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Comparing Fractional Order PI Controllers With Variable Gain and Gain-Order for the Networked Control of a Servomotor ⋆

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Abstract: This paper addresses the use of fractional order control (FOC) in the context of control through networks to minimize or compensate the effect of the time-varying network-induced delay. In particular, two fractional order strategies are presented based on the adaptation of, on one hand, the gains of a fractional order PI controller, and, on the other hand, both its gains and its order in accordance with the variable delay detected in a networked control system (NCS). The essence of these approaches is to make controller gains and both gains and order delay-dependent by minimizing a defined cost function in order to avoid a decreased control performance. Experimental results, related to the control of the angular velocity of a servomotor through the Internet, show the effectiveness of these fractional order adaptive strategies in comparison with the non-scheduled controller. Likewise, a useful application of variable order PI controllers is presented.

Keywords: Networked Control System, Gain Scheduling, Order Scheduling, Variable Delay, Fractional Order PI Controller, Servomotor.

1. INTRODUCTION

Connecting the control system components by means of a network reduces the system's complexity and maintenance costs. However, the insertion of a communication network in the feedback loop poses additional challenges for controlling this kind of systems, called networked control systems (NCS).

Numerous control strategies have been formulated based on several types of network behaviour and configurations in conjunction with different ways to treat network problems and some assumptions (see Hespanha et al. (2007), Tipsuwan and Chow (2003), Yang (2006) for additional information and references). In this line, the NCS literature has recently been dominated by the compensation of time-varying network delays during the controller design process, as in Gao et al. (2008), or in real-time, as in Tipsuwan and Chow (2004a), Tipsuwan and Chow (2004b), Vatanski et al. (2009), Zhi and Weihong (2008), for efficient control purposes. Among these, one of the most important tend is to design approaches which enable an existing controller, since replacing a widely used controller by a new one for efficient networked control capability can be costly and time-consuming (see Tipsuwan and Chow (2003)).

It is well-known that fractional order control (FOC) and its applications are becoming an important research topic since they translate into more tuning parameters, i.e., more adjustable time and frequency responses of the control system, allowing robust performances to be attained.

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In the last few years fractional order strategies have been applied to network-based control (e.g. see Tejado et al. (2011e), Bhambhani et al. (2010), Mukhopadhyay et al. (2009), Bhambhani et al. (2008)). Even though most of those strategies provide satisfactory results –mainly in comparison with those of integer order–, tuning rule developments to treat and compensate a specific network-induced effect remained unknown before our group of papers referred to gain scheduling (Tejado et al. (2011a), Tejado et al. (2011c)) and gain and order scheduling (Tejado et al. (2011b), Tejado et al. (2011d), Tejado et al. (2010)).

Given this context, and based on the initial design of a fractional order controller, this paper collects two real-time scheduled approaches which improve the performance of an NCS without changing the original structure of an existing controller by compensating the actual network condition. Their essence is to make controller gains and both gains and order delay-dependent, respectively, i.e., adjust gains and both gains and order optimally with respect the network delay. A comparison of both strategies for the angular velocity control of a servomotor through the Internet is given.

The remainder of this paper is organized as follows. The problem of network-based applications is stated in Section 2. Section 3 briefly describes the essence of the approaches and the design of the controllers. The application related to the velocity control of a servomotor via the Internet is given in Section 4, including the design of the two controllers and experimental results. Concluding remarks and future work are given in Section 5.

2. PROBLEM STATEMENT

Let us briefly state the NCS problem considered in this paper. In comparison with classical interconnected dynamic systems, networked systems introduce a number of parasitic effects, since an NCS operates over a shared network. These include time-delay, packet losses, and limited communication which are introduced by the communication link, and others that are node-induced, such as quantization, node failure, and synchronization errors. Among these, network delay plays a major role in NCS design and control because it is inevitable and can cause deterioration of the system's performance. In practice, delays are induced by the finite rate allowed by the communication medium and also by the particular structure of the communication protocol. In the literature, there exist several studies of this problem from different points of view, and proposals to reduce its effects (see e.g. Gupta and Chow (2010), Bauer (2008), Tipsuwan and Chow (2003)).

Gain scheduling techniques aim to adapt controller gains with respect to changing operating conditions, so they can be adequate in the network-based control field. As a matter of fact, in the literature it has been demonstrated that gain scheduling can improve the performance of a system, especially in applications in which the replacement of a widely used controller is a hard and time-consuming task (see e.g. Tipsuwan and Chow (2004a)).

However, in basis of FOC, the generalization of traditional PID controllers, i.e., PI$^\alpha$D$^\mu$, can be considered, so not only controller gains K_p, K_i, and K_d can be changed, but also controller orders α and μ. Thus, *will a PI$^\alpha$D$^\mu$ controller with adaptive orders and gains, depending on network conditions, do better?*. Likewise, it is clear that changes in the orders cause modifications in both the gain and phase margins of the controlled system. Hence, the adaptation of not only gains but also order may help to get more adjustable time and frequency responses of the controlled system and, consequently, to improve its performance. For simplicity, we shall focus on the application to fractional order PI controllers.

Hence, this novel strategy arises from the idea of, firstly, applying gain scheduling to a non-scheduled fractional order PI controller and, secondly, extending it to the adaptation of both controller gains and order.

3. APPROACHES DESCRIPTION

This section includes the main issues involved in designing the approaches developed to apply an existing/designed fractional order controller for efficient networked control purposes, avoiding a decreased control performance due to the effect of time-varying induced delays. The fundamentals and full description of them can be found in Tejado et al. (2011a) and Tejado et al. (2011b).

3.1 Overview

Basically, these approaches deal with compensating time-varying delay effects on the basis of FOC by means of the concepts of gain scheduling, which will be referred to as fractional gain scheduling (FGS), and the previous one together with variable order control, which will be called as fractional gain and order scheduling (FGOS). The general structure of gain and order scheduling for NCS is shown in Fig. 1, whose basic components are the following:

- Gain and order schedulers, which modify controller gains –via an external gain β– and order with respect to the current network condition.
- Delay estimator, whose function is to estimate (or measure) the current network condition, which is then utilized by both the gain and the order schedulers.
- Controller, whose original structure is kept. It is important to note that, although this controller can be any designed controller, these approaches are based on using fractional order controllers, and more precisely, a fractional order PI$^\alpha$ (FOPI) controller.
- Remote system.

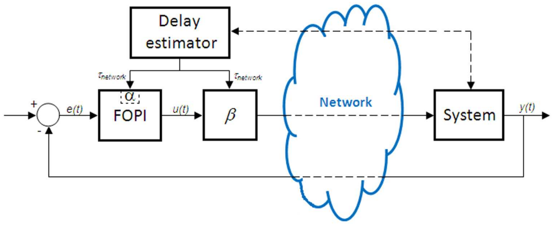

Fig. 1. Structure of gain and order scheduling for NCS

3.2 Tuning development

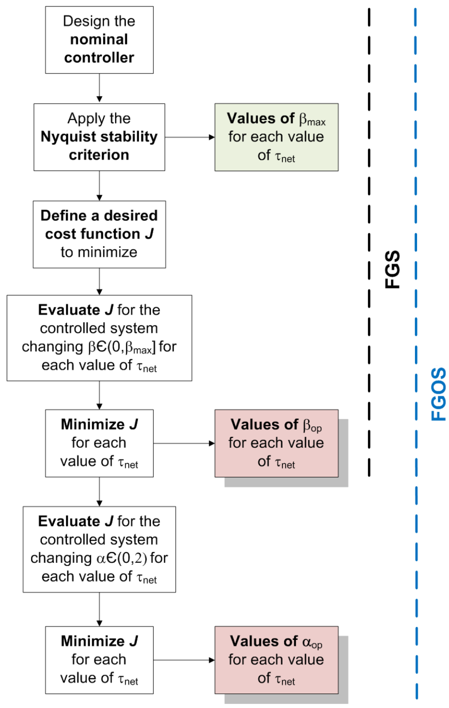

Fig. 2. Summary of FGS and FGOS design

The essence of the two approaches is to search for the optimal values of the external gain β, β_{op}, and the order α, α_{op}, with respect to the current network delay $\tau_{network}$. Obviously, many tuning methods can be used to get suitable controllers. However, FGOS is based on using the new adaptive parameter α to improve the system performance obtained with FGS, i.e., FGOS can be considered as the generalization of FGS[1]. In particular, both parameters β_{op} and α_{op} are determined via an offline closed-loop simulation of the system by minimizing a defined cost function, following the steps outlined in Fig. 2 as follows:

1) To design the nominal PI$^\alpha$ controller.

2) To subdivide the working range of the process into a number of finite intervals, in other words, to define the considered network delay intervals. The delay increment mainly depends on the value of the jitter margin of the controlled system, as well as on the desired precision of the solution.

3) To calculate the maximum value of β for each value of $\tau_{network}$ which guarantees the system's stability by applying the Nyquist stability criterion.

4) To define a cost function J to evaluate the best possible system performance with respect to β under different network conditions.

5) To simulate the closed-loop system changing β from 0 to $\beta_{\max}$ for each value of $\tau_{network}$ (as a constant delay) and evaluate J for each case. The order α is fixed to its nominal value.

6) To get β_{op} as the value of β which minimizes J for each $\tau_{network}$.

7) (Only for FGOS) To repeat steps 5 and 6 changing α for increments[2] of 0.1, $\alpha \in (0, 2)$, when β takes its corresponding optimal value obtained previously. Similarly to FGS, α_{op} is the value of α which minimizes J for each $\tau_{network}$.

Apart from that, both approaches require an estimation or measurement of the current network condition. In accordance with Eriksson (2008), if the exact delay of each measurement is known, one can re-compute the controller at each sampling instant and update the delay value accordingly; or else, (2) if the delay distribution is known rather than the exact delay times, the expectation value of the delay can be used at each sampling instant. In this work, we use the second procedure as follows. Firstly, the network which will close the loop is analyzed by measuring the whole transmission delay –e.g. by measuring the round trip time (RTT)–, in order to determine the best distribution which characterizes that delay. This distribution will be used to estimate the current network delay.

4. EXPERIMENTAL APPLICATION

This section addresses the experimental application of FGS and FGOS to control the angular velocity of the servomotor shown in Fig. 3 via the Internet. The aim is to compare the performance of the system when applying both the gain scheduled controller –will be denoted as FGSC– and the gain and order scheduled controller –will be referred to as FGOSC– in order to show that the use of FGOSC technique can improve the results obtained by applying only gain scheduling.

The design of the controllers and the experimental results will be given next. Some details of the experimental set-up carried out in the MATLAB/Simulink environment will be also included.

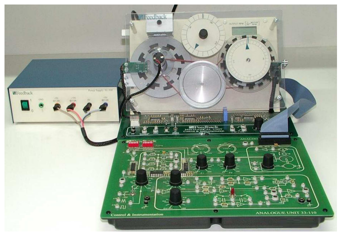

Fig. 3. Picture of the servomotor

4.1 Design of the controllers

The dynamics of the servomotor for its angular velocity is given by the following first order plus delay transfer function:

$$G_s(s) = \frac{K}{Ts+1}e^{-Ls} = \frac{0.92}{0.45s+1}e^{-0.2s}. \quad (1)$$

The nominal fractional order PI$^\alpha$ controller considered in this work is of the form (2), given by the set of tuning rules (3)-(5) –its development is based on a fractional M_s constrained integral gain optimization method (F-MIGO); see the full description in Bhaskaran et al. (2007).

$$FOPI(s) = K_p + \frac{K_i}{s^\alpha} = \frac{K_p(s^\alpha + z_C)}{s^\alpha}, \quad (2)$$

$$K_p = \frac{0.2978}{K(\tau + 0.000307)}, \quad (3)$$

$$K_i = \frac{K_p(\tau^2 - 3.402\tau + 2.405)}{0.8578T}, \quad (4)$$

$$\alpha = \begin{cases} 0.7, & \text{if } \tau < 0.1 \\ 0.9, & \text{if } 0.1 \le \tau < 0.4 \\ 1.0, & \text{if } 0.4 \le \tau < 0.6 \\ 1.1, & \text{if } \tau \ge 0.6 \end{cases}, \quad (5)$$

where $\tau = \frac{L}{L+T}$ is the normalized time delay and $z_C = K_i/K_p$, the ratio between controller gains.

Hence, taking into account the system transfer function (1) and the set of rules (3)-(5), the nominal FOPI controller is given by the expression:

$$FOPI(s) = 1.0510 + \frac{3.9557}{s^{0.9}}. \quad (6)$$

For this application, we consider the following cost function J:

$$J = \omega_1 J_1 + \omega_2 J_2, \quad (7)$$

where J_1 is the overshoot and J_2 is the integral of the squared error (ISE), and ω_1 and ω_2 are the weighting parameters of each subfunction (with the same relative importance in the function J: $\omega_1 = \omega_2 = 0.5$).

Following the aforementioned steps with delay intervals of 20 ms, the FGSC and FGOSC are characterized by the

[1] That is the reason why the minimization process will not be done over α and β simultaneously. This method may provide proper controllers depending on the application but not for the case studied in this paper. In addition, the purpose of the FGOS method presented here is, starting from FGS, to demonstrate that FGOS can improve the system performance obtained using FGS.

[2] The minimization process over the order α is not continuos for $\alpha \in (0, 2)$. Increments of 0.1 are considered in order to reduce the set of optimal values of α and, consequently, make the experiments easier.

optimal values of β and α represented in Fig. 4 and 5, respectively. As can be observed, the set of optimal values of α for $\tau_{network} \in (0, 0.5)$ s is limited and composed of only 5 orders, due to the minimization process established over α with increments of 0.1.

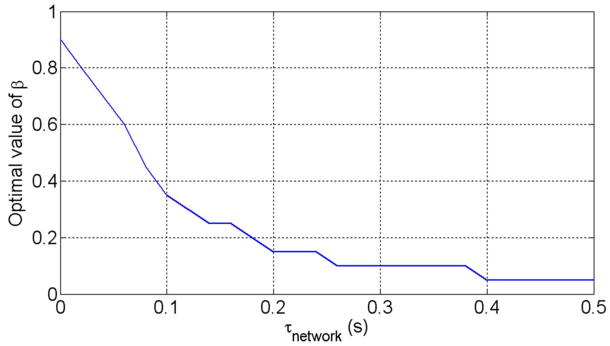

Fig. 4. Optimal values of β for different values of $\tau_{network}$

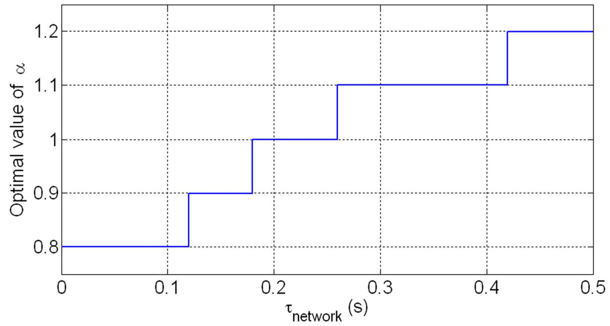

Fig. 5. Optimal values of α for different values of $\tau_{network}$

4.2 Results

Firstly, experimental results of the servomotor will be obtained by applying the FGSC and the nominal (non-scheduled) FOPI controllers. Then, the comparison of the proposed fractional order strategies to compensate time-varying network delays will be given.

The experimental tests can be divided into three scenarios: scenario I and II simulate different network conditions comparing FGSC *versus* FOPI and FGOSC *versus* FGSC, respectively, whereas scenario III is similar to scenario II but assumes a change in the network condition during the experiment, i.e., a change in the mean value of the network delay. All step responses are obtained for an angular velocity reference of 3 rad/s. For a comparison of the results when applying both the nominal FOPI and FGOSC controllers, refer to Tejado et al. (2011d).

Scenario I considers three network conditions: (a) $\tau_{network} \in [0.05, 0.12]$s, (b) $\tau_{network} \in [0.12, 0.18]$ s, and (c) $\tau_{network} \in [0.18, 0.25]$ s. Fig. 6 shows the comparison of the responses obtained by using the nominal controller –green lines– and the FGSC –red lines. The corresponding values of the cost function J are given in Table 1. From these results, it can be stated that the servo performance is significantly better when using FGSC, obtaining an improvement with respect to the non-scheduled controller, given by definition (8), about 67% in the best case.

$$\mathrm{I}\ (\%) = \frac{J_{non-sch} - J_{FGSC}}{J_{non-sch}} 100. \tag{8}$$

Moreover, it is important to note that the improvement in terms of the overshoot is especially greater. Therefore, the efficiency of applying FGS in control over networks with time-varying delays with respect to a non-scheduled controller is demonstrated.

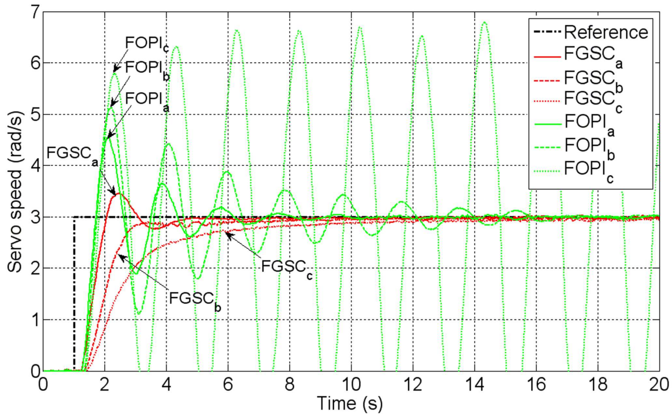

Fig. 6. Experimental responses of the servo when applying the two FGSC and FOPI controllers (Scenario I)

Table 1. Results of Scenario I

Case	(a)	(b)	(c)
FGSC	9.63	3.61	5.65
FOPI	29.07	40.87	–
I (%)	66.87	91.17	100

Similarly to Scenario I, Fig. 7 compares the responses when applying the two fractional order strategies for variable delay compensation, i.e., FGOSC and FGSC controllers, in blue and red, respectively. As can be observed, there exist slight differences in the time responses of the servo for the cases (a) and (c), being a bit faster when using FGOSC. Concerning the variation of the relative stability, it is smaller with FGOSC, which may be caused by the order jumps. Likewise, only slight differences in the performances, in terms of J, can be found, as observed from Table 2. However, it can be seen that FGOSC improves the servo performance in comparison with FGSC.

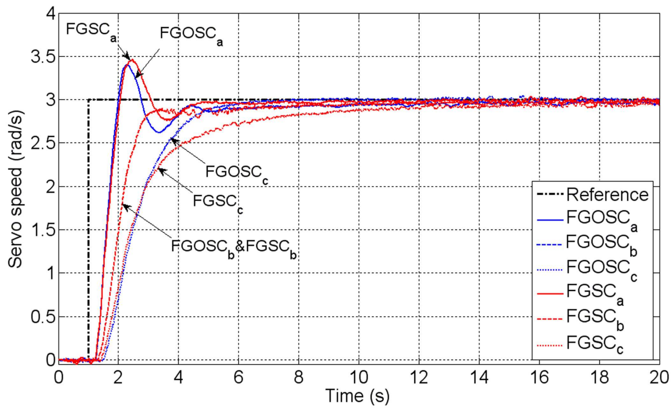

Fig. 7. Experimental responses of the servo when applying the two FGOSC and FGSC controllers (Scenario II)

Next, we consider network delay varying in the interval $[0, 0.05]$s, and, in certain circumstances, a change of its mean value, given by τ_{add}, is assumed at time 5 s. This change could be originated from a network topology change, where a node has fallen off or moved in an ad-hoc network and the path (and delay) has become longer (see e.g. Pohjola (2006) for more details). In particular, other two different cases are simulated: (a) $\tau_{add} = 0.05$ s, (b) $\tau_{add} = 0.12$ s. Figure 8 illustrates the comparison of the servo responses when applying the controllers against changes in the network condition, also summarized in Table 2 in terms of J. As observed in the time domain response, the change of the network condition in the case (b) not only may cause adaptation in the controller order, but also significant changes in the controller gains. That is the reason why the servo responses go through an adaptation at time 5 s –in any case, they reach the reference, i.e., they are stable. Again, only minor differences between the two responses can be found, but FGOSC keeps improving the performance of the system.

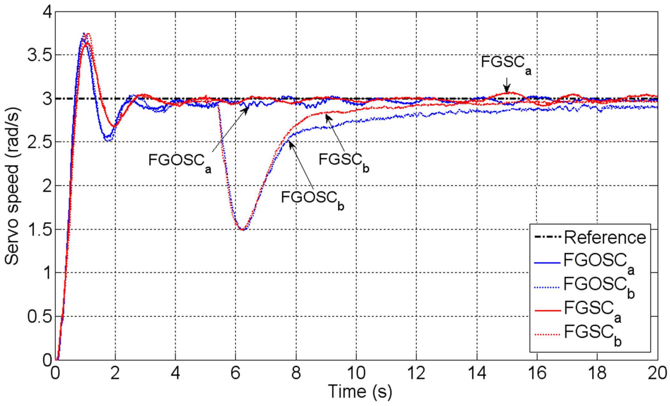

Fig. 8. Experimental responses of the servo when applying the two FGOSC and FGSC controllers (Scenario III)

Table 2. Results of Scenarios II and III

	Scenario II			Scenario III	
Case	(a)	(b)	(c)	(a)	(b)
FGSC	9.63	3.61	5.65	12.32	15.43
FGOSC	9.07	3.61	5.51	12.27	15.06

4.3 Details of the set-up

Fig. 9 shows the connection scheme of the experimental set-up: for transmitting the signal from one system to another through the Internet, the Simulink UDP protocol is used –thanks to the *Instrument Control Toolbox*–, and the acquisition of the servo signals is carried out by the *NI 6259* board (refer to NI Manual (2008) for a detailed information).

In order to preserve the integral effect, the integral part $s^{-\alpha}$ of all fractional order controllers is implemented in Simulink as $s^{-\alpha} = s^{-n}s^{n-\alpha}$, with $n-1 \leq \alpha \leq n$, where the derivative part $s^{n-\alpha}$ is an integer order transfer function of fifth order that fits the frequency response in the range $\omega \in (10^{-3}, 10^3)$, obtained by the modified Oustaloup's method (e.g. refer to Monje et al. (2010) for continuous-time implementations of fractional order operators). On the other hand, the external gain β of FGSC is given by means of a *Lookup Table* block. With respect to FGOSC, for a specific order in Fig. 5, its implementation is performed by means of the approximation of the corresponding controller. Therefore, we have a set of

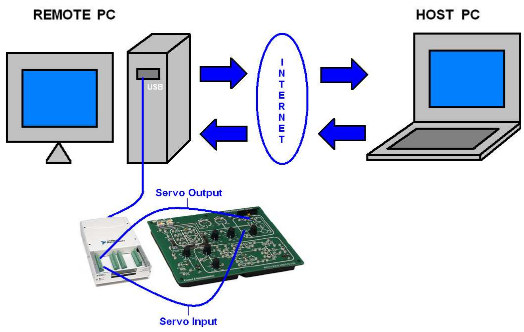

Fig. 9. Experimental set-up for the networked control of the servo

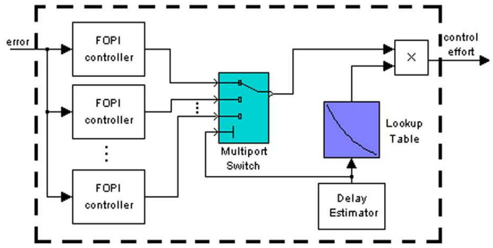

Fig. 10. Scheme of the implementation of FGOSC in Simulink

controllers (approximations) and select the corresponding one depending on the estimation of the current network condition, as shown in Fig. 10. Refer to e.g. Valério and Sá da Costa (2011), Valério and Sá da Costa (2010), Charef and Idiou (2010), Chan et al. (2010), Shyu et al. (2009) for different implementations of fractional variable order controllers.

5. CONCLUSION

This paper has collected two real-time scheduled approaches which improve the performance of an NCS without changing the original structure of a designed controller by compensating the actual network condition. Their essence was to make controller gains and both gains and order delay-dependent, respectively, i.e., adjust gains and both gains and order optimally with respect the network delay. These controllers has been referred to as FGSC and FGOSC, respectively.

A comparison of both strategies for the angular velocity control of a servomotor through the Internet was given. Experimental results have led to the conclusion that FGOSC and FGSC controllers are adequate for efficient network-based control, especially when network conditions become worse in certain circumstances and for applications in which replacing a widely used controller is a hard and time-consuming task. Likewise, there were shown to be advantages in using FGOSC instead of FGSC, although minor. Since, FGOSC has one more delay-adaptive parameter, its order α, the performance of the system can be improved.

The use of fractional order controllers with variable order in NCS can be suitable and effective to improve the system performance. However, further work on their experimental implementation is required, i.e., how to implement finite-dimensional "realizable" controllers. Thus, an in-depth study of implementing this kind of controllers is necessary to obtain even better results. Moreover, the stability of the controlled system should be proved.

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