Design of feedback control for underdamped systems

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Abstract: In practice, there are several processes which are exhibiting oscillatory behaviour. Some representatives are disk-drive heads, robot arms, cranes and power-electronics. One of techniques, aimed at reducing the oscillations, is Posicast Input Command Shaping (PICS) method. The paper combines the PICS method and Magnitude Optimum Multiple Integration (MOMI) tuning method for PID controllers. The combination of both methods significantly improves the speed and stability of the closed-loop tracking responses. Moreover, the proposed approach is relatively simple for implementation in practice and can be used either on process time-response data or on the process model in frequency-domain.

Keywords: PID control, Posicast, MOMI, underdamped systems, controller tuning.

1. INTRODUCTION

Most of the processes in practice are stable and can be controlled by various types of controller structures. The controller parameters are usually not critical and they can vary significantly to achieve stable response. However, some types of processes, like robot arms, disk-drive heads, cranes, power-system electronics and similar (Huey et al., 2008; Singer and Seering, 1990; Singhose, 2009; Li, 2009) exhibit oscillatory behaviour. Such systems require special attention, since stable response can be achieved in significantly smaller controller parameter space. Moreover, the mentioned systems usually require closed-loop response without or with a relatively small overshoot.

Several tuning rules have been proposed so far for oscillatory systems. In general they require relatively precise process model obtained by process identification. One of the methods is so-called Posicast Input Command Shaping (PICS) proposed by Smith (1957). The main idea of the method is to split control signal into direct and delayed paths. When such control signal is applied to the process, the direct and delayed signal paths counteracts and attenuate oscillations at the process output.

If signal splitter is placed inside the close-loop, it modifies the process transfer function by making a sum of undelayed and delayed process model. Most of the tuning methods cannot deal with modified transfer functions, especially if the process transfer functions are not precisely identified. Therefore, the PICS method is mostly used as a reference shaper outside the closed-loop configuration.

Another tuning method for PID controllers, which can be used for moderately oscillatory systems, is Magnitude Optimum Multiple Integration (MOMI) method (Vrančić et al., 2001). However, the method fails to find appropriate controller parameters for highly oscillatory processes. On the other hand, the method requires either process time-response (not necessarily step-response) or process model in order to calculate controller parameters according to the Magnitude Optimum (MO) criteria.

The main idea of this paper is to combine PICS and MOMI method into a new method for tuning oscillatory systems. Namely, PICS can be used in a usual way to obtain less oscillatory process response. Then, MOMI method can be applied to calculate the appropriate PID controller parameters. As will be shown in the paper, the controller parameters can be obtained either from the process time-response or from the process transfer function. Moreover, either time-domain experiment or identification does not have to be repeated after calculating the PICS parameters.

2. PICS Method

Posicast Input Command Shaping method is defined for the following second-order process:

$$G_P(s) = \frac{\omega_n^2}{\omega_n^2 + 2\xi\omega_n s + s^2} \tag{1}$$

where ω_n represents the natural frequency and ξ the damping factor of the process. The under-damped natural frequency is:

$$\omega_d = \omega_n \sqrt{1-\xi^2} \tag{2}$$

Figure 1 represents typical time-response of the process (1) on unity-step input signal. Variable T_{pk} represents the peak time, which is half of oscillation period T_d:

$$T_{pk} = \frac{T_d}{2} = \frac{\pi}{\omega_n \sqrt{1-\xi^2}} \tag{3}$$

The overshoot (δ) of the second-order process can be calculated from the following expression (Seborg et al., 1989):

$$\delta = \frac{y_{pk} - y(\infty)}{y(\infty) - y(0)} = e^{-\frac{\pi\varsigma}{\sqrt{1-\xi^2}}} \tag{4}$$

The main concept of PICS method is to split the process input (u) signal into two parts, as shown in Figure 2. Transfer function of the PICS term is the following (Huey et al., 2008):

$$G_{PICS}(s) = K_1 + (1-K_1)e^{-sT_{dp}} . \tag{5}$$

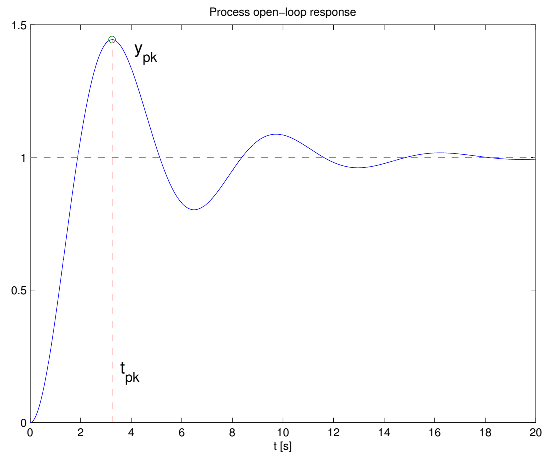

Figure 1. Typical time-response of the second-order system.

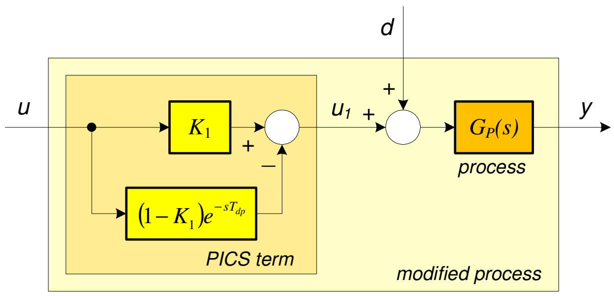

Figure 2. PICS term with a process.

Gain K_1 and time delay T_{dp} are chosen so as to decrease oscillations of the process. For pure second-order process (1), the parameters are the following (Hung, 2007; Huey et al., 2008):

$$K_1 = \frac{1}{1+\delta} \qquad T_{dp} = T_{pk} \tag{6}$$

The parameters K_1 and T_{dp} can also be estimated for the higher-order processes with one pair of complex poles. In this case, three successive peaks (minimums and maximums) from the process open-loop response should be measured, as shown in Figure 3. The Posicast parameters are then calculated from the amplitude and time difference between the peaks:

$$K_1 \approx \frac{d_1}{d_1 + d_2} \qquad T_{dp} \approx t_2 - t_1 \tag{7}$$

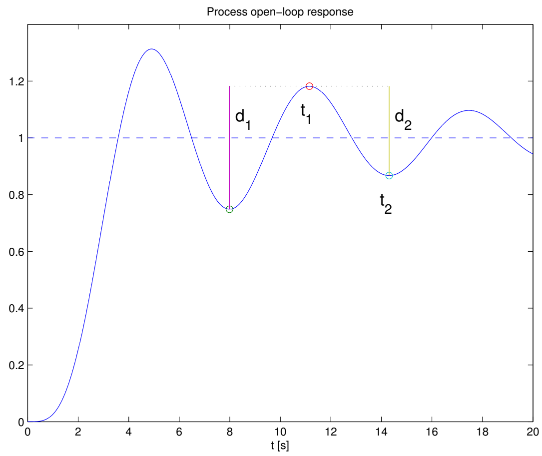

Figure 3. Open loop process response of the higher-order process with one pair of complex poles.

The efficiency of the PICS term will be illustrated on two process models.

Case 1

Consider the following second-order process model:

$$G_P(s) = \frac{1}{1+0.5s+s^2} . \tag{8}$$

According to expressions (1)-(4):

$$T_{pk} = 3.24 \qquad \delta = 0.444 . \tag{9}$$

The PICS term parameters are calculated from (6):

$$K_1 = 0.693 \qquad T_{dp} = 3.24 . \tag{10}$$

Figure 4 shows the process open-loop step response without (broken line) and with PICS term (solid line). It can be seen that the PICS term is very efficient in reducing process oscillations and overshoots.

Case 2

Consider the following fourth-order process model:

$$G_P(s) = \frac{1}{(1+0.2s+s^2)(1+s)^2} . \tag{11}$$

The open-loop response is shown in Figure 5 (see broken line). Since the process is of the higher order, measurement of the difference between the peaks should be performed (compare Figures 3 and 5):

$$
\begin{aligned}
d_1 &= 0.434 \\
d_2 &= 0.3148\,. \\
t_{dp} &= 3.16s
\end{aligned}
\tag{12}
$$

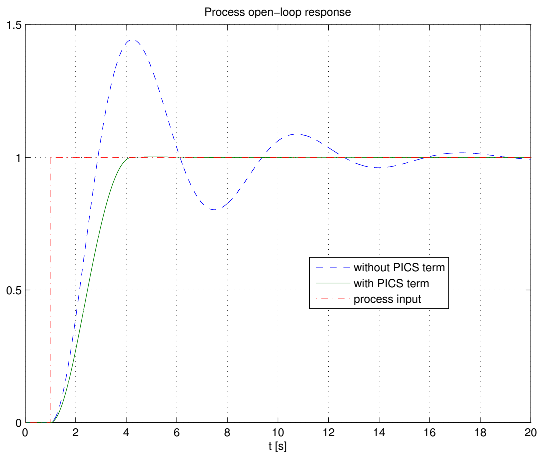

Figure 4. Open loop step-response on the second order process with (__) and without PICS term (---).

The PICS term parameters are then calculated from (7):

$$
\begin{aligned}
K_1 &= 0.58 \\
T_{dp} &= 3.16\,.
\end{aligned}
\tag{13}
$$

Figure 5 shows the process open-loop step response without (broken line) and with PICS term (solid line). Again, the efficiency of the PICS term can be clearly noticed.

Since the PICS term significantly decreases the overshoot and oscillations, it might be beneficial to use it within the closed-loop configuration.

In this case the controller has to be tuned for the following modified process:

$$G_{PP}(s) = \left[K_1 + (1 - K_1)e^{-sT_{dp}}\right]G_P(s)\,. \tag{14}$$

Unfortunately, most of the existing tuning methods for PID controllers are not defined for the above process type, since it consists of two additive terms. Moreover, one of the terms has additional pure time delay.

3. MOMI tuning method

A Magnitude Optimum (MO) tuning method makes the closed-loop amplitude (magnitude) response equal to one for as wide frequency range as possible. The MO criterion is relatively demanding, since it requires accurate process model in the frequency-domain. However, it was shown (Vrančić et al., 2001) that the same criterion can be achieved from time-domain measurement of the process steady-state change response. Since the calculation of controller parameters is based on multiple integrations of the process response, the modified method is called Magnitude Optimum Multiple Integration (MOMI) method.

The chosen PID controller structure is the following:

$$G_C(s) = \frac{K_I + K_P s + K_D s^2}{s(1 + T_F s)}\,, \tag{15}$$

where K_I, K_P and K_D are integral, proportional and derivative controller gains, respectively. Parameter T_F is the first-order filter time constant which filters all controller terms instead of derivative term only (Vrančić et al., 2005). This controller structure permit us to treat the PID controller as an ideal “schoolbook” controller:

$$G_{C0}(s) = \frac{K_I + K_P s + K_D s^2}{s}\,, \tag{16}$$

while filter term can be considered as a part of the process:

$$G_{PF}(s) = G_P(s)\frac{1}{(1 + T_F s)}\,. \tag{17}$$

This assumption significantly simplifies calculation of controller parameters. The PID controller parameters (16) are calculated from the following expression (Vrančić et al., 2001):

$$
\begin{bmatrix} K_I \\ K_P \\ K_D \end{bmatrix} =
\begin{bmatrix} -A_1 & A_0 & 0 \\ -A_3 & A_2 & -A_1 \\ -A_5 & A_4 & -A_3 \end{bmatrix}^{-1}
\begin{bmatrix} -0.5 \\ 0 \\ 0 \end{bmatrix}
\tag{18}
$$

where K_I, K_P and K_D are integral, proportional and derivative controller gains, respectively. Parameters A_0 to A_5 are the so-called process characteristic areas (moments) which can be calculated in time-domain by integrating filtered process G_{PF} (17) input and output signal during the process steady-state change:

$$
\begin{aligned}
&u_0(t) = \frac{u(t) - u(0)}{u(\infty) - u(0)} && y_0(t) = \frac{y(t) - y(0)}{u(\infty) - u(0)} \\
&I_{U1}(t) = \int_0^t u_0(\tau)d\tau && I_{Y1}(t) = \int_0^t y_0(\tau)d\tau \\
&I_{U2}(t) = \int_0^t I_{U1}(\tau)d\tau && I_{Y2}(t) = \int_0^t I_{Y1}(\tau)d\tau \\
&\vdots && \vdots
\end{aligned}
\;.
\tag{19}
$$

The areas can be calculated as follows:

$$
\begin{aligned}
&A_0 = y_0(\infty);\; y_1 = A_0 I_{U1}(t) - I_{Y1}(t) \\
&A_1 = y_1(\infty);\; y_2 = A_1 I_{U1}(t) - A_0 I_{U2}(t) + I_{Y2}(t) \\
&A_2 = y_2(\infty);\; y_3 = A_2 I_{U1}(t) - A_1 I_{U2}(t) + A_0 I_{U3}(t) - . \\
&\quad - I_{Y3}(t) \\
&\quad \vdots
\end{aligned}
\tag{20}
$$

On the other hand, the areas can also be obtained directly from the process transfer function. If filtered process transfer function G_{PF} (17) is described by the following expression:

$$G_P(s) = K_{PR}\frac{1 + b_1 s + b_2 s^2 + \cdots + b_m s^m}{1 + a_1 s + a_2 s^2 + \cdots + a_n s^n}e^{-sT_{delay}} \tag{21}$$

the areas can be calculated as follows (Vrančić et al., 2001):

$$
\begin{aligned}
A_0 &= K_{PR} \\
A_1 &= K_{PR}\left(a_1 - b_1 + T_{delay}\right) \\
A_2 &= K_{PR}\left[b_2 - a_2 - T_{delay}b_1 + \frac{T_{delay}{}^2}{2!}\right] + A_1 a_1 \\
&\vdots \\
A_k &= K_{PR}\left((-1)^{k+1}(a_k - b_k) + \sum_{i=1}^{k}(-1)^{k+i}\frac{T_{delay}{}^i b_{k-i}}{i!}\right) + \\
&+ \sum_{i=1}^{k-1}(-1)^{k+i-1}A_i a_{k-i}
\end{aligned}
\tag{22}
$$

Therefore, the controller parameters can be calculated either from non-parametric measurements of the process in time-domain (not restricted to step-response) or from parametric process model (21).

The MOMI tuning method usually results in a fast and non-oscillatory closed-loop responses for large set of process models. However, the MOMI method fails for some of oscillatory processes, where the calculated controller parameters give unstable closed-loop responses.

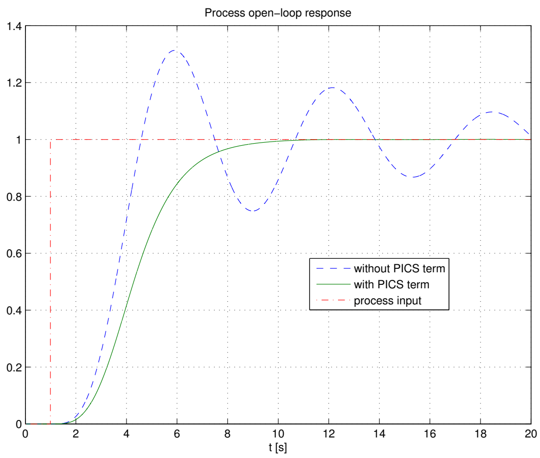

Figure 5. Open loop step-response on the fourth order process with (__) and without PICS term (---).

4. MOMI-PICS tuning method

The main idea of this paper is to apply PICS compensator before the process, as shown in Figure 2. Then the controller parameters can be calculated for the entire process with compensator G_{PP} (14). In time-domain, the areas can be calculated directly from the step-response of the process with PICS compensator (e.g. solid lines in Figures 4 and 5).

If the process is already expressed by a transfer function, the characteristic areas of the process with PICS compensator can be calculated in the following way. The areas of two multiplied transfer functions:

$$
G_{PP}(s) = G_{PF}(s)G_{PICS}(s) \tag{23}
$$

where characteristic areas of the filtered process $G_{PF}(s)$ are denoted as A_i and the areas of the PICS term $G_{PICS}(s)$ are denoted as A_{iPC}, can be calculated as follows:

$$
\begin{aligned}
A_{0PP} &= A_0 A_{0PC} \\
A_{1PP} &= A_0 A_{1PC} + A_1 A_{0PC} \\
A_{2PP} &= A_0 A_{2PC} + A_1 A_{1PC} + A_2 A_{0PC} \\
A_{3PP} &= A_0 A_{3PC} + A_1 A_{2PC} + A_2 A_{1PC} + A_3 A_{0PC} \\
&\vdots
\end{aligned}
\tag{24}
$$

The PICS compensator transfer function (5), when developed into infinite Taylor series, becomes:

$$
\begin{aligned}
G_{PICS}(s) &= 1 - (1-K_1)sT_{dp} + (1-K_1)\frac{s^2T_{dp}^2}{2!} - \\
&- (1-K_1)\frac{s^3T_{dp}^3}{3!} + \cdots
\end{aligned}
\tag{25}
$$

By comparing expressions (25) and (21), the areas (22) of the PICS term are:

$$
\begin{aligned}
A_{0PC} &= 1 \\
A_{1PC} &= (1-K_1)T_{dp} \\
A_{2PC} &= (1-K_1)\frac{T_{dp}^2}{2!} . \\
A_{3PC} &= (1-K_1)\frac{T_{dp}^3}{3!} \\
&\vdots
\end{aligned}
\tag{26}
$$

Inserting expression (26) into expression (24) gives us the characteristic areas of the process with PICS term:

$$
\begin{aligned}
A_{0PP} &= A_0 \\
A_{1PP} &= A_1 + (1-K_1)A_0 T_{dp} \\
A_{2PP} &= A_2 + (1-K_1)\left(A_1 T_{dp} + A_0\frac{T_{dp}^2}{2!}\right) \\
A_{3PP} &= A_3 + (1-K_1)\left(A_2 T_{dp} + A_1\frac{T_{dp}^2}{2!} + A_0\frac{T_{dp}^3}{3!}\right) \\
&\vdots
\end{aligned}
\tag{27}
$$

The PID controller parameters can be calculated from (18) by replacing areas A_i with A_{iPP}.

Let us now calculate the PID controller parameters for the same fourth-order process model as in case 2 (11). The chosen filter time constant of the PID controller was T_F=0.1s. Compensator parameters are given by expression (13), while areas of the process with first-order filter (17) can be calculated from (22):

$$
\begin{aligned}
&A_0 = 1,\ A_1 = 2.3,\ A_2 = 2.67,\ A_3 = 2.56, \\
&A_4 = 3.27,\ A_5 = 4.64
\end{aligned}
\tag{28}
$$

The characteristic areas of the process with PICS term are the following (27):

$$
\begin{aligned}
&A_{0PP} = 1,\ A_{1PP} = 1.33,\ A_{2PP} = 2.097, \\
&A_{3PP} = 2.21,\ A_{4PP} = 1.75,\ A_{5PP} = 1.103
\end{aligned}
\tag{29}
$$

The PID controller parameter can be calculated according to expression (18) by replacing areas A_i by A_{iPP}:

$$
K_I = 0.41,\ K_P = 0.98,\ K_D = 0.64 \tag{30}
$$

The closed-loop time response on step-change of the set-point is shown in Figure 6. The response on the reference change is smooth and without oscillations.

If the closed-loop structure would not include the PICS term, the PID controller parameters can be calculated directly from the process areas (28). However, the calculated controller parameters would lead to unstable closed-loop response.

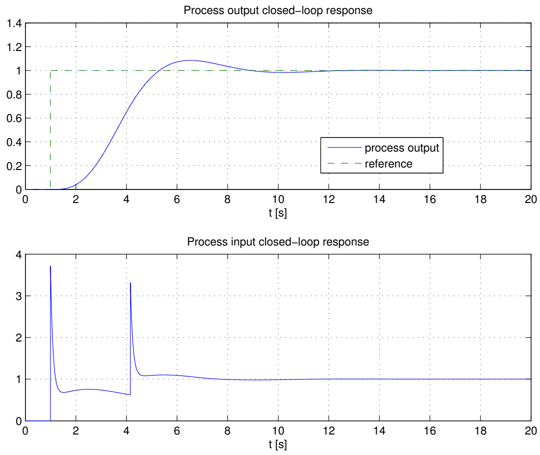

Figure 6. The closed-loop response on a set-point change on the fourth order process with PICS term.

5. EXAMPLES

Consider the following delayed third-order process model:

$$G_{P1}(s)=\frac{e^{-s}}{\left(1+0.5s+2s^2\right)\left(1+s\right)}. \tag{31}$$

The open-loop response of the process is shown in Figure 7 (broken line). By measuring the peaks, the following parameters have been obtained:

$$\begin{aligned} d_1 &= 0.4024 \\ d_2 &= 0.2289 . \\ t_{dp} &= 4.51s \end{aligned} \tag{32}$$

The PICS term parameters are calculated from (7):

$$\begin{aligned} K_1 &= 0.637 \\ T_{dp} &= 4.51 \end{aligned} . \tag{33}$$

The open-loop response of the process with PICS term is shown in Figure 7 (solid line). It is obvious that the PICS term is efficient in reducing oscillations in time-response.

The PID controller parameters have been calculated from the process parameters, a-priori chosen filter time constant T_F=0.1, and PICS term parameters according to procedure given in the previous section:

$$K_I = 0.28, K_P = 0.69, K_D = 0.48 \tag{34}$$

The closed-loop time response on step-change of the set-point is shown in Figure 8. The closed-loop response is smooth with very small overshoot and without noticeable oscillations.

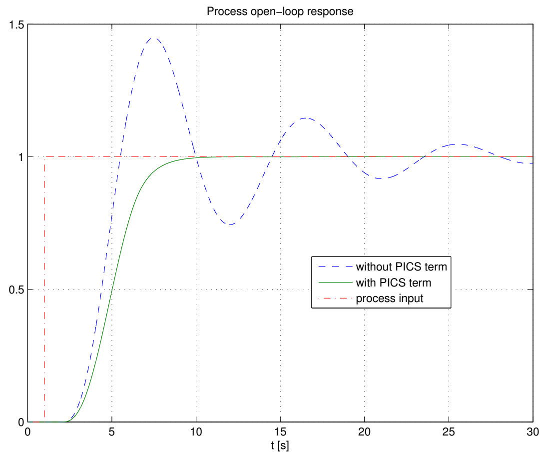

Figure 7. The open-loop response on input step-change of the process G_{P1} with (__) and without (---) PICS term.

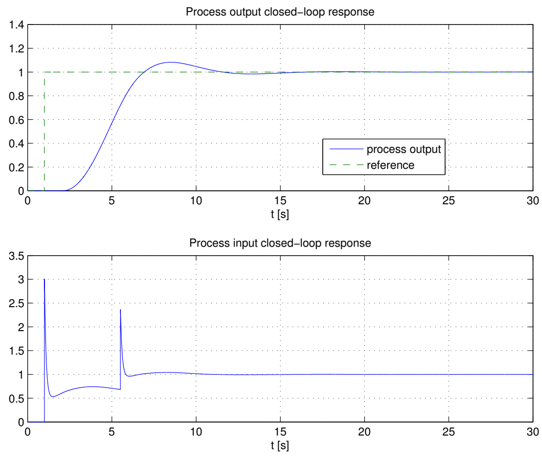

Figure 8. The closed-loop response on a set-point change on the process G_{P1} with PICS term.

The second example employs the sixth-order process model with minimum-phase zero:

$$G_{P2}(s)=\frac{(1-2s)e^{-s}}{\left(1+s+8s^2\right)\left(1+0.5s\right)^4}. \tag{35}$$

The open-loop response of the process is shown in Figure 9 (dashed line). By measuring the peaks, the following parameters have been obtained:

$$\begin{aligned} d_1 &= 0.3243 \\ d_2 &= 0.1845 . \\ t_{dp} &= 9.03s \end{aligned} \tag{36}$$

Then the PICS term parameters are calculated from (7):

$$\begin{aligned} K_1 &= 0.637 \\ T_{dp} &= 9.03 \end{aligned} . \tag{37}$$

The open-loop response of the process with PICS term is shown in Figure 9 (solid line). Similarly as in the previous cases, the PICS term efficiently reduce oscillations in time-response.

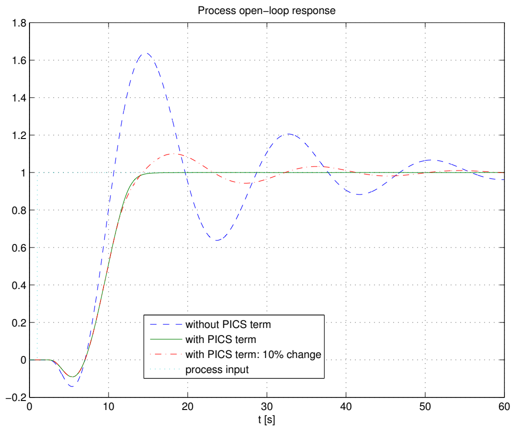

Figure 9. The open-loop response on input step-change of the process G_{P2} with (__, -.-) and without (---) PICS term.

The PID controller parameters have been calculated from the process parameters, a-priori chosen filter time constant T_F=0.1, and PICS term parameters according to procedure given in the previous section:

$$K_I = 0.104, K_P = 0.479, K_D = 0.659 \quad (38)$$

The closed-loop time response on step-change of the set-point is shown in Figure 10 (solid line). The response on the reference change is again smooth with very small overshoot and without noticeable oscillations.

In order to test robustness of the proposed method, the estimated parameter T_{pk} has been reduced by 10%. Since the PICS term is not optimal, there are some residual oscillations in the open-loop response (see dash-dotted line in Figure 9). Due to modified PICS term, the controller parameters became:

$$K_I = 0.094, K_P = 0.348, K_D = 0.361 \quad (39)$$

The closed-loop time response is shown in Figure 10 (dashed line). It can be seen that response is still very good without significant increase of the overshoot.

6. CONCLUSIONS

The results of the experiments showed that the proposed approach with the Posicast term inside the closed-loop configuration can significantly stabilise the process. The PID controller parameters are calculated according to the modified process by using the modified MOMI tuning method.

Test on several process models showed that the proposed approach resulted in a graceful tracking performance even for higher-order processes with a couple of complex poles and non-minimum zero. The method is relatively robust to change of posicast parameters.

Calculation of Posicast parameters can be performed easily from the process open-loop response (not necessarily step-response!) or from the process model. The controller parameters can be then calculated from posicast parameters and the process characteristic areas by appropriate modification of MOMI method, as given in the paper.

Our further research will be concentrated on optimisation of disturbance rejection performance and anti-windup solutions.

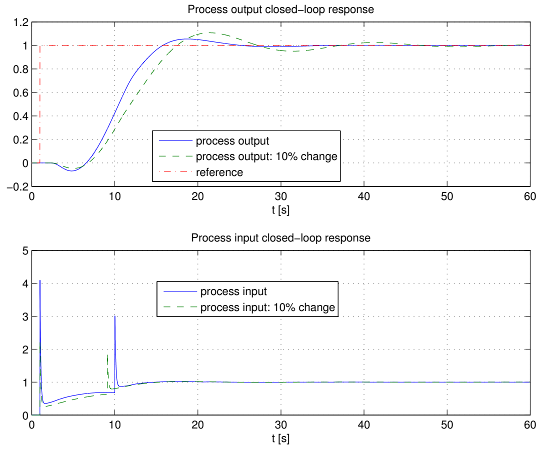

Figure 10. The closed-loop response on a set-point change on the process G_{P2} with PICS term.

REFERENCES

Huey J. R., Sorensen K. L. and Singhose W. E., (2008). Useful applications of closed-loop signal shaping controllers. Control Engineering Practice, Vol. 16, pp. 836-846.

Hung, H. Y., (2007). Posicast Control Past and Present. *IEEE Multidisciplinary Engineering Education Magazine*, Vol. 2, Nº 1, pp. 7-11.

Li Y.W., (2009), Control and Resonance Damping of Voltage-Source and Current-Source Converters with LC Filters. *IEEE Transactions on Industrial Electronics*. Vol. 56, Nº 5, pp. 1511-1521.

Seborg, D. E., Edgar, T. F. and Mellichamp, D. A. (1989). *Process Dynamics and Control*, John Wiley and Sons.

Singer N.C. and Seering W. P. (1990). Preshaping command inputs to reduce system vibration, *Journal of Dynamic Systems Measurement and Control*, Vol. 112 (3), pp. 76-82.

Singhose, W. (2009). Command Shaping for Flexible Systems: A Review of the First 50 Years, *Int. Journal of Precision Eng. and Manufacturing*, Vol. 10, No. 4, pp. 153-168.

Smith O. J. M. (1957). Posicast Control of Damped Oscillatory Systems, *Proc. IRE*, Vol. 45, No. 9, pp. 1249-1255.

Sorensen K. L., Singhose W. and Dickerson S. (2007). A controller enabling precise positioning and sway reduction in bridge and gantry cranes, *Control Engineering Practice* 15, pp. 825–837.

Vrančić D., Kristianssion B., Strmčnik S. and Oliveira P. M. (2005). Improving performance/activity ratio for PID controllers. *IEEE International Conference on Control and Automation*, Budapest, June 27-29, 2005, pp. 834-839.

Vrančić, D., Strmčnik S. and Juričić Đ. (2001). A magnitude optimum multiple integration method for filtered PID controller. *Automatica.* Vol. 37, pp. 1473-1479.