# A Design of Nonlinear PID Controller with Neural net based FRIT

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Abstract: Some design schemes of model-free controllers which do not require any system models have been considered in the last decade. FRIT(Fictitious Reference Iterative Tuning) method that directly computes the control parameters from the operating data have been proposed as the one of model-free controllers. FRIT has some useful practical features. One is that it does not require system identification. Another is that the control parameters can be directly computed using only a set of closed loop input/output data and the desired output signal. The calculations of the control parameters needs the optimization of the cost functions. The ordinary approach is the gradient method. However, this calculations derives only linear parameters. Therefore, the applications of FRIT are limited for linear systems. In this paper, a new approach to the discrete FRIT-based nonlinear PID control is proposed. The neural network is utilized for the optimization of FRIT. PID parameters are adequately adjusted corresponding to the nonlinear properties. The conventional schemes by using the neural networks require the information of system Jacobian to update weighting factors. This proposed method can calculate the control parameters without the information of system Jacobian or system parameters except for the information about the time-delay.

Keywords: PID Control, process control, FRIT, neural networks

# 1. INTRODUCTION

PID control schemes based on the classical control theory, have been widely used for various industrial control systems for a long time[Ziegler and Nichols(1942)],[Chien, Hrons and Reswick(1972)]. The reasons are that PID controllers have simple control structures, and simple maintain and tune. However, since such processes have nonlinear properties and uncertainties caused by modeling errors and process fluctuation, it is difficult to determine 'optimal' PID parameters. Some PID control schemes have been proposed based on the self-tuning control algorithm for the uncertain systems. However, as many self-tuning PID control algorithm calculate their PID parameters based on the estimates by least squares method, it is difficult to employ for the systems which can be used the least squares method. In the real industrial control systems, the linear mathematical model is used to calculate the PID parameters. The parameters of mathematical model are made variable, and these are assigned by using the reference table generated by the priori information. This procedure can clear on the system properties, and can employ the linear control theory for the nonlinear systems. However, it is very difficult to generate the reference table. The generation of this table is required cat-and-try for each processes. Some control schemes for nonlinear systems by using the neural network have been proposed[Haykin(1994)],[Narendra and

Parthasarathy(1990)],[Chen and Chang(1996)]. The conventional neural-net based control schemes can be classified into the two groups. The one is that control input is directly calculated by the neural network. As the control input is given by the output of the neural network, it is easy to employ for the controlled object. However, it is difficult to express the structure of the controller as the transfer function. The other is that control parameters are calculated by the neural network, and this control input can be calculated by these control parameters. Although this scheme makes easy to grasp the physical meanings of control parameters, the properties of controlled object cannot be directly understood. Furthermore, these schemes require the information of system Jacobian to update weighting factors of neural networks. It is difficult to obtain the system Jacobian of such the nonlinear systems.

On the other hand, some design schemes of model-free controllers which do not require any system models have been considered in the last decade. FRIT(Fictitious Reference Iterative Tuning) method that directly computes the control parameters from the operating data have been proposed as the one of model-free controllers[Kaneko, *et al*(2005)],[Soma, *et al*(2004)], [Suehiro, *et al*(2009)]. In particular, the FRIT method is attractive to industry because it is possible to obtain the optimal controller parameters without any system identification. The calculations of the control parameters needs the optimization of the cost

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functions. The ordinary approach is the gradient method. However, this calculations derives only linear parameters. Therefore, the applications of FRIT are limited for linear systems.

In this paper, a new approach to the discrete FRIT-based nonlinear PID control is proposed. The neural network is utilized for the optimization of FRIT. According to the proposed FRIT, PID parameters are adequately adjusted corresponding to the nonlinear properties. This paper is organized as follows. The design scheme of PID controller is explained. Next, the FRIT method is considered. This optimization of PID parameters are calculated by neural networks.

#### 2. CONTROLLER DESIGN

#### 2.1 Problem description

The controlled object to be considered in this paper is following equation:

$$y(k) = z^{-(d+1)}G(z^{-1})u(k),$$
(1)

where, u(t) and y(t) denote the control input signal and the corresponding output signal.  $G(z^{-1})$  denotes the transfer function of the controlled object, which is nonlinear, time-invariant. d is the time-delay which is known.

The following velocity-type PID controller is employed for the controlled object given by eqn.(1):

$$\Delta u(t) = k_c \{ \Delta + \frac{T_s}{T_I} + \frac{T_D}{T_s} \Delta^2 \} e(t), \qquad (2)$$

where  $k_c, T_I$  and  $T_D$  are the proportional gain, the reset time and the derivative time, respectively. And,  $T_s$  denotes the sampling interval. e(t) denotes the control error signal given by

$$e(t) := r(t) - y(t) \tag{3}$$

where, r(k) is the reference signal given as the piecewise constants. These PID parameters in eqn.(2) are strongly depend on the control performance. The design method is considered based on the FRIT method by following procedure.

Next, (2) is rewritten as:

$$C(z^{-1})y(t) + \Delta u(t) - C(z^{-1})r(t) = 0$$
(4)

where

$$C(z^{-1}) = c_0 + c_1 z^{-1} + c_2 z^{-2}$$
  
=  $k_c (1 + \frac{T_s}{T_I} + \frac{T_D}{T_s}) - k_c (1 + \frac{2T_D}{T_s}) z^{-1}$   
+  $\frac{k_c T_D}{T_s} z^{-2}$  (5)

In addition, r(k) is the reference signal with piecewise constants. Here, as the PID control has done by using  $C_0(z-1)$  which is given by the initial PID gains, the following input/output operating data can be obtained:

$$\Delta u_0(k) = C_0(z^{-1})\{r(k) - y_0(k)\} = 0$$
(6)

$$y_0(k) = z^{-d+1} G(z^{-1}) u_0(k), \tag{7}$$

where

$$C_{0}(z^{-1}) = c0_{0} + c0_{1}z^{-1} + c0_{2}z^{-2}$$
  
=  $k_{c0}(1 + \frac{T_{s}}{T_{I0}} + \frac{T_{D0}}{T_{s}}) - k_{c0}(1 + \frac{2T_{D0}}{T_{s}})z^{-1}$   
+  $\frac{k_{c0}T_{D0}}{T_{s}}z^{-2}$  (8)

Next, the desired reference output is given by the following equation:

$$y_m(k) = G_m(z^{-1})r(k),$$
 (9)

where,

$$G_m(z^{-1}) = \frac{z^{-(d+1)}P(1)}{P(z^{-1})},$$
(10)

It is important to design of the user-specified polynomial  $P(z^{-1})$ , because  $P(z^{-1})$  mean the ideal characteristic polynomial of the closed-loop system.

So, the design of  $P(z^{-1})$  is based on the following two criteria, when the reference signal is given by piecewise constant.

- Rise-time.
- Damping oscillation performance.

Here,  $P(z^{-1})$  is defined second-order as follows:

$$P(z^{-1}) = 1 + p_1 z^{-1} + p_2 z^{-2}$$
(11)

where coefficients of  $P(z^{-1})$  are calculated by following equations:

$$p_1 = -2e^{-\frac{\rho}{2\mu}}\cos(\frac{\sqrt{4\mu - 1}}{2\mu}\rho)$$
(12)

$$p_2 = e^{-\frac{\rho}{\mu}} \tag{13}$$

$$\rho = \frac{T_s}{\sigma} \tag{14}$$

$$\mu = 0.25(1 - \delta) + 0.51\delta \tag{15}$$

In these equations,  $\sigma$  is the parameter which is equivalent to the rise-time, and it is desirable to design  $\sigma$  as about 1/3 thru 1/2 times of the time constant based on *priori* information of controlled object. And,  $\delta$  is the coefficient that is related the damping oscillation performance. Practically, it is desirable to set  $0.0 \leq \delta \leq 2.0$ .

The optimal controller  $C^*(z^{-1})$  is designed so that the output signal y(k) tracks the desired reference model output  $y_m(k)$ . It is then necessary that the system parameters are identified by using the least squares method. However, in order to obtain true system parameters input excitation is required. In other words, the desired system parameters cannot be computed by using the closed-loop data. Due to this problem, the FRIT method is becoming increasingly attractive, because the control parameters can be directly computed using the operating data. The discrete-time FRIT is discussed in the following section in which a genetic algorithm is introduced in optimizing the cost function.

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Fig. 1. Schematic figure of a neural network. 2.2 FRIT

So that the output signal tracks the desired reference model output, let the optimal controller be  $C^*(z-1)$ . In addition, the fictitious reference  $\tilde{r}(k)$  is given by

$$\tilde{r}(k) = C^{*-1}(z^{-1})u_0(k) + y_0(k).$$
(16)

Then, the FRIT based on optimizing the following cost function:

$$E_n = \frac{1}{2} \{ y_0(k) - \tilde{y}(k) \}^2$$
(17)

 $\tilde{y}(k)$  is given by

$$\tilde{y}(k) = G_m(z^{-1})\tilde{r}(k), \qquad (18)$$

The discrete FRIT is summarized as follows:

- The initial controller  $C_0(z^{-1})$  is designed.
- Desired reference model  $G_m(z^{-1})$  is also designed.
- Only one-shot experiment data using the controller  $C_0(z^{-1})$  is gathered.
- The fictitious reference signal  $\tilde{r}(k)$  is generated.
- PID gains are computed so that the cost function  $E_n$  is minimized.

## 2.3 Optimization by neural network

In this paper, the neural network as shown in Fig.1 is constructed for the parameter estimation. Note that every unit included in the output layer of the neural network corresponds to the estimated parameter.

In Fig.1, circle at the hidden and the output layers mean the neuron shown in Fig.2.

In Fig.2,  $W_{ji}$ ,  $net_j$  and  $O_j$  mean the weighing factor between the neuron i and the neuron j, the summation of the input to the neuron j and the output from the neuron j, respectively. And  $f(\cdot)$  is the sigmoidal function used by

$$f(x) := \frac{1}{1 + e^{-ax}}.$$
(19)

Since it is assumed that the absolute values of the system parameters are under 1.0, the sigmoidal function described as eqn.(19) can be adopted. The output range of the sigmoidal function (19) should be tuned in proportion to the absolute value in the case where the absolute value in order 1.0.

The weighting factors included in the neural network are updated based on the back-propagation method. The cost function of the learning is cost function of FRIT (17). whose cost function  $E_n$  is given by:

The update rule of  $W_{kj}$  can be derived by the following procedure. First, differentiating  $E_n$  with  $W_{kj}$  yields

$$\frac{\partial E_n}{\partial W_{kj}} = \frac{\partial E_n}{\partial \tilde{y}} \frac{\partial \tilde{y}}{\partial \tilde{r}} \frac{\partial \tilde{r}}{\partial O_k} \frac{\partial O_k}{\partial net_k} \frac{\partial net_k}{\partial W_{kj}} = -\delta_k O_j.$$
(20)

Fig. 2. Mathematical model of a neuron.

Furthermore,  $\delta_k$  is given by

$$\delta_k := -\frac{\partial E_n}{\partial net_k}.$$
(21)

The each terms are obtained as:

$$\frac{\partial E_n}{\partial \tilde{y}} = y_0 - \tilde{y} \tag{22}$$

$$\frac{\partial y}{\partial \tilde{r}} = (1 + p_1 + p_2) \tag{23}$$

$$\frac{\partial O_k}{\partial net_k} = O_k (1 - O_k) \tag{24}$$

$$\frac{\partial net_k}{\partial W_{kj}} = O_j \tag{25}$$

Moreover,

$$\frac{\partial \tilde{r}}{\partial O_{k(1)}} = \frac{1}{c0_0^2} \{ c0_1 y_0(k-1) + c0_2 y_0(k-2) \\ -c0_1 \tilde{r}(k-1) - c0_2 \tilde{r}(k-2) - u_0(k) - u_0(k-1) \}$$
(26)

$$\frac{\partial \tilde{r}}{\partial O_{k(2)}} = \frac{1}{c0_0} \{ y_0(k-1) - \tilde{r}(k-1) \}$$
(27)

$$\frac{\partial \tilde{r}}{\partial O_{k(3)}} = \frac{1}{c0_0} \{ y_0(k-2) - \tilde{r}(k-2) \}$$
(28)

On the other hand, the update rule of  $W_{ji}$  can be derived as follows. Differentiating  $E_n$  with  $W_{ji}$  yields

$$\frac{\partial E_n}{\partial W_{ji}} = \frac{\partial E_n}{\partial net_j} \frac{\partial net_j}{\partial W_{ji}} = -\delta_j O_i, \tag{29}$$

where

$$\delta_j := -\frac{\partial E_n}{\partial net_j} = \sum_k \delta_k W_{kj} O_j (1 - O_j).$$
(30)

Thus, update rules of  $W_{kj}$  and  $W_{ji}$  are given by

$$W_{kj}(t+1) = W_{kj}(t) + \eta \delta_k O_k + \alpha \Delta W_{kj}(t)$$
(31)

$$W_{ji}(t+1) = W_{ji}(t) + \eta \delta_j O_j + \alpha \Delta W_{ji}(t)$$
(32)

Furthermore,  $\eta$  and  $\alpha$  denote the learning rate and the momentum rate, respectively.

The optimization of the FRIT can be realized via the above procedure. Note that according to the proposed scheme, a priori information about the system Jacobean is not necessary.

This algorithm of proposed scheme is summarized below.

[Proposed PID control algorithm]

- **1.** Design the initial controller  $C_0(z^{-1})$ .
- **2.** Obtain the input-output data from the initial controller  $C_0(z^{-1})$ .
- **3.** Design the desired model  $G_m(z^{-1})$ .
- 4. Generate the fictitious reference signal  $\tilde{r}$ .
- 5. Minimize the cost function by using the neural network.
- 6. Obtain the PID parameters as the output of the neural network.



Fig. 3. The block diagram of the closed loop system



Fig. 4. Initial and learned data.

- 7. Calculate u(t) from eqn.(2).
- 8. Update and return to 6..

The block diagram of the closed loop system is shown as Fig.3. The neural network of Fig.3 is used as the off-line manner.

#### **3. SIMULATION EXAMPLE**

This section demonstrates the effectiveness of the proposed scheme by a simulation example.

The following system was used as the controlled object:

$$y(k) = 0.6y(k-1) - 0.1y(k-2) + 1.2x(k-1) + 0.1x(k-2) + \frac{\xi(k)}{\Delta}$$
$$x(k) = 1.5u(k) - 1.5u^{2}(k) - 0.5u^{3}(k)$$
(33)

The initial PID parameters are set as  $k_c = 0.8, T_I = 10.0, T_d = 0.1$ . The initial input and output data were obtained by this parameters. Fig. 4 shows the initial output and learned data by neural network. The solid line and the dotted line mean the initial data  $y_0$  and  $\tilde{y}$ , respectively.

Next, Fig.4 shows the control result of the proposed scheme. And, The trajectries of PID parameters are shown as Fig.5. The good results can be obtained because the PID parameters are tuned corresponding to the nonlinearity of the controlled object.



Fig. 5. Control results by proposed method.



Fig. 6. Trajectories of PID patrameters.

# 4. CONCLUSION

This paper proposes a new approach to the discrete FRIT-based nonlinear PID control. The neural network is utilized for the optimization of FRIT. The conventional schemes by using the neural networks require the information of system Jacobian to update weighting factors. This proposed method can calculate the control parameters without the information of system Jacobian or system parameters except for the informations about the timedelay.

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