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Fragility-Rings - A Graphic Tool for PI/PID Controllers Robustness-Fragility Analysis

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Abstract: The aim of the paper is to present the *Delta 20 Fragility-Rings* plot and its use for robustness-fragility analysis of proportional integral (PI) and proportional integral derivative (PID) controllers. Using the *Delta 20 Fragility Index* and the Nyquist plot it shows the areas on the $L(\mathrm{j}\omega)$ plane corresponding to robustness-fragile, robustness-non-fragile and robustness-resilient controllers providing a visual aid for evaluation of the controller robustness-fragility when its parameters are perturbed.

Keywords: PID controllers, control system robustness, controllers fragility.

1. INTRODUCTION

Since Ziegler and Nichols (1942) presented their tuning rules, a great number of other tuning procedures have been developed for proportional integral (PI) and proportional integral derivative (PID) controllers, as revealed in O'Dwyer (2006) handbook.

At the beginning, only the control system *performance* was taken into account in the controller design, considering a step change either in the set-point, *servo-control* operation, or in the load-disturbance, *regulatory control* operation, as in the classic tuning rules of Cohen and Coon (1953), López et al. (1967), and Rovira et al. (1969), among others (Chien and Fruehauf, 1990; Rivera et al., 1986), for one-degree-of-freedom (1DoF) PI and PID controllers.

Later, the consideration of the control system relative stability, its *robustness* to the changes in the controlled process characteristics, was introduced into the controller design. Initially, considering the control-loop gain and phase margins (A_m, ϕ_m) as in Åström and Hägglund (1984); Fung et al. (1998) and Ho et al. (1995) . More recently, these classic robustness indicators have been replaced by a single value given by the maximum of the magnitude of the sensitivity function, denoted by M_S. This approach has been used in Alfaro et al. (2010); Åström and Hägglund (1995); Hägglund and Åström (2002) and Tavakoli et al. (2005).

There is, however, another consideration that must be taken into account when facing the design of control systems: the effect of the variation of the controller parameters over the control system stability and performance, known as the *controller fragility.* If the control system robustness is an indication of the margin of variation of the process characteristics with a fixed controller, then the controller fragility has a similar meaning but in terms of the variation of the controller parameters considering a fixed controlled process.

The fragility of certain controllers was documented by Keel and Battacharyya (1997). They found that many modern design techniques for optimum and robust controllers under the H_2, H_∞ and l_1 norms would produce extremely fragile, high-order controllers. They observed that in some cases, minimum variations of the parameters of these controllers would make the system unstable. A fragility analysis was included in the PID controller design by Datta et al. (2000), Ho (2000) and Silva et al. (2005).

Although, in control system designs, the assumption is often made that the controller can be implemented exactly, a certain degree of uncertainty inevitably exists in the controller implementation. The controller fragility is affected by the tolerances of its analog components. In its digital version, there are inaccuracies because of the use of fixed-length words and rounded errors of numerical calculations (Whidborne, 2000). In addition, the controller must allow variations of its parameters around their design values, making it easy to *fine-tune* the controller when the control loop is placed in service. The latter is the most probable cause of major variations in the controller parameters from their design, or nominal, values. Effectively, most of the tuning approaches, either based on tuning rules or on optimization methods, provide accurate values for the controller parameters, but due to the inaccuracies associated with the controlled process model used as part of the tuning procedure, normally these parameters should be taken only as a first approximation, and such final *fine-tuning* of the controller is normally required.

Considering the above, modern tuning rules for PI and PID controllers must take into account issues such as, the closed-loop servo and regulatory control *performance*, the

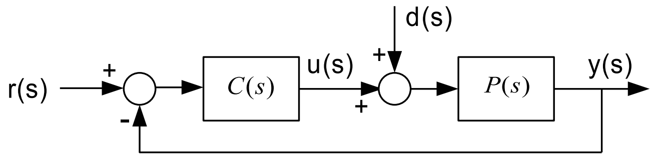

Figure 1. Closed-Loop Control System

control effort requirements, the control system *robustness*, and the controller *fragility*.

If the PID controller design takes into account the closed-loop performance to changes in its inputs, set-point and load-disturbance, and its robustness to changes in the controlled process characteristics, then it is evident that from the designer point of view, it is very important that these characteristics should be preserved regarding this fine-tuning of the controller. In addition, if this is not possible, then there should be at least some sort of information on how such changes in the controller parameters affect the control system robustness and performance.

The fragility-rings plot presented in this publication is based on the PID controller fragility definition introduced by Alfaro (2007) and Alfaro et al. (2009). It provides a graphical interpretation of the *Delta 20 robustness-fragility index* that is a measure of the control system loss of robustness when the controller parameters change.

The rest of the paper is organized as follows: in Section 2 the problem is formulated and the control system robustness and controller fragility indices are presented; the fragility-rings plot is described in Section 3 and its use to analyze the robustness-fragility of PI and PID controllers tuned with several tuning rules is shown in Section 4. The paper end with some conclusions.

2. PROBLEM FORMULATION

Consider the closed-loop control system of Fig. 1, where $P(s)$ and $C(s)$ are the *controlled process model* and the *controller* transfer function respectively. In this system $r(s)$ is the *set-point*, $u(s)$ is the *controller output signal*, $d(s)$ is the *load-disturbance* and $y(s)$ is the *controlled process variable*.

Without the loss of generality, it is supposed that the controller is a Standard PID controller (Visioli, 2006) whose transfer function is as follows:

$$C(s) = K_p \left(1 + \frac{1}{T_i s} + \frac{T_d s}{\alpha T_d s + 1}\right), \qquad (1)$$

where K_p is the controller *proportional gain*, T_i is the *integral time constant*, T_d is the *derivative time constant*. In (1), α is the *derivative filter constant*, usually $\alpha = 0.10$ (Corripio, 2001).

The closed-loop characteristic equation is as follows:

$$1 + L(s) = 1 + C(s)P(s) = 0. \qquad (2)$$

The control system stability depends on the controlled process model $P(s)$, with parameters $\bar{\theta}_p$, and on the controller $C(s)$, with parameters $\bar{\theta}_c = \{K_p, T_i, T_d\}$. The parameters of the controlled process model transfer function will be considered constant for the fragility analysis.

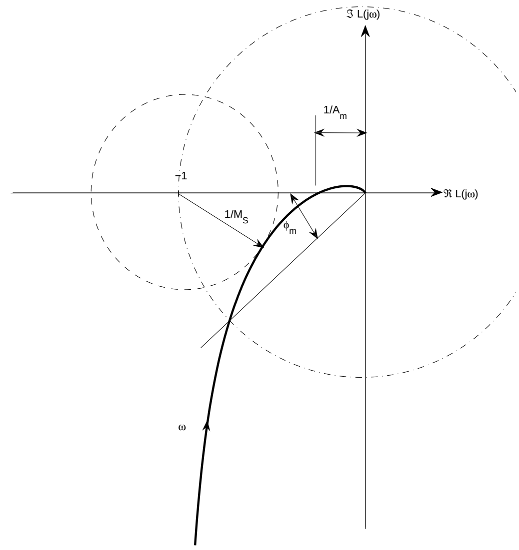

Figure 2. Definition of the Control System Relative Stability Margins

2.1 Control System Robustness Evaluation

There are several quantitative measures of the control system *relative stability* that may be used for the robustness fragility definition, such as the classical *Gain Margin* and *Phase Margin* (A_m, ϕ_m) (Goodwin et al., 2001), that provide an indication of the distance from the open-loop transfer function, $L(\mathrm{j}\omega)$, frequency response, or Nyquist curve, to the critical point (-1,0) on the open-loop polar graph.

Another way to express the system robustness is by using the *Stability Margin*, which is the shortest distance from the Nyquist curve to the critical point (Åström and Murray, 2008). This distance is the reciprocal of the maximum peak of the sensitivity function, or *Maximum Sensitivity* (M_S) (Åström and Hägglund, 1995), defined as follows:

$$M_S \doteq \max_{\omega} |S(\mathrm{j}\omega)| = \max_{\omega} \left| \frac{1}{1 + C(\mathrm{j}\omega)P(\mathrm{j}\omega)} \right|. \qquad (3)$$

The use of the maximum sensitivity as a robustness measure has the advantage that lower bounds to the gain and phase margins can be assured according to the following (Åström and Hägglund, 1995):

$$A_m > \frac{M_S}{M_S - 1}, \quad \phi_m > 2\sin^{-1}\left(\frac{1}{2M_S}\right). \qquad (4)$$

The relations in (4) can be obtained from Fig. 2.

For the controller robustness fragility definitions, we use the maximum sensitivity, M_S, as a measure of the closed-loop control system robustness.

2.2 Delta Epsilon Fragility Indices

The concept of PID controllers fragility, the Delta-Epsilon-Fragility Index $FI_{\Delta\epsilon}$ and their application to define when a controller is considered *fragile*, *non-fragile* or *resilient*, were introduced by Alfaro (2007). In our context, the *PID controller fragility* is an indication of the reduction of the closed-loop control system robustness and/or performance when the controller parameters are perturbed.

For the fragility analysis, the controlled process is represented by a nominal model of the fixed parameters $\bar{\theta}_p^o$, obtained at the control system normal operation point. This model is used for tuning the controller; then, the controller nominal parameters are $\bar{\theta}_c^o$ and their delta epsilon perturbations $\delta\epsilon$. In the following, $\delta\epsilon$ denotes the variation of each individual controller parameter and $\Delta\epsilon$ will be used when all controller parameters are perturbed.

The controller *Delta-Epsilon-Robustness-Fragility Index* relates the control system loss of robustness to its nominal robustness and is given by the following:

$$RFI_{\Delta\epsilon} \doteq \frac{M_{S\Delta\epsilon}^m}{M_S^o} - 1 = \frac{\max\{M_S((1 \pm \delta\epsilon)\bar{\theta}_c^o)\}}{M_S(\bar{\theta}_c^o)} - 1, \quad (5)$$

where $M_{S\Delta\epsilon}^m$ and M_S^o are the control system extreme and the nominal maximum sensitivity, respectively.

The *extreme maximum sensitivity*, $M_{S\Delta\epsilon}^m$, represents the highest loss of robustness of the control system when all the parameters of the controller, $\bar{\theta}_c$, have been perturbed by the same $\delta\epsilon$ amount from their nominal values, $\bar{\theta}_c^o$, considering all the possible combinations of the perturbed parameters.

In the ideal case, for a completely delta epsilon robustness-resilient (or absolutely robustness-non-fragile) controller, $RFI_{\Delta\epsilon} = 0$, the controller would not lose robustness when its nominal parameters, $\bar{\theta}_c^o$, are perturbed by $\delta\epsilon$.

The relative influence of a $\delta\epsilon$ change in the controller parameter p_i over its robustness fragility can be obtained with the *Parametric-Delta-Epsilon-Robustness-Fragility Index* given by the following:

$$RFI_{\delta\epsilon}^{p_i} \doteq \frac{M_{S\delta\epsilon}^{p_i}}{M_S^o} - 1 = \frac{\max\{M_S((1 \pm \delta\epsilon)p_i, \bar{\theta}_c^o)\}}{M_S(\bar{\theta}_c^o)} - 1. \quad (6)$$

The final *fine-tuning* of the control-loop is considered the most probable cause of major variations in the controller parameters, for example, in practice, it is possible to see commissioning changes up to 10% or 20% in their values. Considering this, the *Delta 20 Robustness-Fragility Index* can be defined to measure the maximum loss of the control system robustness when a change of up to 20% occurs in one or more of the nominal controller parameters values and is given by the following:

$$RFI_{\Delta 20} \doteq \frac{M_{S\Delta 20}^m}{M_S^o} - 1. \quad (7)$$

A controller is considered *robustness-fragile* if the control system loses more than 50% of its robustness when all its parameters change up to 20%; otherwise, it is *robustness-non-fragile*. In addition, a controller is *robustness-resilient* if the control system does not lose more than 10% of its robustness when its parameters change up to 20%. A controller with a low robustness-fragility will allow final fine-tuning without a significant reduction in the control system robustness. Therefore, based on the $RFI_{\Delta 20}$, the controller robustness fragility degree is defined as follows:

- *Robustness Fragile PID controller*: a PID controller is robustness-fragile if its delta 20 robustness fragility index is higher than 0.50, $RFI_{\Delta 20} > 0.50$.
- *Robustness Non-Fragile PID controller*: a PID controller is robustness-non-fragile if its delta 20 robustness fragility index is less than or equal to 0.50, $RFI_{\Delta 20} \leq 0.50$.
- *Robustness Resilient PID controller*: a PID controller is robustness-resilient if its delta 20 robustness fragility index is less than or equal to 0.10, $RFI_{\Delta 20} \leq 0.10$.

The selection of a ±20% (Δ20) change in the controller parameters for the robustness fragility definition above considers a 10% reduction in the control system robustness as marginal and a 50% reduction as the maximum allowed limit. Such maximum variation will turn a highly robust system, with M_S lower than 1.4, into one with a minimally acceptable robustness, M_S of approximately 2.0. However, using (5) and (6) it is possible to evaluate the effect of any other particular $\delta\epsilon$ perturbation in one or more controller parameters.

3. ROBUSTNESS-FRAGILITY RINGS PLOT

We will present here the *Delta 20 Robustness-Fragility Rings Plot*, which is a simple tool that uses the open-loop transfer function, $L(\mathrm{j}\omega)$, Nyquist curve of the nominal and Delta 20 perturbed controllers to provide an indication of the control system robustness-fragility. It shows the areas in the $L(\mathrm{j}\omega)$ plane that define when the controller is a robustness-resilient controller (RRC), a robustness-non-fragile controller (RNFC), or a robustness-fragile controller (RFC) as shown in Figure 3.

The plot includes the Nyquist curve of the nominal open-loop transfer function that defines the control system *nominal robustness level*, M_S^o, and states the robustness-fragility rings as follows:

- RRC Ring: $M_S^o \leq M_S \leq 1.1M_S^o$,
- RNFC Ring: $1.1M_S^o < M_S \leq 1.5M_S^o$,
- RFC Area: $1.5M_S^o < M_S$.

It also includes the Nyquist curve corresponding to the *extreme maximum sensitivity*, M_S^m, that gives the delta 20 robustness-fragility index, $RFI_{\Delta 20}$.

For example, the requirement for a robustness-non-fragile controller demands that the control system Nyquist curve does not enter the RFC area when all controller parameters are changed up to ±20%.

4. PI AND PID CONTROLLERS TUNING ROBUSTNESS-FRAGILITY EVALUATION

In the following the robustness-fragility of PI and PID controllers tuned with several well-know tuning rules will be analyzed using the fragility-rings plot.

Consider the following normalized first- and second-order plus dead-time controlled process models:

$$P_1(s) = \frac{\mathrm{e}^{-0.3s}}{s+1}, \; \tau_o = 0.3 \quad (8)$$

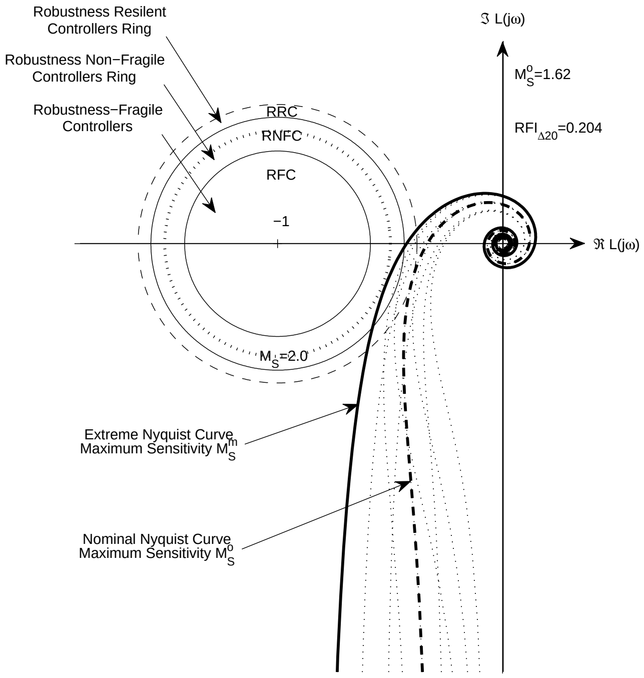

Figure 3. Controller Delta 20 Robustness-Fragility Rings Plot

Table 1. Controllers Parameters

Method	Model	K_p	T_i	T_d
SIMC	P_1	1.667	1	0
SIMC	P_2	0.313	1	0
PO_S	P_3	2.430	1.800	0.444
PO_T	P_3	3.450	1.800	0.444
PI_{2Ms} $(M_S = 2.0)$	P_1	2.279	0.905	0
PI_{2Ms} $(M_S = 1.6)$	P_1	1.707	0.982	0
PI_{2Ms} $(M_S = 1.4)$	P_1	1.270	1.054	0
PI_{2Ms} $(M_S = 2.0)$	P_2	0.665	1.748	0
PI_{2Ms} $(M_S = 1.6)$	P_2	0.467	1.591	0
PI_{2Ms} $(M_S = 1.4)$	P_2	0.308	1.312	0

$$P_2(s) = \frac{\mathrm{e}^{-1.6s}}{s+1},\ \tau_o = 1.6 \tag{9}$$

$$P_3(s) = \frac{\mathrm{e}^{-0,3s}}{(s+1)(0.8s+1)},\ a = 0.8,\ \tau_o = 0.3 \tag{10}$$

where a and τ_o are the model time constants ratio and normalized dead-time, respectively.

Tuning rules to analyze include: *Simple Control* (SIMC) (Skogestad, 2003), *Percent Overshoot* (PO) (Ali and Majhi, 2009) and *Maximum Sensitivity-Based Robust Tuning* (PI_{2Ms}) (Alfaro et al., 2010). The controllers parameters obtained with these rules are listed in Table 1.

4.1 Simple Control (SIMC)

The PI SIMC tuning for FOPDT models produces control systems with a robustness $M_S = 1.59$ for all model normalized dead-times as shown in Figs. 4 and 5. The fragility rings plots show that both SIMC controllers are robustness non-fragile and that their robustness-fragility

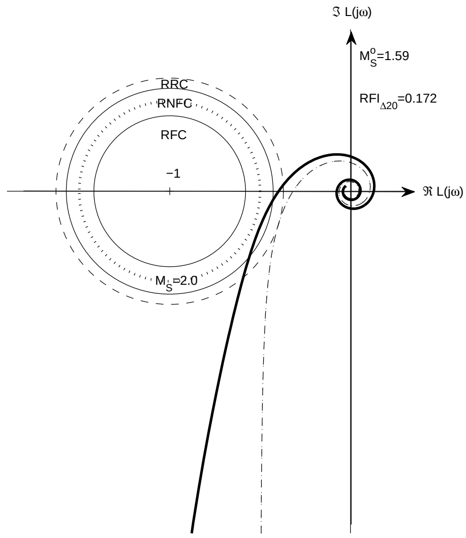

Figure 4. P_1 SIMC PI Robustness-Fragility

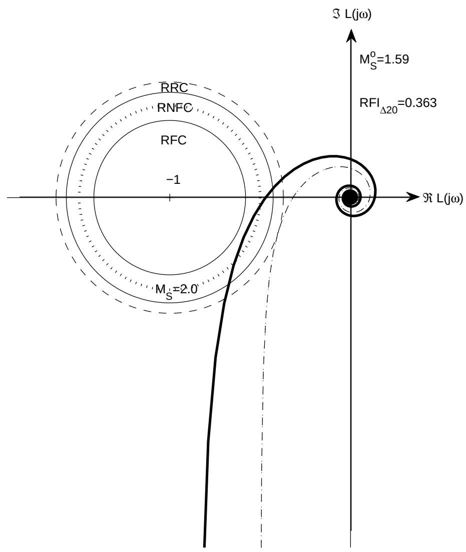

Figure 5. P_2 SIMC PI Robustness-Fragility

is affected by τ_o. For the P_1 model ($\tau_o = 0.3$) $RFI_{\Delta 20} = 0.172$ while for the P_2 model ($\tau_o = 1.6$) $RFI_{\Delta 20} = 0.363$. The controllers turn to more robustness-fragile as the model normalized dead-time increases requiring a more careful final *fine-tuning*.

4.2 Percent Overshoot (PO)

The PO method includes two design criteria for PI (FOPDT) and PID (SOPDT) controllers. A *smooth control* design (PO_S) for 0% OS ($M_S = 1.38$) and a *tight control* (PO_T) design for 10% OS ($M_S = 1.71$). As shown in Figs. 6 and 7 the target robustness are not obtained with the PID controllers. The main reason for this is that the PO method was obtained using an "ideal" non-proper

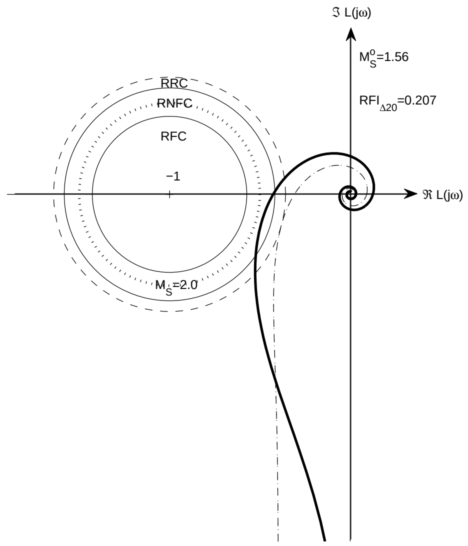

Figure 6. P_3 PO_S PID Robustness-Fragility

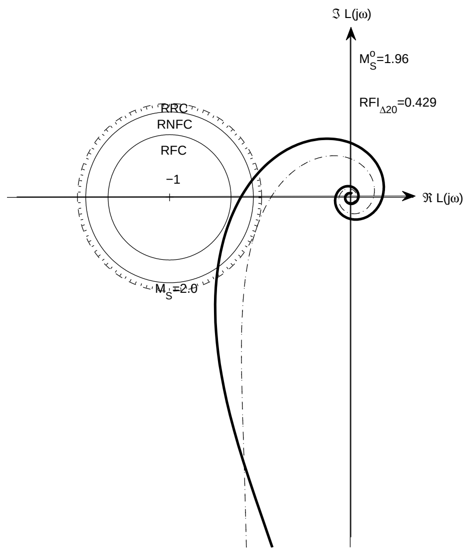

Figure 7. P_3 PO_T PID Robustness-Fragility

PID controller while we tested it using a "real" Standard PID controllers (with derivative filter). Both controllers are robustness-non-fragile but the PO_T controller is less robust and more fragile.

4.3 Maximum Sensitivity-Based Robust Tuning (PI_{2Ms})

The PI_{2Ms} uses a closed-loop model-reference design with a robustness constraint, $M_S \in \{1.4, 1.6, 1.8, 2.0\}$. In Figs. 8 and 9 two extreme cases are shown: a high robustness design ($M_S = 1.4$) for a process with low normalized dead-time (P_1) and a minimum robustness design ($M_S = 2.0$) for a process with high normalized dead-time (P_2).

In the first case $M_S^o = 1.4$ and the controller is nearly robustness-resilient with $RFI_{\Delta 20} = 0.116$. The control

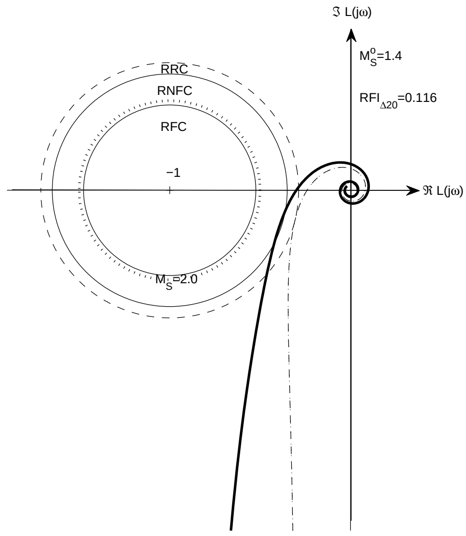

Figure 8. P_1 $M_S^t = 1.4$ PI_{2Ms} PI Robustness-Fragility

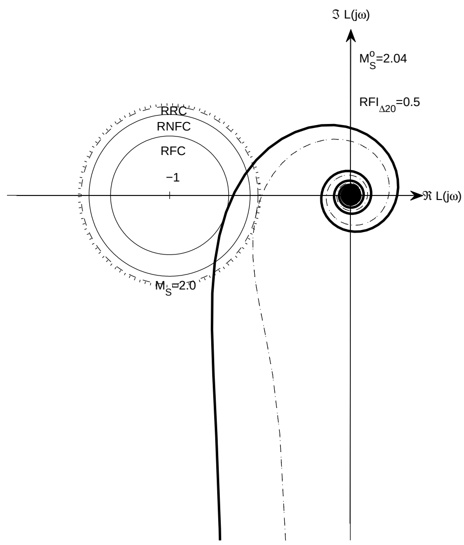

Figure 9. P_2 $M_S^t = 2.0$ PI_{2Ms} PI Robustness-Fragility

system is robust ($M_S < 2$) even in case of a change of up to $\pm 20\%$ in the controller parameters. In the second $M_S^o = 2.04$ and the controller is at the border of the robustness-fragile controllers area ($RFI_{\Delta 20} = 0.5$). The control system may turn non-robust if the controller parameters are perturbed.

5. CONCLUSIONS

Based on the Robustness-Fragility Indices the *Delta 20 Fragility-Rings* plot provides information of the controller robustness-fragility in case its parameters are changed as in the controller final fine-tuning. It is a valuable tool for control system design, which allows anticipating and quantifying the possible loss of robustness.

The delta 20 fragility-rings plot shows that even a tuning rule may provide control systems with same robustness for models with a wide range of normalized dead-times, τ_o, the controller robustness-fragility depends on τ_o turning more fragile as the model normalized dead-time increases.

The controller robustness-fragility depends not only on the model dead-time but also on the control system nominal robustness. Then, the selection of the design robustness level must consider, in addition to the expected changes in the controlled process dynamics, the controller robustness-fragility for the tuning rule used.

The PI and PID controller design should take into account not only the existing *performance/robustness* trade-off but also the controller *robustness-fragility*.

As an extension of the robustness-fragility-rings an analysis of the controller fragility in the frequency domain will provide an in-depth knowledge of its implication on the control system performance and robustness.

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