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Acoustic Impedance Matching using Loop Shaping PID Controller Design

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Abstract: For several decades Proportional-Integral-Derivative control (PID) has been successfully used for a wide variety of industrial processes and remains the most used method. Recent work concerning the tuning of PID control coefficients has been proven to provide both robust and near-optimal performance using a Frequency Loop Shaping (FLS) procedure. The FLS tuning method minimizes the difference between the actual and the desired target loop transfer function. Such a control design procedure is ideal for problems in which the desired closed loop frequency response is predetermined over a specific frequency band. This paper explores the possibilities and trade-offs of applying the FLS control strategy in Active Noise Control (ANC) problems. The use of the FLS design is ideal for the problem of noise suppression in ducts, because the required acoustic impedance for the elimination of reflecting sound waves in the one-dimensional case is well defined. Hence, by controlling locally the reflecting boundary structure, a global cancelation of the undesired noise can be accomplished.

Keywords: PID, Frequency Loop Shaping, Active Noise Control.

1. INTRODUCTION

The use of active noise control (ANC) in order to reduce the reflections of sound inside acoustic ducts has been investigated thoroughly by the scientific community and a large number of control schemes have been proposed (Kuo and Morgan, 1999). Classical ANC control procedures concerning cancelation of reflected noise often make use of distributed microphones and loudspeakers in order to generate appropriate cancellation. Such designs often use Filtered-X Least Mean Square (FXLMS) algorithms, and examples can be found in *Yu et al. (2006)*; these control procedures can lead to complex solutions to implement and also generate measurement noise. In this paper a control scheme that is simple to implement and is focused on using local measurements in contrast to the remote error microphone required in FXLMS designs is proposed. In order to achieve a reduction in the reflection of sound the approach here is to directly control the dynamics of a terminating boundary surface inside the acoustic duct.

Recent work in the field of ANC has been focused on designing actuator setups that will enable active structural acoustic control (ASAC) of low frequency noise radiated by vibrating structures, Zhu *et al.* (2003), for example. The work described by these authors explores the development of thin panels that can be controlled electronically so as to provide surfaces with desired reflection coefficients. Such panels can be used as either perfect reflectors or absorbers. The development of the control system is based on the use of wave separation algorithms that separate incident sound from reflected sound. The reflected sound is then controlled to desired levels. The incident sound is used as an acoustic reference for feedforward control and has the important property of being isolated from the action of the control system speaker. The suggested control procedure makes use of a half-power FXLMS algorithm and therefore requires installation of microphones in order to be applicable and the use of low pass filters which adds significant complexity to the solution of the primary problem. Another approach in the field of ASAC and can reduce the inherent complexity of Zhu *et al.* solution is examined by Lee *et al.* (2002). Specifically, this research investigated the application of a low frequency volume velocity vibration control procedure for a smart panel in order to reduce sound transmission. The control algorithm makes use of a simple velocity feedback controller in order to add damping to the resonant frequencies of the controlled panel. The addition of damping will reduce the vibration that occurs when an incident acoustic wave impacts the panel and will thereby reduce the acoustic radiation efficiency.

In this paper, the aim is to develop a feedback controller for a general acoustic duct system as illustrated in Fig. 1. The control scheme will make use solely of local measurements (velocity of terminating surface) of the reflecting boundary surface in order to suppress the undesired reflecting sound waves that occur in the presence of an incident disturbance sound wave. In order to reduce the reflected sound and so avoid problematic acoustic resonance, the feedback controller, is required to achieve a match between the characteristic impedance of the fluid in the duct and the specific acoustic impedance of the reflecting surface. By definition, the characteristic impedance of a lossless fluid is the product between the density of the fluid (ρ_0) and the speed of sound in the fluid (c_0), $Z_0=c_0\rho_0$.Furthermore, the specific acoustic impedance of a boundary reflecting surface Z, is the frequency dependent ratio of the fluid's pressure (p) over the volume velocity (u) of the surface. When the specific acoustic impedance of the boundary surface perfectly matches the characteristic impedance of the surrounding fluid (considered to be air, in this paper) no reflection will not occur and sound propagates as if contained within an infinitely long duct.

The desired target frequency response of the controlled surface is therefore well defined and the problem of impedance matching can be approached using a frequency loop shaping design procedure. Recent work concerning PID controller design has shown that a robust and near-optimal controller can be developed that makes use of the frequency domain model of the system to be compensated, Grassi *et al.* (2001). This tuning method minimizes the difference between the actual and the desired target loop transfer function, in a L_∞ sense. This loop-shaping method has been adopted here in order to tune the frequency response of the controlled reflecting surface's specific acoustic impedance (actual transfer function) to the characteristic impedance of the bounding fluid (target loop transfer function). In Section 2 a model for the acoustic duct system that is considered is presented. In Section 3 the PID loop-shaping controller design approach that is used to cancel the undesired reflecting sound wave is detailed. In section 4 the formulation of the control loop and simulation results illustrating the design and performance are presented. Finally, Section 7 provides some concluding remarks and plans for future work.

2. DEVELOPMENT OF A SYSTEM MODEL

2.1 Acoustic duct Model

In order to simplify the modelling procedure it is convenient to assume a one-dimensional case. Accurate transfer function models of the acoustic duct have been developed over the past decades by a number of authors, Zimmer *et al*. (2003), Hull *et al.* (1993), Pota & Kelkar (2001), Hull & Radcliffe (1991), Levine & Schwinger (1948), Hu (1995), Birdsong & Radcliffe (1999) and Lane & Clark (1998), for example. In order to simplify the acoustic duct problem the following assumptions are made when modelling: (i) the system is assumed to be adiabatic (ii) the mean flow in the duct is zero (iii) the duct cross section is uniform (iv) there are negligible air viscosity effects (v) the duct has hard walls with dissipation only occurring at the termination end (vi) the duct diameter is small compared to the length so that wave propagation is planar and finally (vii) for circular ducts of radius a; these assumptions are valid for frequencies bellow the cut off frequency $0.293c/a$ with c being the speed of sound in the medium, Hull *et al.* (1993). The layout of the acoustic duct for which a model will be derived and subsequently used in simulation is shown in Fig. 1.

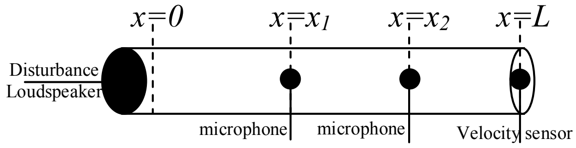

Fig. 1: Rigid duct of total length equal to $L=2m$ with a source loudspeaker at one end and an open end at the other, herein called the disturbance and open ends, respectively. The duct has a circular cross section of radius equal to α and a spatial coordinate represented by x. There are sensor microphones located at positions x_1 and x_2 solely to monitor the reflecting sound wave. At x=L there is a velocity sensor that is used for feedback.

The following fundamental equations describe the acoustic pressure inside a duct:

$$\frac{\partial^2 p(x,t)}{\partial x^2}-\frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2}=\frac{1}{\pi A^2}\rho_0\frac{\partial V(x,t)}{\partial t} \tag{1}$$

$$\rho_0\frac{\partial}{\partial t}u(x,t)=-\frac{\partial}{\partial t}p(x,t) \tag{2}$$

$$\frac{1}{c_0^2}\frac{\partial p}{\partial t}=-\frac{\partial u}{\partial x}\rho_0+\frac{1}{\pi A^2}\rho_0 V(x,t) \tag{3}$$

Where A is the cross-section of the duct, c is the speed of sound in the medium, $p(x,t)$ is the pressure in the duct, $u(x,t)$ is the particle velocity, ρ_0 is the density of air and $V(x,t)$ is a volume velocity source per unit length of the duct due to the controlled boundary end surface. The control approach discussed in this paper involves altering terminating boundary conditions in order to approximate the desired characteristic impedance. Hence, Zimmer's model, which takes into consideration variations in the terminating boundary conditions, is ideal for use.

The Laplace transforms of equations (2) and (3) with respect to time is equivalent to a linear boundary value problem and can be solved with a standard Green's function method and will lead to a transfer function model that relates the pressure measured at point x to the voltage applied to the disturbance loudspeaker, Zimmer et al. (2003):

$$G_d(x,s)=e^{-xs/c}G_{d0}(x,s) \tag{4}$$

$$G_{d0}(x,s)=\frac{Bl\rho_0 c(1+a_0(s))}{2R_{coil}Z_0(s)(1-a_0(s)a_L(s)e^{-2Ls/c})(1+a_L(s)e^{2(x-L)s/c})} \tag{5}$$

The reflecting coefficients a_0(s) and a_L(s) of the disturbance loudspeaker and reflecting boundary surface are given in (6) and (7), $Z_0(s)$ is the acoustic impedance of the disturbance loud speaker.

$$a_0(s) = \frac{Z_0(s) - \rho_0 c A_D}{Z_0(s) + \rho_0 c A_D} \tag{6}$$

$$a_L(s) = \frac{Z_L(s) - \rho_0 c}{Z_L(s) + \rho_0 c} \tag{7}$$

The values of the coefficients used in equations (4) – (7) are fully described in table 1.

2.2 Disturbance loudspeaker

The boundary condition at the disturbance end of the duct is treated in a variety of ways in the literature. The disturbance loudspeaker has been considered to be a source of pressure, Zimmer *et al*. (2003). However, a loudspeaker is closer to a volume velocity source than a pressure source. The disturbance speaker cone velocity can be used as the input to the duct transfer function, Birdsong & Radcliffe (1999) and Lane & Clark (1998). Feedback is introduced to a loudspeaker so that its response is close to that of a pure volume velocity source. However, this approach does not include the interaction between the loudspeaker and the duct. Even when inactive, the loudspeaker acts as a mechanical mass-spring-damper system and there is coupling between the duct and the loudspeaker. Thus, a system model, which assumes a pure pressure or volume velocity source, neglects this coupling. A full electro-mechanical model of the loudspeaker should be coupled to the duct model to properly represent the disturbance end. If a mechanically analogous system is used to describe the impedance of the speaker, the following impedance equation can be derived, Guicking & Karcher (1984):

$$Z_0(s) = \frac{\pi a^2}{A_D s}(m_D s^2 + c_D s + k_D) \tag{8}$$

The coefficients of equation (8) are related to the loudspeaker, specifically A_D is the disturbance speaker's effective cross section, m_D is the disturbance speaker's cone effective mass, c_D is the disturbance speaker's damping coefficient and k_D is disturbance speaker's cone suspension stiffness. In this paper the values from Zimmer et al. (2003) are used as detailed in Table 1.

2.3Reflecting surface

Various terminating boundary conditions are employed in the literature to describe the reflecting surface. From Hull *et al.* (1993), Pota & Kelkar (2001), Hull & Radcliffe (1991) and Levine & Schwinger (1948) mixed absorptive/reflective boundary conditions are used at the open end of the duct. This leads to nonzero constant impedance. However, Levine & Schwinger (1948) show that the analytical solution for the open-end impedance of a duct is highly frequency dependent. Hu (1995), derives a transfer function for a duct with variable impedances at each end. The boundary-end of the acoustic duct can be modelled as a simple mechanical baffle (a mechanical mass-spring-damper system) as for the disturbance speaker (Fig. 2).

The following assumptions concerning the boundary condition are considered: (i) zero friction between the baffle and the acoustic duct (ii) acoustic pressure leakage is not present at the connection between the duct and the boundary surface. The frequency dependent specific acoustic impedance of a mechanical mass-spring-damper structure is therefore:

$$Z_L(s) = \frac{Ms^2 + Cs + K}{s} \tag{9}$$

Where *M, C* and *K* are the mass, absolute damping and stiffness of the system.

2.3. Calculation of the reflecting sound wave

The control procedure aims to cancel the reflecting sound wave. Equation (4) describes the total acoustic pressure due to the disturbance speaker (incident and reflecting sound waves) therefore it is necessary to apply a separation technique in order to verify that the control method cancels the reflecting sound wave. Two simultaneous pressure values at two pre-defined points in the duct can be used to derive the following equation for the pressure of a wave travel in the reflected direction, Guicking & Karcher (1984):

$$p_r(s) = \frac{e^{-2\tau} p_2(s) - p_1(s)}{e^{-s2\tau} - 1} \tag{10}$$

$p_1(s)$ and $p_2(s)$ are the total acoustic pressure at locations $x = x_1$ and $x = x_2$ and τ is sound propagation time between these two locations, thus $\tau = \Delta x/c$. Hence if equations (4) and (10) are combined the reflected sound wave from the desired termination in the model can be derived in order to monitor the efficiency of the control scheme.

3. PID CONTROL

3.1 Definition and background in PID control design

An ideal feedback PID controller is described by the following transfer function in the continuous s-domain:

$$C(s) = K_p + \frac{K_i}{s} + K_d s \tag{11}$$

Where K_p is the proportional gain, K_i is the integral coefficient and K_d is the derivative coefficient. In practice, the following realisation is usually employed to reduce noise amplification:

$$C(s) = K_p + \frac{K_i}{s} + \frac{K_d s}{(sT_n + 1)} \tag{12}$$

The PID realisation given by (12) is known as the parallel form and admits complex zeros. Tuning procedures employed for the design of a PID controller has been a popular field of investigation and many techniques have been developed, for example, Liu & Daley (2000) and Cominos & Munro (2002). The most popular approaches utilise the Ziegler-Nichols method, the Kappa-tau tuning and the pole placement method.

3.2 FLS PID control design

The controller design in this paper is carried out in the frequency domain, hence it is natural to focus on frequency loop shaping methods which aim to provide a desirable sensitivity function, $S(s)=1/(1+C(s)G(s))$ and complementary sensitivity function, $T(s)=1-S(s)$. $G(s)$ represents the transfer function of the plant and $C(s)$ is the transfer function of the feedback PID controller used to shape the plant's response. In general the performance characteristics can be described as bounds on S and T in the frequency domain. For standard control performance good disturbance rejection and set-point following is required and so the sensitivity function S must be small for low frequencies. For good noise attenuation and for robustness, related to high-frequency unmodelled dynamics, one wants the complementary sensitivity T to be small. Convex optimisation techniques using L_∞ or L_2 norms can be used to tune the controller. The specifications are usually given in the form of a desired loop transfer function (LTF). A method that integrates identification and tuning for PID control is presented by Grassi *et al*. (2001). In the Frequency Loop Shaping (FLS) PID tuning method the PID coefficients K_p, K_i and K_d are tuned so that the compensated open-loop transfer function is close to the target transfer function $L(s)$ in a L_∞, sense. Here a more specific use of the approach is made since the desired loop transfer function is defined by the impedance matching requirement. The PID controller transfer function is rearranged in the following form:

$$C(s) = \frac{K_1 s^2 + K_2 s + K_3}{s(\tau s + 1)} \tag{13}$$

With $K_p = K_2 - K_3\tau$, $K_i = K_3$ and $K_d = K_1 - K_2\tau + K_3\tau^2$.

An advantage of such parameterisation is that any function of the following form $\|W(G(s)C(s) - L)\|_\infty$, which will be used as a cost function in order to tune the PID controller, with W a fixed weighting function, is convex in the new design parameters, Grassi *et al* (2001). The FLS tuning of the PID controller can then be formulated as the following optimisation problem:

$$\min_{K_1, K_2, K_3 \in K} \|W(s)S_0(s)(G(s)C(s) - L(s))\|_{L\infty} \tag{14}$$

where S_0 is the target sensitivity and K is a convex set of constraints for the PID parameters. A more detailed description of this PID tuning method and some applications of it can be found in Grassi *et al*. (2001).

4. IMPEDANCE MATCHING WITH FLS PID CONTROL

4.1 Feedback control arrangement

The physical implementation of a feedback PID controller on a boundary surface such as the one described in section 2.3 with the addition of a control force relative to the acceleration of the mass is illustrated in Fig. 2. The transfer function that relates the force due to the acoustic disturbance to the velocity of a surface is:

$$G(s) = \frac{s}{Ms^2 + Cs + K} \tag{15}$$

Because the PID tuning is required to provide impedance matching between the reflecting surface and the specific acoustic impedance of the fluid in which the waves travel (considered here to be air) it is necessary to express relation between equation (15) and the impedance of the surface, therefore:

$$Z_L(s) = \frac{1}{AG(s)} \tag{16}$$

Where A is the cross section of the acoustic duct, Fig. 1.

5.2 Definition of minimisation rule and simulations

The next step is to define the terms of the minimization rule that will allow calculation of the controller. Based on equation (14) $W(s)$ is defined as a weighting function to emphasize over what frequency range the controller is to be tuned, in this simulation the range is chosen to cover 1-1000 Hz which is the cut-off frequency at which the model of the duct is valid. As mentioned in the previous section in order to achieve a robust controller $S_0(s)$ the sensitivity of the desired closed loop response is added. Because we are interested in matching the controlled surface to a specific response, the open loop system from Fig. 3 should approximate the desired open loop response, hence $L(s)=\rho_0 c_0$ ($\rho_0 c_0$ is the characteristic acoustic impedance of the surrounding fluid that is assumed here to be air). Now that it is possible to calculate the controller, from (16) the acoustic impedance of the reflecting boundary surface of the closed loop system equation is:

$$Z_{CLSE}(s) = \frac{1}{AG_{CLSE}(s)} \tag{17}$$

Where $G_{CLSE}(s)$ is the closed loop transfer function of the controlled system illustrated in (Fig. 3).

Equation (17) can generate a simulated response of the specific acoustic impedance of the reflecting surface when the feedback control is applied to the surface. Fig. 4 shows the acoustic impedance for the controlled and uncontrolled cases and it can be seen that the reflecting boundary surface with the aid of the controller can approximate the characteristic impedance of air across a frequency band between 200-1000 Hz, with 1000Hz being the cut off frequency at which the model is accurate (section 2 assumptions of modeling). The frequency range in which the surface approximates the characteristic acoustic impedance of air results in the reduction of the undesired reflecting sound that arises inside an acoustic duct. Using equation (4) and (10) the response of the reflected sound wave for the controlled system can be calculated. The result is shown inFig. 5, where the reflecting sound wave in the modeled acoustic duct for the case with the actively controlled surface and the case without the local feedback control scheme applied is displayed. It will be noted that a significant reduction of the reflecting noise is evident in the 200 - 1000 Hz range as expected from the impedances plotted in Fig. 4. The reduction is up to 60 dB at some frequencies, in particular towards the end of the plotted range. This confirms the potential of the methodology.

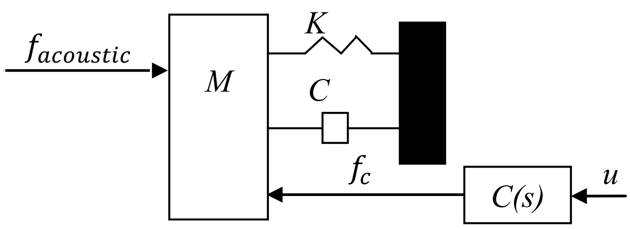

Fig. 2: Mass-spring-damper element used to model the reflecting surface of the acoustic duct with addition of the PID feedback controller *C(s)*. *M* is the mass of the reflecting surface, *K* is the stiffness, *C* is the absolute damping, $f_{acoustic}$ is the external force due to the acoustic wave, f_c is the control force applied on the mass by the local velocity feedback control loop.

Table 1.

L duct length	2 (m)
a duct radius	0.101 (m)
r density of air	1.20 (km/m^3)
c speed of sound in air	341 (m/s)
mD disturbance speaker's cone effective mass	0.015 (kg)
kD disturbance speaker's cone suspension stiffness	810.87 (N/m)
Rcoil electrical resistance of voice coil disturbance	6.0 (V)
Bl magnetic voice coil motor disturbance	5.6 (N/A)
rd disturbance speaker's effective radius	0.087 (m)
A_D disturbance speaker's effective cross section	0.024 (m^2)

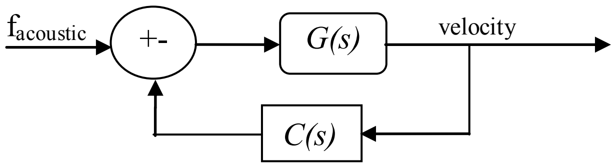

Fig. 3: Block diagram of control loop of reflecting surface with a control force related to the velocity of the reflecting surface. The force due to the incident acoustic wave is $f_{acoustic}$, *G(s)* is the transfer function relating the external force to the velocity of the mass of the reflecting surface (16), *C(s)* is the PID controller designed to reduce the undesired reflecting sound waves (13). The acoustic impedance of the controlled surface, (17), uses the closed loop equation of the block diagram with $G_{CLSE}(s) = G(s)/(1+G(s)C(s))$.

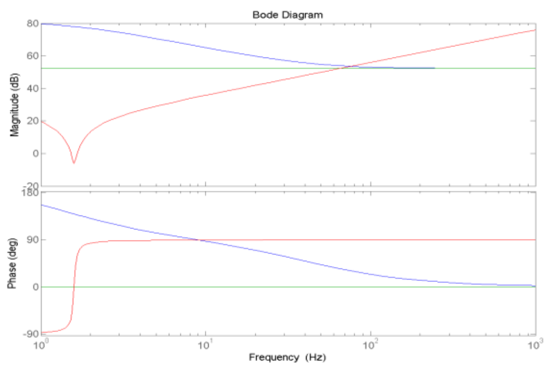

Fig. 4: Theoretical specific acoustic impedance of an actively controlled surface composed of a mass-spring-damper element (blue), theoretical specific acoustic impedance of the mass-spring-damper element without control (red) and the characteristic acoustic impedance, i.e. the desired response, of air (green). For these simulations the mass of the reflecting surface is M = 0.02 (Kg), C=0.05 (Ns/m) and K=10 (N/m).

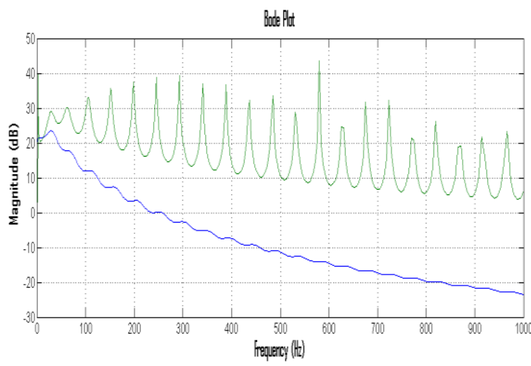

Fig. 5: Simulated response of reflecting sound wave in an acoustic duct with an actively controlled reflecting surface as the boundary surface (blue), Simulated response of reflecting sound wave in the same acoustic duct with the same reflecting surface without the active control applied (green).

5. CONCLUSIONS

In this paper a novel approach in the field of ANC was developed in order to achieve a reduction of the reflecting sound waves in a numerical simulation of a one-dimensional acoustic duct problem Fig. 1. This method made use of a robust and near-optimal FLS PID controller and was able to reduce the undesired reflected sound waves within a frequency bandwidth that reached the limits of the cut-off frequency (0 - 1000 Hz) of the model accuracy. In contrast to classical ANC the suggested feedback procedure is a locally based control design. The approach utilises only a local measurement of the velocity of the boundary reflecting surface in order to produce the control signal and does not therefore require any remote measurements, such as microphones placed in the fluid. In practice this would reduce the physical size and cost of the control system and moreover reduces the complexity of the compensator. Since the current work is entirely analytical in nature the true practical viability will be assessed in forthcoming experimental verification work.

7. REFERENCES

Birdsong, C. & Radcliffe, C. R. (1999), 'A compensated acoustic actuator for systems with strong dynamic pressure coupling', ASME J. Vibr. Acoust. 121, 89–94.

Cominos, P. & Munro, N. (2002), PID controllers: recent tuning methods and design to specification, *in* 'Control Theory and Applications, IEE Proceedings-', pp. 46--53.

Grassi, E.; Tsakalis, K.; Dash, S.; Gaikwad, S.; MacArthur, W. & Stein, G. (2001), 'Integrated system identification and PID controller tuning by frequency loop-shaping', Control Systems Technology, IEEE Transactions on 9(2), 285--294.

Guicking, D. & Karcher, K. (1984), 'Active impedance control for one-dimensional sound', Journal of vibration, acoustics, stress, and reliability in design 106(3), 393--396.

Hu, J. S. (1995), 'Active sound attenuation in finite-length ducts using closeform transfer function models', ASME J. Dyn. Syst., Meas., Control 117, 143-154.

Hull, A. & Radcliffe, CJ. (1991), 'An Eigenvalue Based Acoustic Impedance Measurement', Journal of Vibrations and Acoustics 113, 250--254.

Hull, A. Radcliffe, J. & Southward, S. (1993), 'Global active noise control of a one-dimensional acoustic duct using a feedback controller', Journal of Vibration and Acoustics 115, 488--494.

Kuo, S. & Morgan, D. (1995), Active noise control systems: algorithms and DSP implementations, John Wiley & Sons, Inc. New York, NY, USA.

Kuo, S. & Morgan, D. (1999), 'Active noise control: a tutorial review', *Proceedings of the IEEE* **87**(6), 943--973.

Lane, S. & Clark, R. (1998), 'Improving loudspeaker performance for active noise control applications', Journal of the Audio Engineering Society 46(6), 508--519.

Lee, Y.; Gardonio, P. & Elliott, S. (2002), 'Volume velocity vibration control of a smart panel using a uniform force actuator and an accelerometer array', Smart materials and structures 11, 863.

Levine, H. & Schwinger, J. (1948), 'On the radiation of sound from an unflanged circular pipe', Physical Review 73(4), 383--406.

Liu, G. & Daley, S. (2000), 'Optimal-tuning nonlinear PID control of hydraulic systems', Control Engineering Practice 8(9), 1045--1053.

Nelson, P.; Elliott, S. (1993), Active control of sound, Academic Press Inc.

Pota, H. & Kelkar, A. (2001), 'Modeling and control of acoustic ducts', Journal of Vibration and Acoustics 123, 2.

Yu, X.; Zhu, H.; Rajamani, R. & Stelson, K. (2007), 'Acoustic transmission control using active panels: an experimental study of its limitations and possibilities', *Smart Materials and Structures* **16**, 2006.

Zhu, H.; Rajamani, R. & Stelson, K. (2003), 'Active control of acoustic reflection, absorption, and transmission using thin panel speakers', The Journal of the Acoustical Society of America 113, 852.

Zimmer, B.; Lipshitz, S.; Morris, K.; Vanderkooy, J. & Obasi, E. (2003), 'An improved acoustic model for active noise control in a duct', Journal of Dynamic Systems, Measurement, and Control 125(382).

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