Tuning rules for a reset PI compensator with variable reset

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Abstract: The PI+CI compensator is a simple reset compensator, its base system is a PI, which has been shown to be effective in a number of practical applications. One fundamental parameter to be tuned is the reset ratio; for lag dominant systems or systems with integrators it is known to give good results overcoming PI compensation. In this work, a systematic method for the tuning of a variable reset ratio (variable at the reset instants) is developed for first and second order plants.

Keywords: hybrid control systems, reset control systems, PID tuning rules, PI+CI compensator

1. INTRODUCTION

Reset control systems were started to be developed fifty years ago with the founding work of Clegg (Clegg (1958)), that introduced a nonlinear integrator based on a reset action. Basically, since the integrator output is set to zero when its input is zero, a faster system response without excessive overshoot may be expected, thus avoiding limitation of its LTI counterpart. The seminal works (Krishnan and Horowitz (1974); Horowitz and Rosenbaum (1975)) developed for the first time control synthesis methods for reset compensator based on the Clegg integrator (CI) and the First Order Reset Element (FORE). More recently (see the monograph (Baños and Barreiro (2012))), reset control systems have started to be an attractive approach to improve stability and performance of linear and time invariant (LTI) compensators.

This work is focused on a specific reset compensator, referred to as the PI+CI compensator (Baños and Vidal (2011)). PI+CI is a simple modification of a PI compensator, which includes a Clegg integrator (CI) in parallel. It has been shown that PI+CI compensation gets better performance indices than PI compensation in some specific cases: in particular, in lag dominant systems and systems with integrators (Baños and Barreiro (2012)). In general, a limitation of reset compensation, and also of the PI+CI, is the appearance of undesirable undershoots that limits its performance in control practice. Several modifications of the PI+CI has been already considered in Baños and Vidal (2011), including a variable reset ratio to improve the setpoint traking, specifically reducing the undershoot of the response step. In this case, PI+CI parameters like the reset ratio is tuning following an heuristic method based on extensive simulation.

In this work, the goal is to obtain a systematic method for PI+CI tuning, including the case of variable reset. By simplicity, basic plants including first order and second order systems (with and without integrators) will be considered. More general plants, including plants with time delays will be treated elsewhere.

The outline of this paper is as follows. The PI+CI compensator and the resulting closed loop system are introduced in Section 2. In Section 3, a description of a reset system as a set of LTI systems is given; this representation allows the analysis of the time response of an nonlinear/hybrid system by using standard techniques like the root locus. Finally, in Section 4, tuning rules are devoloped for first order plants; and in Section 5, for second order plants.

2. PRELIMINARIES

2.1 The PI+CI controller

The PI+CI compensator is simply a parallel connection of a PI compensator and a Clegg integrator (Fig. 1). The main motivation of this setup is to overcome the performance/robustness properties of a PI compensator without increasing the *cost of feedback*. By cost of feedback (Horowitz (1993)) it is meant the increasing on the sensitivity of a control system with respect to the sensor noise, which is specially important in designs with derivative terms in the compensator.

As a result, the PI+CI compensator will have three terms as shown in the blocks diagram structure of Fig. 1: k_p and τ_I are the proportional gain and the integral time constant, and p_r is the reset ratio that represents the part of the integral term over which the reset action is applied. Note that for $p_r = 0$ a PI compensator is obtained (it will be referred to as *PI base compensator*), and that for $p_r = 1$ the result is a full reset P+CI compensator. In general, the reset is not applied on the whole of the integral term, because the fundamental asymptotic property of the integral term would be lost, for example the steady-state error of the step response would not disappear for plants with no integrators.

In the state-space, the PI+CI compensator can be expressed by a two dimensional state $\mathbf{x}_r = (x_i, x_{ci})^\top$: x_i is

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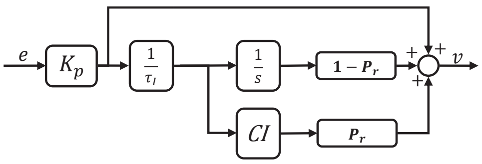

Fig. 1. PI+CI controller structure

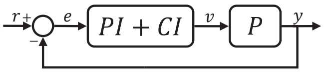

Fig. 2. PI+CI controller applied to a LTI plant P

the integral term state, and x_{ci} corresponds to the Clegg integrator term. A state-space realization of the PI+CI compensator with state $\mathbf{x}_r = (x_i, x_{ci})^\top$ is given by the impulsive differential equation:

$$PI + CI : \begin{cases} \dot{\mathbf{x}}_r(t) = A_r \mathbf{x}_r(t) + B_r e(t), & e(t) \neq 0 \\ \mathbf{x}_r(t^+) = A_\rho \mathbf{x}_r(t), & e(t) = 0 \\ v(t) = C_r \mathbf{x}_r(t) + D_r e(t) \end{cases} \quad (1)$$

where $x_r(t^+)$ or x_r^+ is the value $x_r(t+\epsilon)$ with $\epsilon \to 0^+$, and the matrices A_r, B_r, C_r, D_r and A_ρ are given by:

$$A_r = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad B_r = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad C_r = \frac{k_p}{\tau_i} (1 - p_r \quad p_r)$$

$$D_r = k_p \quad A_\rho = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

2.2 Reset control system

Consider the feedback control system given by Fig. 2, where the plant P is a finite dimensional linear and time invariant system described by:

$$P : \begin{cases} \dot{\mathbf{x}}(t) = A_p \mathbf{x}_p(t) + B_p v(t) \\ y(t) = C_p \mathbf{x}_p(t) \end{cases}$$

and the PI+CI controller is given by (1). The closed loop state is $\mathbf{x} = \left(\mathbf{x}_p^\top, \mathbf{x}_r^\top\right)^\top$, with dimension $n = n_p + 2$ (here n_p is the dimension of the state x_p). Now, a state-space realization of the reset control system is:

$$\begin{cases} \dot{\mathbf{x}}(t) = A\mathbf{x}(t) + Br(t), & \mathbf{x}(t) \notin \mathcal{M} \\ \mathbf{x}(t^+) = A_R \mathbf{x}(t), & \mathbf{x}(t) \in \mathcal{M} \\ y(t) = C\mathbf{x}(t) \end{cases} \quad (2)$$

where

$$A = \begin{pmatrix} A_p - B_p D_r C_p & B_p C_r \\ -B_r C_p & A_r \end{pmatrix} \quad (3)$$

$$B = \begin{pmatrix} B_p D_r \\ B_r \end{pmatrix} \quad (4)$$

$$C = (\, C_p \;\; 0 \,) \quad (5)$$

$$A_R = diag\left(I_{n_p}, A_\rho\right) \quad (6)$$

and the *reset surface* $\mathcal{M}$ is given by

$$\mathcal{M} \triangleq \{x(t) \in \mathbb{R}^n \;:\; Cx(t) = r(t)\}$$

In general, the CI state is reset to zero at the crossing *reset instants* t_i, $i = 1, 2, \cdots$, given by $\mathbf{x}(t_i) \in \mathcal{M}$, that will depend both on the initial condition and the reference. It will be assumed that the *reset instants* are all well-posed in the sense that they are well-defined and are distinct, and thus the reset control system is well-posed (Baños and Barreiro (2012)). Note that it may exist a finite or a infinite number of reset instants for a given pair of initial condition and reference. Since in this work the focus is on the tracking problem, a zero initial condition will be assumed.

3. CONTROL RESET SYSTEM ANALYSIS

The basic idea of this work is that the response of a reset control system to a reference input can be given as a sequence of impulse responses of a family of LTI systems. In this way, standard design tools, for example root locus, can be adapted to design reset control systems. Note that the approach is exact, no approximation is performed in any analysis/design step.

Firstly, the solution of the impulsive differential equation (2) in the interval $(t_k, t_{k+1}]$, $k = 0, 1, \cdots$ ($t_0 = 0$ by definition) is simply given by the LTI base system solution:

$$\mathbf{x}(t) = e^{A(t-t_k)} \mathbf{x}(t_k^+) + \int_{t_k}^{t} e^{A(t-\tau)} Bu(\tau) d\tau$$

and at the reset instants $\mathbf{x}(t_k^+) = A_R \mathbf{x}(t_k)$.

3.1 Impulse response

Specifically, if the input $u(t)$ is an impulse $\delta(t)$, for zero initial conditions the state $\mathbf{x}(t)$ at the time interval $t \in [0, t_1]$ is given by:

$$x(t) = e^{At} B$$

and

$$x(t) = e^{A(t-t_k)} x(t_k^+)$$

for $t \in (t_k, t_{k+1}]$ with $k \geq 1$.

A key point is that the state time evolution at $t \in (t_k, t_{k+1}]$, with $k \geq 0$, can be considered like a *time-shifted* state impulse response of an auxiliary linear system H_k:

$$H_k : \begin{cases} \dot{x}(t) = Ax(t) + B_k \delta(t) \\ y(t) = Cx(t) \end{cases}$$

where A and C are given by (3) and (5) respectively, and

$$B_k = \begin{cases} B, & k = 0 \\ x(t_k^+), & k > 0 \end{cases}$$

In addition, the systems H_k can be expressed by the transfer function

$$\begin{aligned} H_k(s) &= C(sI - A)^{-1} B_k \\ &= \frac{G_k (s + z_{k_1})(s + z_{k_2}) \cdots (s + z_{k_{n_z}})}{(s + p_1)(s + p_2) \cdots (s + p_{n-1})} \end{aligned} \quad (7)$$

here the poles p_i, $i = 1, \cdots, n - 1$ are the same as base system poles. Note that the realization (2) is not

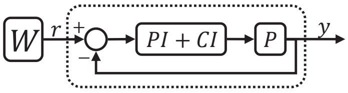

Fig. 3. Autonomous system

minimal and thus a pole/zero cancellation happens when the transfer function (7) is computed.

On the other hand, assuming (without loss of generality) that the observer canonical form is used for the plant P, the matrix B_k for $k > 0$ is given by:

$$B_k = \begin{pmatrix} x_{p_1}(t_k) \\ \vdots \\ x_{p_{n_p-1}}(t_k) \\ 0 \\ x_i(t_k) \\ 0 \end{pmatrix}$$

3.2 More general reference inputs

The above approach can be generalized to more general references inputs by using a exogenous system (exosystem) whose impulse response corresponds to the given reference (see Fig. 3). The exogenous system is given by the state-space representation:

$$W : \begin{cases} \dot{\mathbf{w}}(t) = A_w \mathbf{w}(t), \ \mathbf{w}(0) = \mathbf{w}_0 \\ r(t) = C_w \mathbf{w}(t) \end{cases}$$

where $\mathbf{w}_0$ is the exogenous system initial state. Note that signals like steps, ramps, sinusoids, $\cdots$ can be produced in this way (in general any Bohl function can be synthesized (Trentelman et al. (2001))). The output of the autonomous system is equal to the impulsive response of an extended system. Now, consider an extended state $\mathbf{z} = (\mathbf{x}^\top, \mathbf{w}^\top)^\top$. The reset control system is given by:

$$Z : \begin{cases} \dot{\mathbf{z}}(t) = A_z \mathbf{z}(t) + B\delta(t), & C_{z_c}\mathbf{z}(t) \neq 0 \\ \mathbf{z}(t^+) = A_{z_R}\mathbf{z}(t), & C_{z_c}\mathbf{z}(t) = 0 \\ y(t) = C_{\mathbf{z}}\mathbf{z}(t) \end{cases}$$

where $B = \mathbf{z}(0)$ and

$$A_z = \begin{pmatrix} A & B & 0 \\ 0 & A_w \end{pmatrix} \tag{8}$$

$$C_z = (\, C \ \ 0_{n_w} \,) \tag{9}$$

$$C_{z_c} = (\, -C \ \ 1 \ \ 0_{n_w-1} \,) \tag{10}$$

$$A_{z_R} = diag\,(A, I_{n_w}) \tag{11}$$

with n_w the dimension of the state w. The impulsive response of the extend system can be described by H_k where the input matrix B_k for $k > 0$ is given by:

$$B_k = \begin{pmatrix} x_{p_1}(t_k) \\ \vdots \\ x_{p_{n_p-1}}(t_k) \\ w_1(t_k) \\ x_i(t_k) \\ 0 \\ w_1(t_k) \\ \vdots \\ w_{n_w}(t_k) \end{pmatrix} \tag{12}$$

assuming $C_w = (1, 0, \cdots, 0_{n_w-1})$.

On the other hand, the error signal $e_k(t)$ at any time interval $t \in (t_k, t_{k+1}]$, is given in the s-domain by:

$$E_k(s) = C_{z_c}(sI - A_z)^{-1}B_k \tag{13}$$

In the following, functions $E_k(s)$ are used as a key information to obtain tuning rules for the parameter p_r.

4. FIRST ORDER PLANTS

Consider the feedback system given by Fig. 2, where P is now a first order plant given by:

$$P : \begin{cases} \dot{x_p}(t) = -a_0 x_p(t) + b_0 v(t) \\ y(t) = x_p(t) \end{cases}$$

where x_p is the plant state, and $x_p(0)$ is assumed 0. The closed loop matrices (3), (4), (5) and (6) are given by:

$$A = \begin{pmatrix} -a_0 - b_0 k_p & \dfrac{b_0 k_p (1-p_r)}{\tau_i} & \dfrac{b_0 k_p p_r}{\tau_i} \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} b_0 k_p \\ 1 \\ 1 \end{pmatrix} \quad C = (\, 1 \ 0 \ 0 \,) \quad A_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

4.1 Step response

The exogenous system matrices A_w and C_w for a step input are simply given by:

$$A_w = 0 \tag{14}$$

$$C_w = 1 \tag{15}$$

and the initial state w_0 is the height of the step. Thus, the extend system matrices (8), (9), (10), (11) and (12) are given by:

$$A_z = \begin{pmatrix} -a_0 - b_0 k_p & \dfrac{b_0 k_p (1-p_r)}{\tau_i} & \dfrac{b_0 k_p p_r}{\tau_i} & b_0 k_p \\ -1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C_z = (\, 1 \ 0 \ 0 \ 0 \,) \quad C_{z_c} = (\, -1 \ 0 \ 0 \ 1 \,)$$

$$A_{z_R} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad B_k = \begin{pmatrix} w_0 \\ x_i(t_k) \\ 0 \\ w_0 \end{pmatrix}$$

Therefore, for the step response, using (13), $E_k(s)$ is:

$$E_k(s) = \frac{a_0 w_0 - \frac{b_0 k_p x_i(t_k)(1-p_r)}{\tau_i}}{\left(s^2 + (a_0 + b_0 k_p)\, s + \frac{b_0 k_p}{\tau_i}\right)} \tag{16}$$

4.2 PI+CI tuning rules for first order plants

The gain k_p and the time integral τ_i are first tuned for the base PI compensator using one of the many available tuning methods. A fast step response (it does not matter much the percentage of overshoot) can be used. In the following, it will be shown how a proper election of the parameter p_r can remove the overshoot at the first reset instant, producing a flat response and reaching the steady-state a that instant. In this case, overshooting can be simply eliminated after the first reset time if $E_k = 0$, $k = 1, 2, \cdots$. Therefore, making $E_k(s) = 0$ in (16) it is easily obtained

$$p_r(t_k) = 1 - \frac{a_0 \tau_i w_0}{b_0 k_p x_i(t_k^-)} \tag{17}$$

which will relate the reset ratio at the reset instant t_k, $p_r(t_k)$, with the plant gain $k = \frac{b_0}{a_0}$, the integral time τ_i, the proportional gain k_p, and the step height w_0, and also the integral state x_i at the reset instant t_k. Note that from the plant side the p_r just depends on the plant gain, which gives a very robust design against uncertainty over the plant time constant.

It should be noted that since $x_i(t)$ is constant for $t \geq t_1$, then p_r is constant after the first reset instant. In addition, for a given base control system the parameter p_r does not depend of the step height ($w_0 \neq 0$), since $x_i(t)$ is directly related to w_0. However, as we can see in (17) the parameter p_r changes the value into 1 when the reference is 0 ($w_0 = 0$).

In Baños and Vidal (2011), a tuning of PI+CI, with fixed p_r, that removes the overshooting after the second reset instant is given. The new tuning rule (17), based on a variable p_r, produces a zero overshoot for an arbitrarily fast step response.

Example Consider the tuning of a PI+CI compensator for a unit step as a reference signal, and for the plant P:

$$P : \begin{cases} \dot{x_p}(t) = -0.5 x_p(t) + 1.5 v(t) \\ y(t) = x_p(t) \end{cases}$$

The next two settings for the base PI controller are chosen:

Table 1. PI parameters

	k_p	τ_i
1	2	0.15
2	1.5	0.5

As can be seen in Fig. 5 reset ratio, p_r, is calculated every reset time by (17) in order to reach the steady-state just after the first reset time for each step. Fig. 4 shows the step responses for both tuning of the compesator PI and PI+CI. Note that responses of PI and PI+CI are identical until first reset instant, and that the overshooting is eliminated by PI+CI compensation at first reset time. The control signal of the PI+CI remains constant from the first reset instant, as it is shown in Fig. 6. Thus the PI+CI needs less energy to achieve the steady-state.

5. SECOND ORDER PLANTS

Consider a second order plant P described by:

$$P(s) = \frac{b_0}{s^2 + a_1 s + a_0} \tag{18}$$

and with a state space representation (observer form) given by:

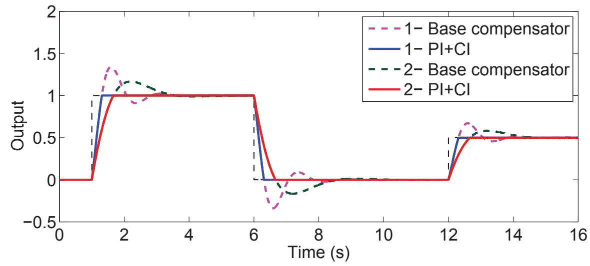

Fig. 4. System responses for PI and PI+CI compensators

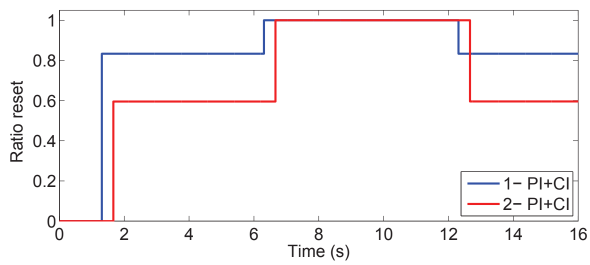

Fig. 5. Ratio reset for PI+CI compensators

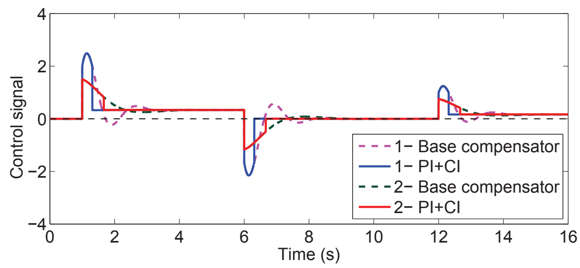

Fig. 6. Control signal for PI and PI+CI compensators

$$P : \begin{cases} \dot{\mathbf{x}}_p(t) = \begin{pmatrix} 0 & -a_0 \\ 1 & -a_1 \end{pmatrix} \mathbf{x}_p(t) + \begin{pmatrix} b_0 \\ 0 \end{pmatrix} v(t) \\ y(t) = \begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x}_p(t) \end{cases}$$

Computing the extend system matrices (8), (9), (10), (11) and (12) and taking the exogenous system matrices (14) and (15), the error functions $E_k(s)$, $k = 1, 2, \cdots$, for the step response are given by:

$$E_k(s) = \frac{(x_{p_1}(t_k) - a_1 w_0)\, s - a_0 w_0 - \frac{b_0 k_p x_i(t_k)(p_r - 1)}{\tau_i}}{s^3 + a_1 s^2 + (a_0 + b_0 k_p)\, s + \frac{b_0 k_p}{\tau_i}} \tag{19}$$

In contrast to (16), now the step response error $E_k(s)$ is strongly influenced by the position of its zero and its DC-gain. Note that, in general, it is imposible to make zero $E_k(s)$ by just changing the position of the zero and the DC-gain, since the term $x_{p_1}(t_k) - a_1 w_0 = -\dot{e}(t_k)$ in (19) can not be cancelled. Clearly, there exist three different cases depending of the position of the zero z_k of $E_k(s)$ (see Appendix A).

(1) $E_k(s)$ is non-minimum phase ($z_k > 0$): The maximum value of the absolute error signal is reduced for $t \in (t_k, t_k + 1]$.
(2) $E_k(s)$ is minimum phase ($z_k < 0$): The maximum value of the absolute error signal is increased for $t \in (t_k, t_k + 1]$.

(3) The zero is at the origin ($z_k = 0$): The maximum value of the absolute error signal is between the above cases.

5.1 PI+CI tuning rules for second order plants

The goal of designing a PI+CI compensator is to find a suitable reset ratio to improve the performance of a previously well-designed PI compensator. As it has been discussed above, the overshoot or undershoot can be reduced by getting a nonminimun phase zero at each reset interval, which depends on the derivative of the error signal. However, it is a known fact that the derivative is very sensitive to sensor noise, thus a more conservative strategy may be used.

A. Zero of $E_k(s)$ at the origin. By a suitable election of p_r, it is possible to locate the z_k at the origin. As a result the tuning of p_r does not depend on the derivative of the error signal. In this case,

$$\frac{b_0 k_p x_i(t_k)(p_r(t_k) - 1)}{\tau_i} = -a_0 w_0$$

and thus (an identical tuning to the first order case is obtained)

$$p_r(t_k) = 1 - \frac{a_0 \tau_i w_0}{b_0 k_p x_i(t_k)} \tag{20}$$

Therefore, it is not difficult to show that reset instants $\{t_1, t_2, \dots\}$ are periodic with fundamental period Δ_{reset} given by solving the next equation:

$$Ce^{A\Delta_{reset}} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0$$

where A and C are the closed loop base system matrices.

B. Zero of $E_k(s)$ in the right-half plane. An improvement of the above tuning method for reducing the overshoot/undershoot have to be based on moving the zero of E_k to the right-halft plane. The zero of E_k is on the right-half plane if the reset ratio satisfies

$$p_r(t_k) < 1 - \frac{a_0 \tau_i w_0}{b_0 k_p x_i(t_k)} \text{ if } \dot{e} > 0$$

$$p_r(t_k) > 1 - \frac{a_0 \tau_i w_0}{b_0 k_p x_i(t_k)} \text{ if } \dot{e} < 0$$

for $k > 0$. This can be achieved, using the Clegg integrator state before reset instants, by

$$p_r(t_k) : \begin{cases} 1 - \dfrac{a_0 \tau_i w_0}{b_0 k_p x_i(t_k)} + \delta \dfrac{x_{ci}(t_k^-)}{w_0}, & w_0 \neq 0 \\ 1, & w_0 = 0 \end{cases} \tag{21}$$

where $\delta \geq 0$ is a tuning parameter and $x_{ci}(t_k^-)$ is the value $x_{ci}(t_k - \epsilon)$ with $\epsilon \to 0^+$. In this case, p_r is a function of δ, so that now we have three parameters to fix (k_p, τ_i and δ).

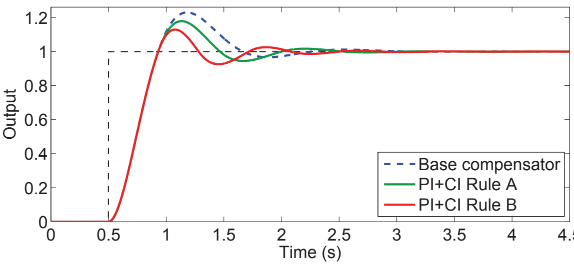

Fig. 7. System responses for PI, PI+CI and P+CI compensators

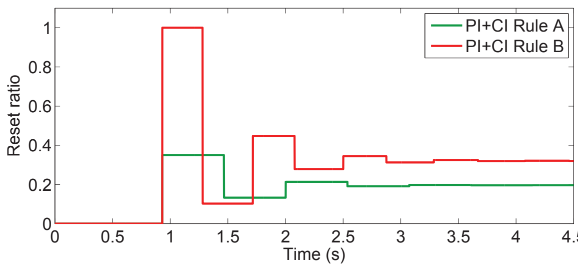

Fig. 8. Ratio reset for PI+CI compensator

A suitable value of δ is given when $p_r(t_1) \approx 1$ (first overshoot) and $p_r(t_2) \approx 0$ for (first undershoot). To this end, an initial value of δ is:

$$\delta \geq \frac{8 a_0 \tau_i}{b_0 k_p t_r^2} \tag{22}$$

where t_r is the rise time of the base control system. Note that the tunning rule (20) would be obtained if $\delta = 0$.

5.2 Example

Here the above PI+CI tuning methods (21) and (20) are applied to a second order plant with low damping:

$$P(s) = \frac{5}{s^2 + 0.24s + 5}$$

The PI_{base} setting is obtained by SIMC tuning rules (Skogestad (2003)). The constant values for the compensator are $k_p = 5.5$ and $\tau_i = 0.8$. The parameter $p_r(t_k)$, $k = 1, 2, \cdots$, is calculated by using (20) and (21). For the last tuning method the parameter δ is obtained by (22). In this case the rise time of the base control system is approximately 0.43, thus $\delta = 6.29$. Fig. 7 shows the step response for the base compensator and both tunings methods. Note that the overshoot is reduced by both proposed tunings rules. The ratio reset of the PI+CI compensators are shown in Fig. 8 and the control signals of the three compensators in Fig. 9. As we can see the oscillations of the ratio reset are greater for the second tunning rule than the first rule.

In order to get a better comparison between the different compesators, performance indices such as the integral of absolute error (IAE), the integral time absolute error (ITAE) and integral of absolute variation of control signal (IAVU), are obtained in table 2. From these indices, it can be concluded that the method given by (21) gets the

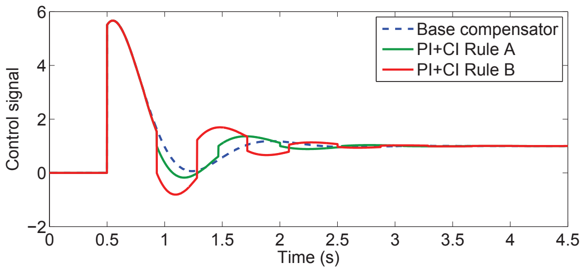

Fig. 9. Control signals for PI, PI+CI and P+CI compensators

Table 2. Performance indices

	ITAE	IAE	IAVU	Overshoot	Undershoot
PI	0.317	0.356	5.05	23.0	3.31
PI+CI A	0.275	0.325	5.25	17.8	5.52
PI+CI B	0.243	0.301	5.05	13.2	7.3

smallest overshoot, just by making the undershoot lightly worse than PI compensator. As for the control signal the three compensators use approximately the same energy.

5.3 Second order plants with integrator

This is a particular case of (18) for $a_0 = 0$. Thus, the PI+CI tuning rule can be directly applied by making $a_0 = 0$ in the equation (20). In this case the suitable reset ratio is $pr(t_k) = 1$, $k = 1, 2, \cdots$, that is a P+CI compensator is used. On the other hand, the asymptotic property of the compensator is kept for a step reference, since the plant has an integrator.

6. CONCLUSIONS

The tuning of the reset PI+CI compensators has been developed in previous works by using approximate analysis methods. In this work, a systematic method for PI+CI tuning is developed, the main point has been the analysis of the closed loop reset system as the impulse response of an extended system (by using an exosystem to produce reference inputs). It has been shown that the step response between two consecutive reset instants can be analyzed by the impulse response of LTI system in a given set. As a results, simple tuning rules have been developed for first and second order plants. For first order plants, the tuning method provides a flat step response that reaches the steady-state in the first reset instant; for second order plants, several tuning methods have been developed, based on the location of a zero in the right-half plane, and although do not give a flat response, they produce significant improvements over previous results. This work is a first step for the development of systematic and efficient PI+CI tuning rule. In subsequent works, application to plants with time delays will be investigated, as well as important practical issues like disturbance rejection (a similar approach based on a exosystem can be used) and robustness against plant uncertainty.

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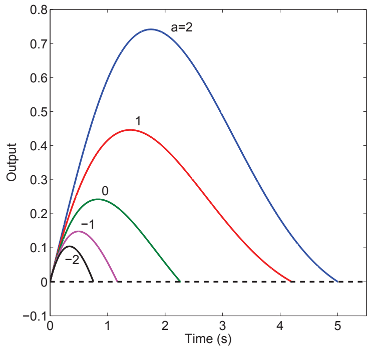

Fig. A.1. Impulse response of a third order system plus zero

Appendix A. IMPULSE RESPONSE OF A THIRD ORDER SYSTEM PLUS ZERO

Consider the normalized ($w_n = 1$) third order system given by:

$$F(s) = \frac{s + a}{(s^2 + 2\xi s + 1)(\gamma s + 1)}$$

where the parameter a has effect on the DC-gain and the location of the zero. The impulse response of the tranfer function F is plotted in Fig. A.1 for $\xi = 0.5$, $\gamma = 1.5$ and different values of a. As it can be seen, the overshoot decreases as a decreases and takes negative values. Thus, we can reduce the overshoot by moving the zero into right-half plane and getting a nonminimun phase system. Note that the responses of Fig. A.1 will be the response of a reset control system between two reset instants: the instant $t = 0$ corresponds with the reset instant t_k, and the first positive instant corresponding with zero output is t_{k+1}. On the other hand, it is important to note that the derivative of the response at t_{k+1} increases as far as a decreases. Hence, the undershoot of the reset system will be greater at the next reset interval if the same value of a (corresponding to the same value of p_r) is used.