substantially reduced.

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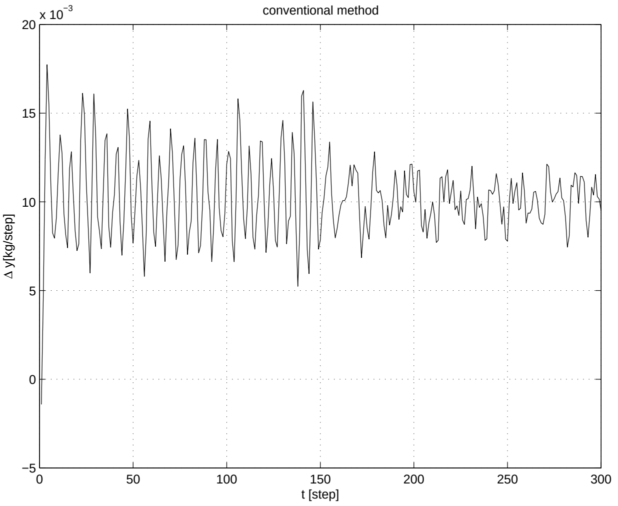

Fig. 2. Flow-rate based on the variance of control error

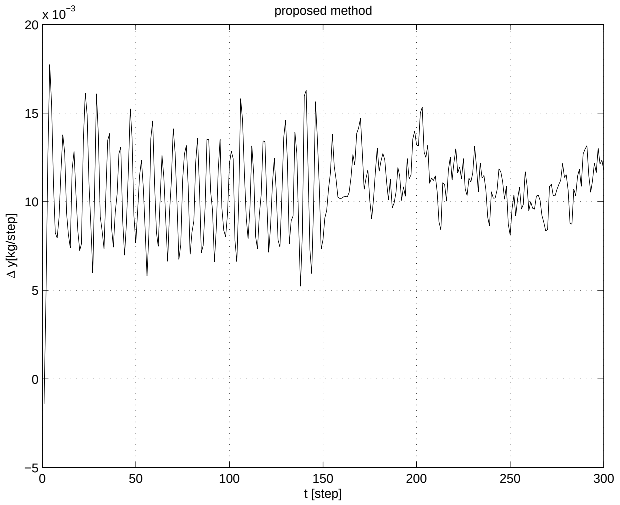

Fig. 5. Flow-rate based on the variance of the difference in control input

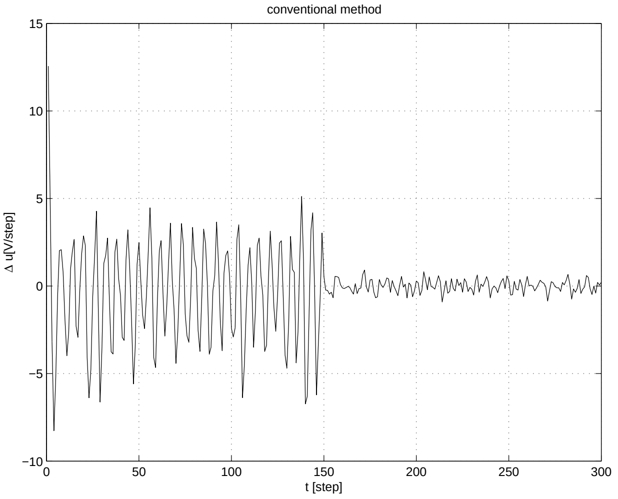

Fig. 3. Difference in input voltage based on the variance of control error

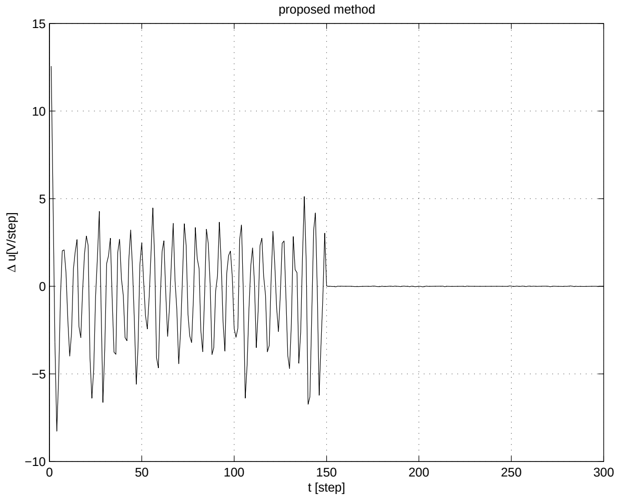

Fig. 6. Difference in input voltage based on the variance of the difference in control input

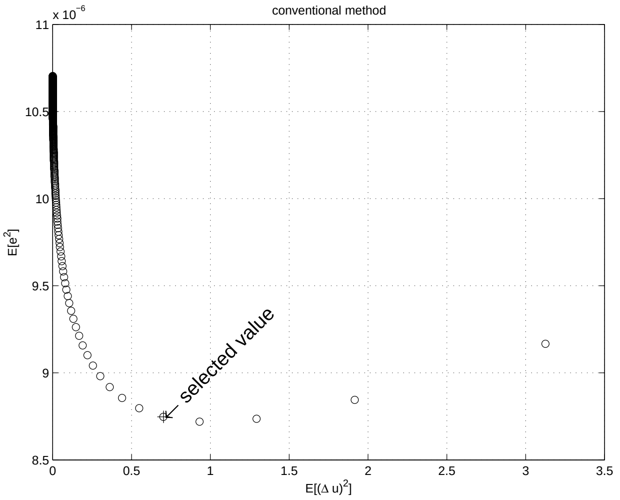

Fig. 4. Trade-off curve based on the variance of control error

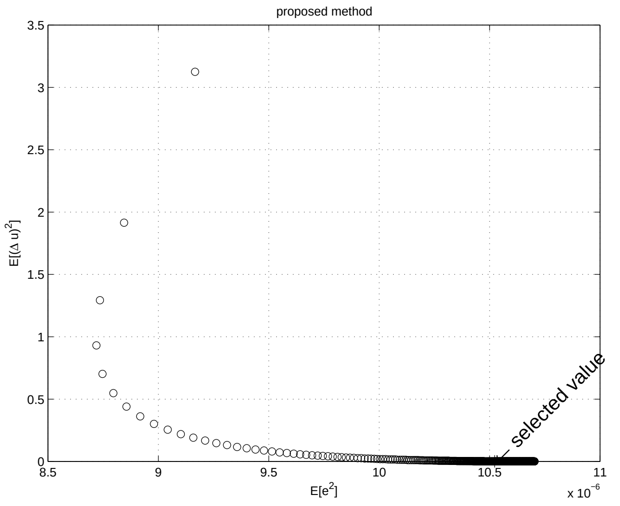

Fig. 7. Trade-off curve based on the variance of the difference in control input