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Optimal Robust Tuning for 1DoF PI/PID Control Unifying FOPDT/SOPDT Models

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Abstract: The aim of the paper is to present tuning equations for one-degree-of-freedom (1DoF) proportional integral (PI) and proportional integral derivative (PID) controllers. These are based on a performance/robustness trade-off analysis with first- and second-order plus dead-time models. On the basis of this analysis a tuning method is developed for 1DoF PI and PID controllers for servo and regulatory control that allows designing closed-loop control systems with a specified M_S robustness that at the same time have the best possible IAE performance. The control system robustness is adjusted varying only the controller proportional gain.

Keywords: PID controllers, one-degree-of-freedom controllers, servo/regulatory control, performance/robustness trade-off.

1. INTRODUCTION

As it has been widely reported, proportional integral derivative (PID) type controllers are with no doubt, the controllers most extensively used in the process industry. Their success is mainly due to their simple structure, easier to understand by the control engineer than other most advanced control approaches.

In industrial process control applications, the set-point normally remains constant and good load-disturbance rejection (regulatory control) is required. There are also applications where the set-point following (servo-control) is the more important control task.

Although from their commercial introduction in 1940 (Babb, 1990) the original three-term PID control algorithm has evolved into the actual four- or five-term two-degree-of-freedom (2DoF) PID control algorithms the vast majority of the controllers still in use are of one-degree-of-freedom (1DoF) type.

Since Ziegler and Nichols (1942) presented their PID controller tuning rules, a great number of other procedures have been developed as revealed in O'Dwyer (2006) review. Some of them consider only the system performance (López et al., 1967; Rovira et al., 1969), its robustness (Åström and Hägglund, 1984), or a combination of performance and robustness (Ho et al., 1999).

There are tuning rules optimized for regulatory control operation (López et al., 1967) or optimized for servo-control operation (Tavakoli and Tavakoli, 2003). There are also authors that present separate sets of rules for each operation (Zhuang and Atherton, 1993; Kaya, 2004). For the servo-control operation there is an important group of tuning rules based on zero-pole cancellation, Internal Model Control (IMC), and direct synthesis techniques (Martin et al., 1975; Rivera et al., 1986; Alcántara et al., 2011).

Due to the constraints imposed by the 1DoF control algorithm it is necessary to develop separate tuning rules for servo and regulatory control. In addition, the control-system design procedure is usually based on the use of low-order linear models identified at the control system normal operation point. Due to the non-linear characteristics found in most industrial processes, it is necessary to consider the expected changes in the process characteristics assuming certain relative stability margins, or robustness requirements, for the control system.

Therefore, the design of the closed-loop control system with 1DoF PI and PID controllers must consider the main operation of the control system (*servo-control* or *regulatory control*) and the *trade-off* of two conflicting criteria, the time response *performance* to set-point or load-disturbances, and the *robustness* to changes in the controlled process characteristics. If only the system performance is taken into account, by using for example an integrated error criteria (IAE, ITAE or ISE) or a time response characteristic (overshoot, rise-time or settling-time) as in Huang and Jeng (2002), and Tavakoli and Tavakoli (2003), the resulting closed-loop control system probably will have a very low robustness. On the other hand, if the system is designed to have high robustness as in Hägglund and Åström (2002) and if the performance of the resulting system is not evaluated, the designer will not have any indication of the *cost* of having such highly robust system. Control performance and robustness are taken into account in Shen (2002), and Tavakoli et al. (2005) optimizing its IAE or ITAE performance but they just guarantee the usual minimum level of robustness.

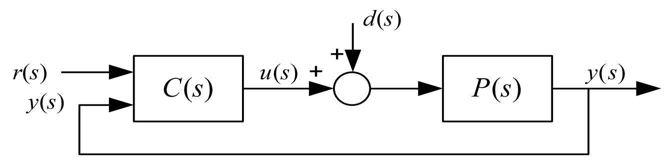

Figure 1. Closed-Loop Control System

To have an indication of the performance loss when the control system robustness is increased, using M_S as a measure, a performance/robustness analysis was conducted for 1DoF and 2DoF PI and PID control systems with first- (FOPDT) and second-order plus dead-time (SOPDT) models (Alfaro et al., 2010).

Based on this performance/robustness analysis, tuning rules are proposed for servo and regulatory 1DoF PI and PID controllers for four M_S robustness levels in the range from 1.4 to 2.0, to design robust closed-loop control systems that at the same time have the best possible performance under the IAE criteria. The presented tuning rules integrate in a single set of equations the tuning of controllers for first- and second-order plus dead-time process models.

The rest of the paper is organized as follows: the transfer functions of the controlled process model, the controller, and the closed-loop control system are presented in Section 2; the performance/robustness analysis is summarized in Section 3; the proposed *Optimal and Robust Tuning* is presented in Section 4 and particular examples of the performance/robustness trade-off are shown in Section 5. The paper ends with some conclusions.

2. PROBLEM FORMULATION

Consider a closed-loop control system, as shown in Fig. 1, where $P(s)$ and $C(s)$ are the controlled process model and the controller transfer function, respectively. In this system, $r(s)$ is the set point; $u(s)$, the controller output signal; $d(s)$, the load disturbance; and $y(s)$, the controlled process variable.

The controlled process is represented by an SOPDT model given by the general transfer function

$$P(s) = \frac{Ke^{-Ls}}{(Ts+1)(aTs+1)},\ \tau_o = \frac{L}{T}, \tag{1}$$

where K is the gain; T, the main time constant; a, the ratio of the two time constants ($0 \leq a \leq 1.0$); L, the dead-time; and τ_o, the *normalized dead time*. The model transfer function (1) allows the representation of FOPDT processes ($a = 0$), over damped SOPDT processes ($0 < a < 1$), and dual-pole plus dead-time (DPPDT) processes ($a = 1$).

The process is controlled with a 1DoF PID controller whose output is as follows (Åström and Hägglund, 1995):

$$u(s) = K_p\left\{\left(1+\frac{1}{T_i s}\right)e(s) - \left(\frac{T_d s}{\alpha T_d s+1}\right)y(s)\right\}, \tag{2}$$

where K_p is the controller *proportional gain*; T_i, the *integral time constant*; T_d, the *derivative time constant*; and α, the *derivative filter constant*. Then the controller parameters to tune are $\theta_c = \{K_p, T_i, T_d\}$. Usually, $\alpha = 0.10$ (Corripio, 2001).

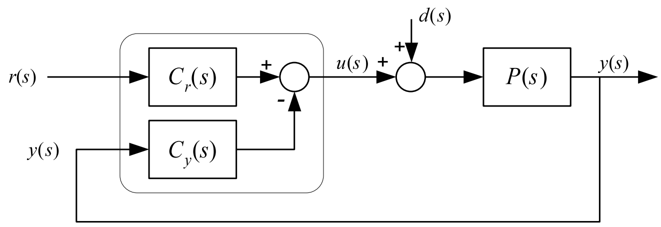

Figure 2. PID Closed-Loop Control System

Equation (2) may be rearranged, for analysis purposes, as follows

$$\begin{aligned} u(s) =& K_p\left(1+\frac{1}{T_i s}\right)r(s) \\ &- K_p\left(1+\frac{1}{T_i s}+\frac{T_d s}{0.1T_d s+1}\right)y(s), \end{aligned} \tag{3}$$

or in the compact form shown in Fig. 2 as

$$u(s) = C_r(s)r(s) - C_y(s)y(s), \tag{4}$$

where $C_r(s)$ is the *set-point controller* transfer function and $C_y(s)$ is the *feedback controller* transfer function.

The output of the closed-loop control system varies with a change in any of its the inputs as:

$$y(s) = \frac{C_r(s)P(s)}{1+C_y(s)P(s)}r(s) + \frac{P(s)}{1+C_y(s)P(s)}d(s), \tag{5}$$

or

$$y(s) = M_{yr}(s)r(s) + M_{yd}(s)d(s), \tag{6}$$

where $M_{yr}(s)$ is the transfer function from the set-point to the controlled process variable and is known as the *servo control* closed-loop transfer function; $M_{yd}(s)$ is the transfer function from the load disturbance to the controlled process variable and is known as the *regulatory control* closed-loop transfer function.

The *performance* of the closed-loop control system is evaluated using the IAE cost functional given by

$$J_e \doteq \int_0^\infty |e(t)|\,\mathrm{d}t = \int_0^\infty |y(t)-r(t)|\,\mathrm{d}t. \tag{7}$$

The controller parameters in the servo-control closed-loop transfer function, M_{yr}, are the same than the controller parameters in the regulatory control closed-loop transfer function, M_{yd}. Therefore it is not possible to obtain a single set of controller parameters θ_c that optimize, at the same time, the control system response to a set-point step change and the control system response to a load-disturbance step change.

The performance (7) is evaluated for a step change in the set-point, J_{er} and in the load-disturbance, J_{ed}.

The peak magnitude of the sensitivity function is used as an indicator of the system *robustness* (relative stability). The maximum sensitivity for the control system is defined as

$$M_S \doteq \max_\omega |S(\mathrm{j}\omega)| = \max_\omega \frac{1}{|1+C_y(\mathrm{j}\omega)P(\mathrm{j}\omega)|}. \tag{8}$$

If the system robustness (8) is not taken into account for the design, the controller parameters may be optimized to maximize the system performance or to achieve the minimum value of the cost functional in (7), using M_{yr}

for set point changes (J_{er}^o) and M_{yd} for load disturbance changes (J_{ed}^o).

Because of the control system performance/robustness trade-off, if a robustness constraint is included into the design then, it is expected that the actual system performance will be reduced ($J_e \geq J_e^o$). Then, the *performance degradation factor* defined as

$$F_p \doteq \frac{J_e^o}{J_e}, \; F_p \leq 1, \tag{9}$$

is used to evaluate the performance/robustness trade-off.

3. PERFORMANCE/ROBUSTNESS TRADE-OFF ANALYSIS

To evaluate the performance degradation when the system robustness is increased, the following steps, as they were presented in Alfaro et al. (2010), were followed.

3.1 1DoF Controllers Optimum Performance

For the 1DoF servo- and regulatory-control performance-optimized PI and PID controllers, the parameters $\theta_c^o = \{K_p^o, T_i^o, T_d^o\}$ were obtained using the cost functional (7) such that

$$J_e^o \doteq J_e(\theta_c^o) = \min_{\theta_c} J_e(\theta_c), \tag{10}$$

for (1) with $a \in \{0, 0.25, 0.5, 0.75, 1\}$ and ten τ_o in the range from 0.05 to 2.0, for set-point and load-disturbance step changes. The robustness of the control systems that deliver the optimal performance was evaluated by using M_S.

3.2 1DoF Controllers Degraded Performance

To increase the control-loop robustness, a target performance degradation factor, F_p^t, was included in the cost functional, as follows

$$J_{F_p} \doteq J(\theta_c, F_p^t) = \left| \frac{J_e^o}{J_e(\theta_c)} - F_p^t \right|, \tag{11}$$

for obtaining the PI and PID (servo and regulatory control) parameters θ_c^{o1} such that

$$J_{F_p}^o \doteq J_{F_p}(\theta_c^{o1}, F_p^t) = \min_{\theta_c} J_{F_p}(\theta_c, F_p^t). \tag{12}$$

When F_p^t was decreased, the control-system robustness was increased to the target level, M_S^t.

With starting point as the original unconstrained (from the point of view of robustness) optimal parameters θ_c^{o1}, a second optimization was conducted using the cost functional

$$J_{M_S} \doteq J(\theta_c, M_S^t) = \left| M_S(\theta_c) - M_S^t \right|, \tag{13}$$

in order to achieve the target robustness. The robust controller parameters, θ_c^{o2}, are such that

$$J_{M_S}^o \doteq J_{M_S}(\theta_c^{o2}, M_S^t) = \min_{\theta_c} J_{M_S}(\theta_c, M_S^t). \tag{14}$$

For the analysis, four target robustness levels were considered, $M_S^t \in \{2, 1.8, 1.6, 1.4\}$.

Finally, the performance degradation factor required for obtaining M_S^t in (14) was evaluated as follows

$$F_p(M_S^t) = \frac{J_e^o}{J_e(\theta_c^{o2})}. \tag{15}$$

Therefore, the second optimization provided the controller parameters θ_c^{o2} required to formulate a system with the target robustness (8), M_S^t, and with the best performance allowed when using the IAE criteria (7), J_{er} or J_{ed}.

The performance/robustness analysis of the resulting in PI and PID closed-loop control systems pointed out the existing *trade-off* between them. As shown in Alfaro et al. (2010), in general performance optimized 1DoF PI controllers are more robust than the PIDs but their optimal performance is lower. The performance optimized regulatory control systems, for both PI and PID, are less robust than the servo-control ones, requiring also more performance degradation, lower *degraded performance factor*, to reach the same robustness level.

4. UNIFIED SIMPLE OPTIMAL ROBUST TUNING FOR 1DOF PI AND PID CONTROLLERS ($USORT_1$)

One of the purposes of this contribution is try to capture in a single set of equations the performance/robustness trade-off. This is with no doubt a novel feature as the first- and second-order models are considered at once, without forcing a distinction with respect to neither the model used nor the controller structure. The other purpose is that these robust tuning equations be as simple as possible.

Analysis of the regulatory and servo-control PI and PID controllers parameters shows that for a model with a given time constants ratio a, increasing the control system robustness by decreasing M_S^t, results in a substantial reduction in K_p. However, this increase in the robustness has negligible effect on T_i and T_d, except in the case of models with a very low τ_o (when high robustness is required).

On the basis of this observation, equations *that are independent of the target robustness level* can be obtained for the controller integral time constant and derivative time constant, as follows:

$$T_i = \mathbf{F}(T, \tau_o, a), \quad T_d = \mathbf{G}(T, \tau_o, a). \tag{16}$$

With these equations at hand, the controller proportional gains are readjusted to match a target robustness to obtain equations given by the following

$$K_p = \mathbf{H}(K, \tau_o, a, M_S^t). \tag{17}$$

For FOPDT and SOPDT models with τ_o in the range from 0.1 to 2.0 and four M_S^t values the normalized 1DoF PI and PID controller parameters can be obtained using the process model parameters, $\theta_p = \{K, T, a, L, \tau_o\}$, for servo-control and regulatory control from the following relations:

- Regulatory control operation:

$$\kappa_p \doteq K_p K = a_0 + a_1 \tau_o^{a_2}, \tag{18}$$

$$\tau_i \doteq \frac{T_i}{T} = b_0 + b_1 \tau_o^{b_2}, \tag{19}$$

$$\tau_d \doteq \frac{T_d}{T} = c_0 + c_1 \tau_o^{c_2}, \tag{20}$$

Table 1. Regulatory Control PI Tuning

	Controlled process time constants ratio a				
	0.0	0.25	0.50	0.75	1.0
	Target robustness $M_S^t = 2.0$				
a_0	0.265	0.077	0.023	-0.128	-0.244
a_1	0.603	0.739	0.821	1.035	1.226
a_2	-0.971	-0.663	-0.625	-0.555	-0.517
	Target robustness $M_S^t = 1.8$				
a_0	0.229	0.037	-0.056	-0.160	-0.289
a_1	0.537	0.684	0.803	0.958	1.151
a_2	-0.952	-0.626	-0.561	-0.516	-0.472
	Target robustness $M_S^t = 1.6$				
a_0	0.175	-0.009	-0.080	-0.247	-0.394
a_1	0.466	0.612	0.702	0.913	1.112
a_2	-0.911	-0.578	-0.522	-0.442	-0.397
	Target robustness $M_S^t = 1.4$				
a_0	0.016	-0.053	-0.129	-0.292	-0.461
a_1	0.476	0.507	0.600	0.792	0.997
a_2	-0.708	-0.513	-0.449	-0.368	-0.317
b_0	-1.382	0.866	1.674	2.130	2.476
b_1	2.837	0.790	0.268	0.112	0.073
b_2	0.211	0.520	1.062	1.654	1.955

- Servo-control operation:

$$\kappa_p \doteq K_p K = a_0 + a_1 \tau_o^{a_2}, \tag{21}$$

$$\tau_i \doteq \frac{T_i}{T} = \frac{b_0 + b_1 \tau_o + b_2 \tau_o^2}{b_3 + \tau_o}, \tag{22}$$

$$\tau_d \doteq \frac{T_d}{T} = c_0 + c_1 \tau_o^{c_2}, \tag{23}$$

The value of the constants a_i, b_i, and c_i in (18) to (23) are listed in Tables 1 to 4. As noted in these Tables only the a_i constants for K_p calculation depend on the robustness level M_S.

Equations (18) to (23) provide a direct controller tuning for the FOPDT ($a = 0$) and the DPPDT ($a = 1$) models. In the case of the SOPDT models with $a \notin \{0.25, 0.5, 0.75\}$ the set of controller parameters must be obtained by linear interpolation between the two sets of parameters obtained with the adjacent a values used in the optimization.

The performance/robustness analysis also shows that the PI controllers with performance optimized parameters for servo-control operation produce control systems with a robustness $M_S \approx 1.8$. Then, the minimum robustness level of $M_S = 2.0$ is exceeded in this case.

With a maximum absolute deviation from the target robustness M_S^t of 4.09% and an average deviation of only 0.70% the proposed $uSORT_1$ tuning may be considered as a *global robust tuning method* with levels $M_S^t \in \{2.0, 1.8, 1.6, 1.4\}$ for FOPDT and SOPDT models with normalized dead-times in the range from 0.1 to 2.0.

Equations (18) to (20) and (21) to (23) were obtained for tuning Standard PID controllers. It is know that an equivalent Serial PID controller only exists if $T_i/T_d \geq 4$. As can be seen from Fig. 3 for the $uSORT_1$ regulatory control $\tau_i/\tau_d < 4$, then there is no Serial PID equivalent in this case, and that for the $uSORT_1$ servo-control in general $\tau_i/\tau_d \geq 4$ for time constant dominant models ($\tau_o \leq 1.0$). In the particular case of FOPDT controlled process models the servo-control Serial PID equivalent exists for $\tau_o \leq 1.4$.

Table 2. Regulatory Control PID Tuning

	Controlled process time constants ratio a				
	0.0	0.25	0.50	0.75	1.0
	Target robustness $M_S^t = 2.0$				
a_0	0.235	0.435	0.454	0.464	0.488
a_1	0.840	0.551	0.588	0.677	0.767
a_2	-0.919	-1.123	-1.211	-1.251	-1.273
	Target robustness $M_S^t = 1.8$				
a_0	0.210	0.380	0.400	0.410	0.432
a_1	0.745	0.500	0.526	0.602	0.679
a_2	-0.919	-1.108	-1.194	-1.234	-1.257
	Target robustness $M_S^t = 1.6$				
a_0	0.179	0.311	0.325	0.333	0.351
a_1	0.626	0.429	0.456	0.519	0.584
a_2	-0.921	-1.083	-1.160	-1.193	-1.217
	Target robustness $M_S^t = 1.4$ †				
a_0	0.155	0.228	0.041	0.231	0.114
a_1	0.455	0.336	0.571	0.418	0.620
a_2	-0.939	-1.057	-0.725	-1.136	-0.932
	†Valid only for $\tau_o \geq 0.40$ if $a \geq 0.25$				
b_0	-0.198	0.095	0.132	0.235	0.236
b_1	1.291	1.165	1.263	1.291	1.424
b_2	0.485	0.517	0.496	0.521	0.495
c_0	0.004	0.104	0.095	0.074	0.033
c_1	0.389	0.414	0.540	0.647	0.756
c_2	0.869	0.758	0.566	0.511	0.452

Table 3. Servo-Control PI Tuning

	Controlled process time constants ratio a				
	0.0	0.25	0.50	0.75	1.0
	Target robustness $M_S^t = 1.8$				
a_0	0.243	0.094	0.013	-0.075	-0.164
a_1	0.509	0.606	0.703	0.837	0.986
a_2	-1.063	-0.706	-0.621	-0.569	-0.531
	Target robustness $M_S^t = 1.6$				
a_0	0.209	0.057	-0.010	-0.130	-0.220
a_1	0.417	0.528	0.607	0.765	0.903
a_2	-1.064	-0.667	-0.584	-0.506	-0.468
	Target robustness $M_S^t = 1.4$				
a_0	0.164	0.019	-0.061	-0.161	-0.253
a_1	0.305	0.420	0.509	0.636	0.762
a_2	-1.066	-0.617	-0.511	-0.439	-0.397
b_0	14.650	0.107	0.309	0.594	0.625
b_1	8.450	1.164	1.362	1.532	1.778
b_2	0.0	0.377	0.359	0.371	0.355
b_3	15.740	0.066	0.146	0.237	0.209

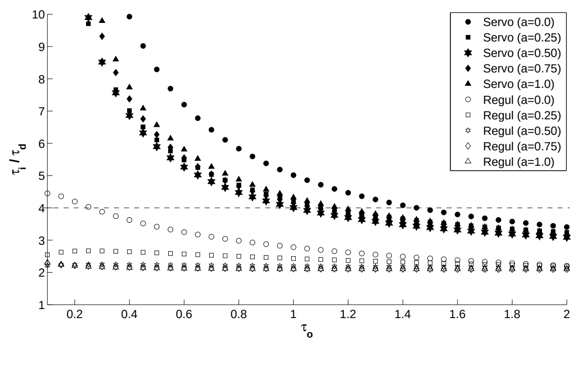

Figure 3. Servo and Regulatory Control τ_i/τ_d Ratio

Table 4. Servo-Control PID Tuning

	Controlled process time constants ratio a				
	0.0	0.25	0.50	0.75	1.0
	Target robustness $M_S^t = 2.0$				
a_0	0.377	0.502	0.518	0.533	0.572
a_1	0.727	0.518	0.562	0.653	0.728
a_2	-1.041	-1.194	-1.290	-1.329	-1.363
	Target robustness $M_S^t = 1.8$				
a_0	0.335	0.432	0.435	0.439	0.482
a_1	0.644	0.476	0.526	0.617	0.671
a_2	-1.040	-1.163	-1.239	-1.266	-1.315
	Target robustness $M_S^t = 1.6$				
a_0	0.282	0.344	0.327	0.306	0.482
a_1	0.544	0.423	0.488	0.589	0.622
a_2	-1.038	-1.117	-1.155	-1.154	-1.221
	Target robustness $M_S^t = 1.4$				
a_0	0.214	0.234	0.184	0.118	0.147
a_1	0.413	0.352	0.423	0.575	0.607
a_2	-1.036	-1.042	-1.011	-0.956	-1.015
b_0	1687	0.135	0.246	0.327	0.381
b_1	339.2	1.355	1.608	1.896	2.234
b_2	39.86	0.333	0.273	0.243	0.204
b_3	1299	0.007	0.003	-0.006	-0.015
c_0	-0.016	0.026	-0.042	-0.086	-0.110
c_1	0.333	0.403	0.571	0.684	0.772
c_2	0.815	0.613	0.446	0.403	0.372

Table 5. P_1 Servo-Control Operation

	$uSORT_1$ M_S^d				MEB
	2.0	1.8	1.6	1.4	IAE
	PI Controller				
K_p	-	0.778	0.646	0.482	-
T_i	-		2.546		-
M_S^r	-	1.81	1.61	1.40	-
$J_{er}/\Delta r$	-	2.947	3.282	4.392	-
	PID Controller				
K_p	1.132	1.003	0.846	0.642	1.174
T_i			3.022		3.085
T_d			0.495		0.589
M_S^r	2.0	1.80	1.60	1.40	2.21
$J_{er}/\Delta r$	2.458	2.512	2.976	3.918	2.481

5. EXAMPLES

For comparison of the performance and robustness obtained with the proposed $uSORT_1$ method we use the Madhuranthakam et al. (2008) [MEB] tuning rules for Standard PID controllers that optimize the IAE criteria for servo- and regulatory control operation.

First, we consider the FOPDT process given by

$$P_1(s) = \frac{1.2\mathrm{e}^{-1.5s}}{2s+1}.$$

The controller parameters and the control system performance and robustness for servo-control and regulatory control operation of P_1 are listed in Table 5 and Table 6, respectively.

As a second model we consider the SOPDT process given by

$$P_2(s) = \frac{1.2\mathrm{e}^{-1.5s}}{(2s+1)(s+1)}.$$

The controller parameters and the control system performance and robustness for servo-control and regulatory control operation of P_2 are listed in Table 7 and Table 8, respectively.

Table 6. P_1 Regulatory Control Operation

	$uSORT_1$ M_S^d				MEB
	2.0	1.8	1.6	1.4	IAE
	PI Controller				
K_p	0.885	0.779	0.651	0.500	-
T_i		2.576			-
M_S^r	2.01	1.81	1.61	1.42	-
$J_{ed}/\Delta d$	2.910	3.305	3.960	5.156	-
	PID Controller				
K_p	1.108	0.984	0.829	0.626	1.293
T_i		1.867			1.971
T_d		0.614			0.569
M_S^r	2.02	1.82	1.61	1.40	2.36
$J_{ed}/\Delta d$	1.969	2.215	2.593	3.303	1.666

Table 7. P_2 Servo-Control Operation

	$uSORT_1$ M_S^d				MEB
	2.0	1.8	1.6	1.4	IAE
	PI Controller				
K_p	-	0.711	0.590	0.441	-
T_i	-		3.421		-
M_S^r	-	1.83	1.62	1.41	-
$J_{er}/\Delta r$	-	4.311	4.831	6.469	-
	PID Controller				
K_p	1.110	0.989	0.839	0.625	1.497
T_i		4.264			5.121
T_d		0.921			0.812
M_S^r	1.98	1.79	1.61	1.40	2.78
$J_{er}/\Delta r$	3.385	3.596	4.234	5.687	3.798

Table 8. P_2 Regulatory Control Operation

	$uSORT_1$ M_S^d				MEB
	2.0	1.8	1.6	1.4	IAE
	PI Controller				
K_p	0.838	0.740	0.613	0.461	-
T_i		3.743			-
M_S^r	2.03	1.83	1.62	1.42	-
$J_{ed}/\Delta d$	4.466	5.059	6.102	8.098	-
	PID Controller				
K_p	1.037	0.951	0.801	0.620	1.539
T_i		2.454			2.971
T_d		1.108			0.883
M_S^r	1.93	1.79	1.60	1.41	2.94
$J_{ed}/\Delta d$	2.848	3.094	3.605	4.456	2.141

From Tables 5 to 8 it is noted that for same robustness design level (M_S^d) the PID controllers deliver more performance than the PI controllers. They also show the performance/robustness trade-off, an increment in control system robustness always reduces its performance. For example, to increase the robustness reducing M_S^d from 1.8 to 1.6 produces a 11 to 20% reduction in the control system performance.

It is also noted that the performance optimized MEB control systems have low robustness, $M_S > 2.0$ in all cases. Although the MEB controllers are performance optimized the servo-control $uSORT_1$ PID controllers for $M_S^d = 2.0$ produce control systems that are more robust and that at the same time have better performance.

The P_2 control system responses to a 10% set-point and load-disturbance step changes are shown in Fig. 4 and Fig. 5, respectively.

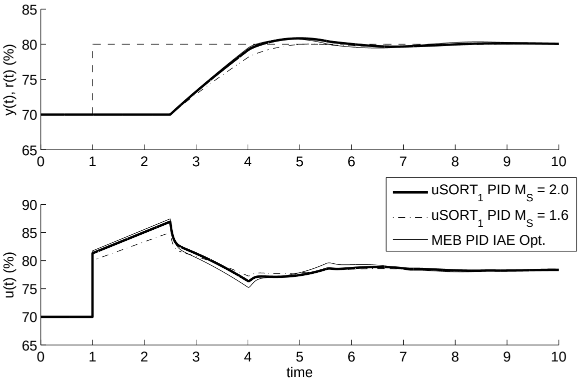

Figure 4. Model P_2 Servo-Control Responses

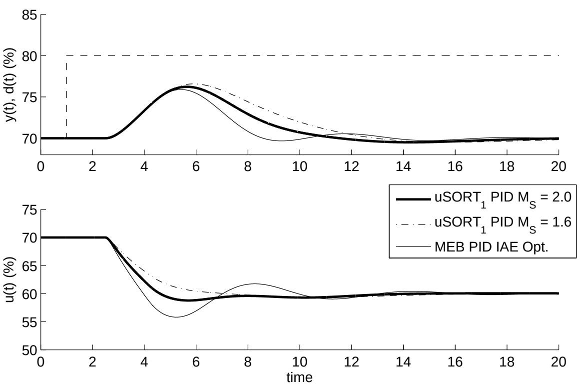

Figure 5. Model P_2 Regulatory Control Responses

6. CONCLUSIONS

Based on a performance (IAE) - robustness (M_S) analysis tuning relations are proposed that unifies the treatment of one-degree-of-freedom (1DoF) PI and PID controllers and the use of first- and second-order plus dead-time (FOPDT, SOPDT) models for servo- and regulatory control systems.

The proposed *Unified Simple Optimal and Robust Tuning* for 1DoF PI/PID controllers ($uSORT_1$) allows to adjust the control system robustness varying only the controller proportional gain.

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