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Computing PID tuning regions based on fractional-order model set

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Abstract: The paper describes a new PID tuning approach suitable for both researcher and industrial practice. It justifies the authors' previous work where only intervals for particular controllers parameters were developed. Compared to other ones, the robustness regions method provides an admissible area of all controller parameters satisfying the required closed loop performance for exactly defined class of fractional processes. In contrast to common time domain tuning methods the process is characterized by three moments of its impulse response. The resulting areas serve as a common playground for future development of feature based tuning rules as shown in the illustrative example. The described procedure was partly implemented into the interactive Java applet freely accessible at `www.pidlab.com`.

Keywords: PID controllers, fractal systems, self-tuning control, robust control, Nyquist diagram, model set.

1. INTRODUCTION

There was a lot of PID tuning techniques developed since the popular Ziegler-Nichols method (Ziegler and Nichols (1942)) appeared. Some of them were evaluated in Ho et al. (1996). They are alltogether not very reliable (Schlegel (2002)) as they are only heuristic and based on one nominal process model. One of the most reputable tuning rules were authored by Åström and Hägglund (2006) where a large benchmark set of processes was integrated into the design procedure. Unfortunately, each method offers only nominal controller parameters. None of them provides an area of all parameters guaranteeing some required closed loop robustness and bandwidth.

The novel approach suggested in this paper grew up from the authors' previous results in the field of process identification, robustness regions method and fractional-order systems (Schlegel and Čech (2005)). They were bound together into an exact tuning procedure starting with a simple identification experiment and resulting into analytically computed regions of controller parameters ensuring fulfillment of common design specifications. All the actions are done in the frequency domain where one can get the biggest benefit from the fractional-order model set. Although this work concentrates on step/pulse identification experiment, the similar approach is applicable also for relay process identification. The paper justifies the results of authors' previous work Čech and Schlegel (2011) where only the intervals for particular controller parameters were provided. Instead of two ultimate processes, the whole model set boundary is taken into account during the design procedure. Hence the regions obtained are corrected. Moreover, the set of design specification is reduced to the fundamental ones. The pre-computed areas may serve in the future as the basis for development of advanced feature based controller parameterization.

The paper keeps the following structure: The fractional-pole process identification based on model set is briefly outlined in Section 2. The controller design problem is formulated in Section 3 where also the solution procedure – robustness regions method – is sketched out. Resulting controller parameter areas are partly documented in Section 4 together with a possible way out to feature based parameterization. Concluding remarks and ideas for further work are summarized in Section 5.

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2. IDENTIFICATION – MODEL SET APPROACH

Nowadays, when designing a robust controller in the frequency domain, it is usually assumed that the frequency response of the 'true' system lies for each frequency in regions of the complex plane (Rotstein et al. (1998)). These regions, called value sets, quantify the amount of model uncertainty at a given frequency. The problem of obtaining hard bounds of the value sets from experimental data has been considered before, see e.g. Helmicki et al. (1991); Milanese et al. (1996). The model set approach briefly summarized below deals with similar problems. This identification technique is efficient for subsequent robust design of any controller with fixed structure. The key idea is the combination of *a priori* assumption and experimental data (Fig. 1).

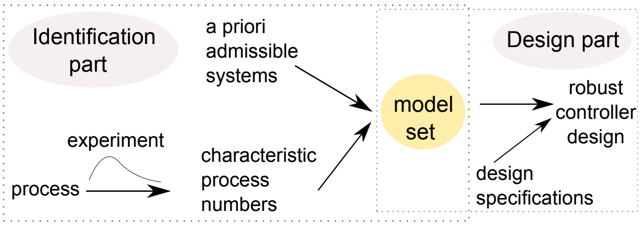

Fig. 1. General idea of the model set approach

2.1 *A priori admissible systems*

It was shown in Charef et al. (1992) that to cover the huge number of real processes, one has *a priori* to consider the transfer function in the form

$$P(s) = \frac{K}{\prod\limits_{i=1}^{p}(\tau_i s + 1)^{n_i}}, \qquad (1)$$

where p is arbitrary integer number and K, τ_i, n_i $i = 1, 2, \ldots, p$ are positive real numbers. The transfer function 1 describes very well the majority of essentially monotone processes (see Åström and Hägglund (2004) for definition).

Remark 1. If all n_i, $i = 1, 2, \ldots, p$ are integer numbers, one obtains a classical integer-order transfer function in a Bode's form.

Remark 2. If $p \to \infty$ then the set of all transfer functions (1) contains also processes with dead time and approximates several processes with transcendent transfer functions (like heat transfer, chemical processes, etc.).

2.2 *Characteristic numbers – experimental data*

Three-parameter time domain process description is well accepted in the control community. The authors' previous works vindicate the usage of first three moments m_i of the impulse response $h(t)$ instead of numbers obtained from the step response using its tangent line in the inflexion point. The application of time moments in control firstly appeared in Maamri and Trigeassou (1993). They are defined as

$$m_i = \int\limits_0^\infty t^i h(t) \mathrm{d}t, \qquad i = 0, 1, 2 \qquad (2)$$

and may be converted to another more suitable group of numbers $\{\kappa,\ \mu,\ \sigma^2\}$ (Schlegel et al. (2003)) defined as

$$\kappa = \int\limits_0^\infty h(t)\mathrm{d}t = m_0, \qquad \mu = \frac{\int_0^\infty t h(t)\mathrm{d}t}{\int_0^\infty h(t)\mathrm{d}t} = \frac{m_1}{m_0},$$

$$\sigma^2 = \frac{\int_0^\infty (t-\mu)^2 h(t)\mathrm{d}t}{\int_0^\infty h(t)\mathrm{d}t} = \frac{m_2}{m_0} - \frac{m_1^2}{m_0^2}. \qquad (3)$$

It can be proved (Čech (2008)) that for transfer function (1), it holds

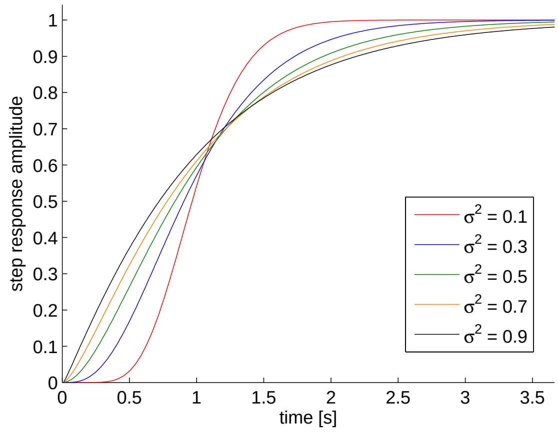

Fig. 2. Step response shaping by parameter σ^2

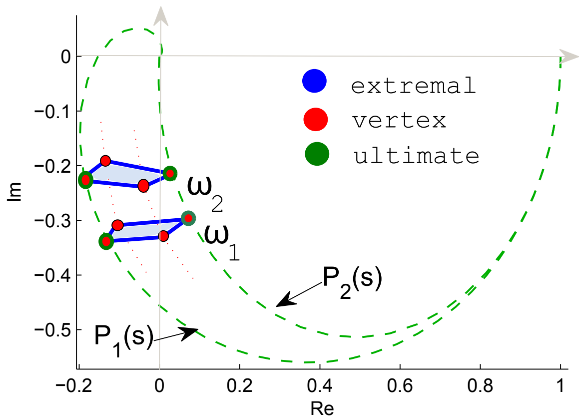

Fig. 3. Extremal, vertex and ultimate processes frequency responses

$$\kappa = K, \qquad \mu = \sum_{i=1}^{p} \tau_i n_i, \qquad \sigma^2 = \sum_{i=1}^{p} \tau_i^2 n_i. \qquad (4)$$

From a control point of view, κ is equal to process static gain and μ represents the residual time constant. Without loss of generality, the process can be normalized in gain and time, thus $\kappa = 1$ and $\mu = 1$. The remaining parameter σ^2 then has a meaning similar to normalized dead time. It shapes the step response from first order to pure dead time process as shown in Fig. 2.

Remark 3. The impulse response moments (2) or equivalently the numbers (3) can be obtained from the process step response or rectangular pulse response (Schlegel et al. (2003)). They may be also estimated from process input/ouput data.

Assumption 4. In the following let us assume that we have measured precisely the numbers (3) and other information about the process is not available.

2.3 *Model set, extremal, vertex and ultimate processes*

To make the paper more self-contained let us briefly remind basic definitions and lemmas.

Definition 5. (Model set). The transfer function $P(s)$ is *admissible* if and only if

(i) $P(s)$ is in the form (1), $n_i \geq m, \forall i, \sum_{i=1}^{p} n_i \leq n$, where $n \in \mathbb{R}^+$ is the total order of the process and $m \in \mathbb{R}^+$ is the minimum allowed order of each fractional pole.
(ii) $P(s)$ is consistent with experimental data, thus fulfills (4). The set of all admissible transfer functions will be called *model set* and denoted as $\mathcal{S}^{n,m}(\kappa, \mu, \sigma^2)$.

The following lemma (proved in Čech (2008)) answers the question, when the model set is not empty.
Lemma 6. Let $n \geq 2m$, then the model set $\mathcal{S}^{n,m}(\kappa, \mu, \sigma^2)$ is not empty if and only if

$$\frac{1}{n} \leq \frac{\sigma^2}{\mu^2} \leq \frac{1}{m}. \tag{5}$$

If the inequality (5) is satisfied then the model set contains for given characteristic numbers κ, μ, σ^2 infinite number of processes. Fortunately, these processes create after mapping into frequency domain a connected area called value set for each frequency $\omega > 0$.
Definition 7. (Value set). The set $\mathcal{V}_\omega^{n,m}(\kappa, \mu, \sigma^2) = \left\{P(\mathrm{j}\omega) : P(s) \in \mathcal{S}^{n,m}(\kappa, \mu, \sigma^2)\right\}$ will be called the *value set* of $\mathcal{S}^{n,m}(\kappa, \mu, \sigma^2)$ at the frequency $\omega > 0$.

The value set boundary is generated by so called extremal transfer functions.
Definition 8. (Extremal transfer function). The admissible transfer function $P(s) \in \mathcal{S}^{n,m}(\kappa, \mu, \sigma^2)$ will be called *extremal*, if there exists $\omega > 0$ such, that $P(\mathrm{j}\omega) \in \partial\mathcal{V}_\omega^{n,m}(\kappa, \mu, \sigma^2)$, where $\partial\mathcal{V}_\omega^{n,m}(\kappa, \mu, \sigma^2)$ denotes the value set boundary in complex plane. Let us denote the set of all extremal transfer functions as $\mathcal{S}_E^{n,m}(\kappa, \mu, \sigma^2)$.
Remark 9. For the *a priori* assumption (1) and condition (4) the set $\mathcal{S}_E^{n,m}(\kappa, \mu, \sigma^2)$ is independent on frequency ω.

The value set boundary is composed of finite number of smooth curves which end-points are created by *vertex* processes and the whole band is bounded by two *ultimate* processes $P_1(s)$ and $P_2(s)$ (see Fig. 3). In Section 3 the set of extremal processes will be used for robust controller design.

In authors previous works (Čech (2008); Schlegel et al. (2003)), the analytical relations for computing value set boundaries (extremal processes) were derived for both integer-order *(IO)* and fractional-order *(FO)* model set. In Fig. 4 one can examine that omitting fractional-order processes reduces the uncertainty markedly [1].
Remark 10. It is acceptable to define the minimum pole order as $m = 1$. Processes of order less than one do not have an equivalent in the real word. Besides, they extend more and more the model set uncertainty. On the contrary, the maximum process order need not to be restricted because the model set uncertainty (value sets size) converges very quickly for $n \to \infty$ and the generated extremal processes are quite easier to simulate in the time domain. Therefore, the normalized model set dependent only on σ^2 and denoted as $\mathcal{S}^{\infty,1}(\sigma^2)$ and the

[1] The dynamics of real distributed parameter processes is closed to fractional. Hence considering only *IO* processes in robust design may lead to non-satisfactory closed loop behavior.

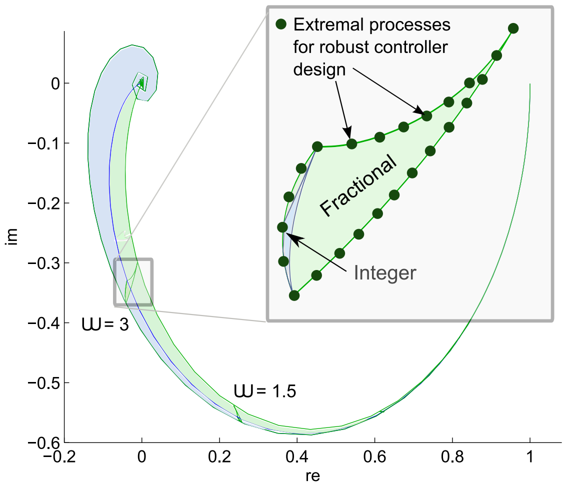

Fig. 4. Comparison of integer (blue) and fractional order (green) model set uncertainty in frequency domain ($\sigma^2 = 0.7$)

set of processes creating its value set boundary $\mathcal{S}_E^{\infty,1}(\sigma^2)$ will be further considered.
Corollary 11. It flows out from Lemma 6 that $\mathcal{S}^{\infty,1}(\sigma^2)$ is nonempty if and only if $\sigma^2 \in \langle 0, 1\rangle$.
Theorem 12. ($\partial\mathcal{V}^{\infty,1}$ parameterization). Let $n \geq 3m$, then for any $\omega > 0$ is the value set boundary $\partial\mathcal{V}_\omega$ of the normalized model set $\mathcal{S}^{\infty,m}(1, 1, \sigma^2)$ created by three arcs $P_i(s = \mathrm{j}\omega, \alpha)$, $\alpha \in I_i$, $i = 1, 2, 3$, which are defined as:

$$P_1(s, \alpha) = \frac{\mathrm{e}^{-(1-\sigma\sqrt{\alpha})s}}{\left(\frac{\sigma}{\sqrt{\alpha}}s + 1\right)^{\alpha}}, I_1 = \left\langle m, \frac{1}{\sigma^2}\right\rangle \tag{6}$$

and $P_i(s, \alpha)$, $i = 2, 3$ have common form

$$P_i(s, \alpha) = \frac{1}{(\tau_1(\alpha)s + 1)^{n_1(\alpha)}\,(\tau_2(\alpha)s + 1)^{n_2(\alpha)}}, \tag{7}$$

where for $i = 2$ it holds:

$$n_1 = \alpha, \quad n_2 = m, \quad I_2 = \left\langle \max\left\{m, \frac{1 - m\sigma^2}{\sigma^2}\right\}, \infty\right)$$

$$\tau_1(\alpha) = \frac{\alpha - \sqrt{\alpha m[(m+\alpha)\sigma^2 - 1]}}{\alpha(m+\alpha)}, \tag{8}$$

$$\tau_2(\alpha) = \frac{m + \sqrt{\alpha m[(m+\alpha)\sigma^2 - 1]}}{m(m+\alpha)} \tag{9}$$

and for $i = 3$ it holds:

$$n_1 = m, \quad n_2 = \alpha, \quad I_3 = \left\langle \max\left\{m, \frac{1 - m\sigma^2}{\sigma^2}\right\}, \frac{1}{\sigma^2}\right\rangle$$

$$\tau_1(\alpha) = \frac{m - \sqrt{\alpha m[(m+\alpha)\sigma^2 - 1]}}{m(m+\alpha)}, \tag{10}$$

$$\tau_2(\alpha) = \frac{\alpha + \sqrt{\alpha m[(m+\alpha)\sigma^2 - 1]}}{\alpha(m+\alpha)}\ . \tag{11}$$

The proof is available in Čech (2008) and is omitted for brevity.

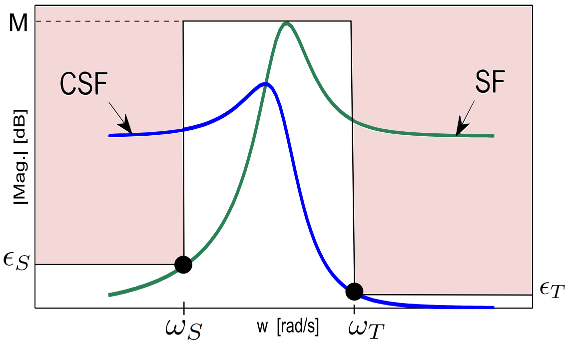

Fig. 5. Essential sensitivity functions limits

3. CONTROLLER DESIGN PROCEDURE

3.1 Problem formulation

Firstly, let us remind that for one nominal process $P(s)$, arbitrary controller $C(s)$ and after denoting $L(s) = P(s)C(s)$ (open loop) one can define sensitivity function $S(s)$ and complementary sensitivity function $T(s)$ as follows

$$S(s) = \frac{1}{1 + L(s)}, \quad T(s) = \frac{L(s)}{1 + L(s)}. \tag{12}$$

Our aim is to find for each σ the area $\mathcal{R}_\sigma$ of all controller parameters $C \in \mathcal{R}_\sigma$ satisfying the design specifications $\mathcal{DS} = \{M, \epsilon_T, \omega_T, \epsilon_S, \omega_S, M_{MAX}\}$ defined as follows:

$\forall C \in \mathcal{R}_\sigma \ \ \exists P(s) \in \mathcal{S}^{\infty,1}(\sigma^2):$

A. Sensitivity functions M-limit

$$\sup_\omega |S(\mathrm{j}\omega)| \le M \quad \wedge \quad \sup_\omega |T(\mathrm{j}\omega)| \le M, \tag{13}$$

B. Maximum bandwidth

$$|T(\mathrm{j}\omega)| \le \epsilon_T, \quad \omega \in \langle \omega_T, \infty) , \tag{14}$$

C. Low frequency disturbance rejection

$$|S(\mathrm{j}\omega)| \le \epsilon_S, \quad \omega \in \langle 0, \omega_S \rangle , \tag{15}$$

$\forall C \in \mathcal{R}_\sigma \ \ \forall P(s) \in \mathcal{S}^{\infty,1}(\sigma^2):$

D. Robust stability limit

$$\sup_\omega |S(\mathrm{j}\omega)| \le M_{MAX} \wedge \sup_\omega |T(\mathrm{j}\omega)| \le M_{MAX}. \tag{16}$$

Remark 13. In A, B and C, the $\exists$ quantifier is used as one wants to ensure that for any $P(s) \in \mathcal{S}^{\infty,1}(\sigma^2)$ the controller parameters leading to the closed loop 'touching' the defined limits can be found in $\mathcal{R}_\sigma$ as shown in Fig 5. Having in mind that one never knows which $P(s) \in \mathcal{S}^{\infty,1}(\sigma^2)$ describes the real plant, the minimum stability robustness must be ensured for the whole model set. Thus the $\forall$ quantifier is used in D.

3.2 Robustness regions method

The problem defined in Section 3.1 is analytically solvable only for low order controllers or compensators. In authors recent papers, the robustness regions method based on classical D-partition (Neimark (1948)) was extended to solve the problem for PID controller with filtered derivative part described as

$$C(s) = K\left(1 + \frac{1}{T_I s} + \frac{T_D s}{\frac{T_D}{N}s + 1}\right), \quad K_I = \frac{K}{T_I}. \tag{17}$$

For engineers, it is practical to figure the region of admissible controller parameters in $K - K_I$ plane [2]. For PID controller in the form 17 one has to fix the remaining two parameters. Fortunately, the ratio $f = T_I/T_D$ is usually near to 0.25 (Ziegler and Nichols (1942)) and the filter in derivative part is often chosen in the interval $N \in \langle 2, 10 \rangle$ according to the signal/noise ratio in the individual control application.

Remark 14. The values of f and N have a strong influence on the region shape and consequently on resulting controller parameters. For further demonstration of the approach $N = 4$ and $f = 0.25$.

Remark 15. After varying f within proper limits one can visualize the 3D PID tuning area in $K - K_I - K_D$ space.

Firstly, let us show the method basic principle for condition (13) and one process $P(s) \in \mathcal{S}^{\infty,1}(\sigma^2)$. It is well-known that (13) can be wrapped by the condition that the Nyquist plot $L(\mathrm{j}\omega)$ should not enter a M-circle with a given origin $c \in \mathbb{R} = -(2M^2 - 2M + 1)/(2M(M-1))$ and radius $r \in \mathbb{R}^+ = (2M-1)/(2M(M-1))$ denoted as $U(c, r)$. Further let us denote

$$L(\mathrm{j}\omega) \triangleq u + \mathrm{j}v, \qquad \frac{\mathrm{d}}{\mathrm{d}\omega}L(\mathrm{j}\omega) \triangleq u_1 + \mathrm{j}v_1. \tag{18}$$

Proposition 16. Choice of any of the limit controller parameters at the region boundary will lead to the Nyquist plot $L(\mathrm{j}\omega)$ plot which is tangential to a given circle $U(c, r)$ at some frequency $\omega > 0$, thus

$$(u - c)^2 + v^2 = r^2, \qquad (u - c)u_1 + vv_1 = 0. \tag{19}$$

To get the pair of controller parameters K, K_I from (18) and (19) for each $\omega > 0$ [3], the eight order polynomial equation [4] must be numerically solved. Only the positive values of controller parameters are selected from the set of all roots. In the analogous way, the regions for all design specifications (13–16) may be computed.

Remark 17. The design specifications $\mathcal{DS}$ are contradictory. The fulfillment of condition (15) need some minimum control power, while the rest of conditions restrict the control gains. Hence, the admissible region of controller parameters lies for the conditions (15) 'outside' the computed boundary. Actually, for particular controller structure and values of design specifications $\mathcal{DS}$ one can get an empty region of admissible controller parameters.

[2] The $K - K_I$ plane is applicable also for PI controllers which are the most employed in process control.

[3] For depicting regions, the suitable set (usually logspace-type) of discrete values of ω must be chosen.

[4] The full relations of polynomial coefficients exceed the page limit of this paper.

3.3 Robust design

Roughly speaking, it can be proved that to find a solution of a problem defined in Section 3.1 it is sufficient to deal with extremal processes $\mathcal{S}_E^{\infty,1}(\sigma^2)$ exactly defined by the Theorem 12 which create the boundary of the value set. To be more precise:

Lemma 18. Let us denote the region of all controller parameters satisfying design specification X for one nominal process P as $\mathcal{R}_\sigma^X(P)$. Then the region R_σ can be computed only using extremal processes as follows

$$\mathcal{R}_\sigma = \bigcup_{\forall P \in \mathcal{S}_E^{\infty,1}(\sigma^2)} \mathcal{R}^A(P) \cap \mathcal{R}^B(P) \cap \mathcal{R}^C(P) \bigcap_{\forall P \in \mathcal{S}_E^{\infty,1}(\sigma^2)} \mathcal{R}^D(P). \tag{20}$$

The lemma is based on the representative subset principal which is proved in Schlegel (2000) under the assumption that the value set is a compact area.

Remark 19. For numerical computation of the region, the value set boundary is sampled as shown in Fig. 4.

3.4 Choice of design specifications

The regions may be computed for any particular values of design specifications $\mathcal{DS}$. The following design specifications suitable for most of real applications were chosen to depict the tuning areas: $M = 1.4$ (inspired by Åström and Hägglund (2006)). Further let us assume that there is known the maximum closed loop bandwidth ω_T^{MAX} where the loop with PID controller and process $P(s)$ is on the stability limit, e.g. $L(\mathrm{j}\omega)$ passes the point $(-1, \mathrm{j}0)$. Then

$$\begin{aligned} \omega_T &= \omega_T^{MAX}/1.5,\ \epsilon_T = 0.707 = -3 \text{ [dB]}, \\ \omega_S &= \omega_T^{MAX}/100,\ \epsilon_S = 0.1 = -20 \text{ [dB]}. \end{aligned} \tag{21}$$

At last the robust stability limit which has to be fulfilled for any $P(s) \in \mathcal{S}_E^{\infty,1}(\sigma^2)$ after selecting any controller $C(s)$ from $\mathcal{R}_\sigma$ was chosen as $M_{MAX} = 2.3$.

4. PID TUNING REGIONS

Using the procedure described in Section 3 the robustness regions for discrete values $\sigma = 0.1, 0.2, \ldots, 0.9$ (chosen according to lemma 6) were computed. An example of resulting region is shown in Fig. 6. The developed regions serve as fundamental limitations for controller parameter. One can choose the controller parameters inside these areas according to additional requirements. One utilization was presented in Čech and Schlegel (2011) where the intervals for controller gains were estimated. The following example shows the direction to get controller tuning parameter defining the closed loop bandwidth.

Example 1. Suppose that the real plant is described by the fractional-order transfer function

$$P(s) = \frac{3}{(0.38s+1)^{3.4}(0.72s+1)}. \tag{22}$$

From the identification experiment one should obtain according to (4) the characteristic numbers

$$\kappa = 3, \quad \mu = 2.01, \quad \sigma^2 = 1.01. \tag{23}$$

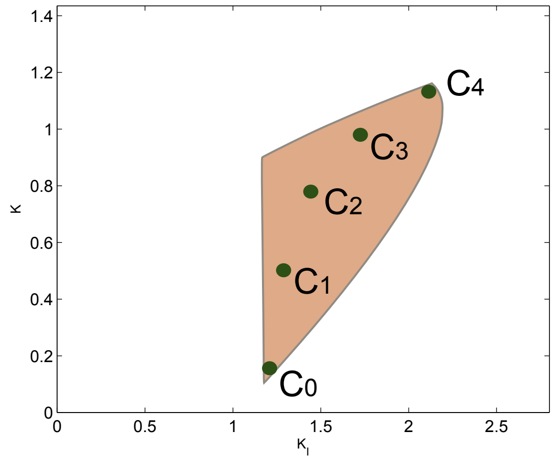

Fig. 6. Example of the normalized PID tuning region for $\sigma = 0.5$; Illustrative example normalized controller parameters $\overline{K}$, $\overline{K_I}$

After computing the normalized $\overline{\sigma} = \sigma/\mu \doteq 0.5$ the controller parameters may be chosen in the appropriate region. The choices shown in Fig. 6 lead to the closed loops with different bandwidths. The normalized parameters $\overline{K}, \overline{K_I}$ of all controllers $C_0 - C_4$ can be finally denormalized in gain and time as follows

$$K = \overline{K}/\kappa, \quad K_I = \overline{K_I}/(\kappa\mu) \tag{24}$$

which to the required set of controller parameters:

$$\begin{aligned} C_0 &: K = 0.06, K_I = 0.20, \\ C_1 &: K = 0.17, K_I = 0.22, \\ C_2 &: K = 0.26, K_I = 0.24, \\ C_3 &: K = 0.32, K_I = 0.28, \\ C_4 &: K = 0.37, K_I = 0.35. \end{aligned} \tag{25}$$

The responses of closed loop with controllers (25) applied to the process (22) are shown in Fig. 7. The amplitude Bode plots of resulting control loops are shown in Fig 8. It is evident that choosing different controllers inside the arising regions lead to bandwidth variations (about 1/3 of decade) while the hard limits of closed loop performance are preserved.

Remark 20. The time-domain behavior may be further optimized by the feed-forward control part which does not affect the closed loop robustness.

Hence the tuning procedure may be summarized into a following algorithm [5] :

Algorithm 1. (Tuning procedure). Obtaining characteristic numbers (3) from pulse identification experiment → normalization $\overline{\sigma} = \sigma/\mu$ → choice of controller parameters inside the area $\mathcal{R}_{\overline{\sigma}}$ → denormalization of controller parameters (24).

[5] The similar approach dealing with integer-order processes was already proved suitable in real autotuners

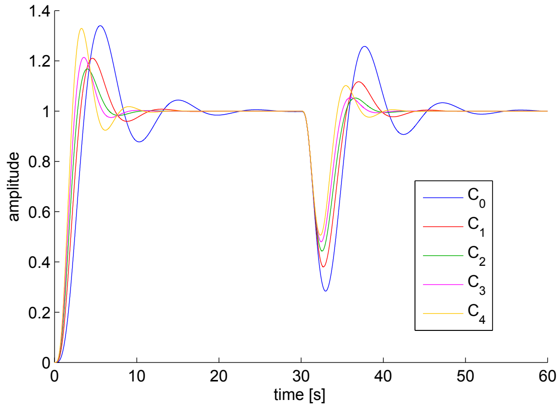

Fig. 7. Closed loop step responses and load disturbance responses for different controllers C_i (Example 1)

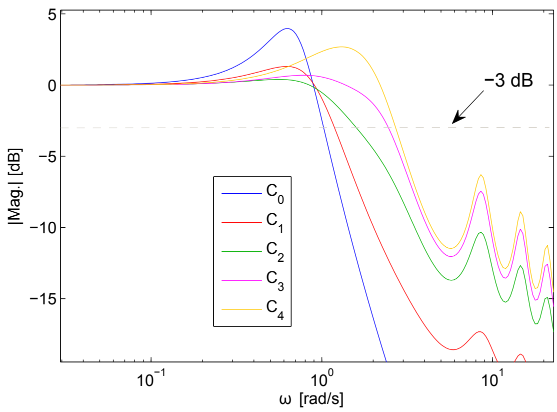

Fig. 8. Closed loop bandwidth different controllers C_i (Example 1)

5. CONCLUSION

In this paper a new PID tuning technique is presented. It is based on model set approach combining *a priori* assumption about the process transfer function together with a few characteristic numbers obtained from the real measurement. To cover the majority of real process plants, also the fractional-order-pole processes were included into the class of *a priori* admissible systems. The process characteristic numbers – moments of the impulse response – may be obtained simply by a rectangular pulse identification experiment. Further, the robustness regions method is used to find the controller parameters. In contrast to other PID tuning methods the suggested approach gives an area of all controller parameters ensuring fulfillment of fundamental frequency domain requirements (proper bandwidth, sensitivity function limits, etc.). The procedure was partly implemented and packed into the interactive Java applet freely accessible at `www.pidlab.com`. In the future, the developed areas will serve as a basis for development several feature based controller parameterization rules. For instance, industrial practitioners working in process control field can benefit from an additional tuning knob specifying the closed loop bandwidth.

REFERENCES

Åström, K. and Hägglund, T. (2004). Revisiting the Ziegler - Nichols step response method for PID control. *Journal of Process Control*, 14(6), 635 – 650.

Åström, K. and Hägglund, T. (2006). *Advanced PID control*. ISA - Instrumentation, Systems and Automation Society. ISBN: 1-55617-942-1.

Čech, M. (2008). *Robust controller design for fractional-order systems*. Ph.D. thesis, University of West Bohemia, Pilsen.

Čech, M. and Schlegel, M. (2011). Interval PID tuning rules for a fractional-order model set. In *Proceedings of IFAC World Congress 2011*. Milano, Italy.

Charef, A., Sun, H., Tsao, Y., and Onaral, B. (1992). Fractal system as represented by singularity function. *IEEE Transactions on Automatic Control*, 37, 1465–1470.

Helmicki, A., Jacobson, C., and Nett, C. (1991). Control oriented system identification: a worst case deterministic approach in H_∞. *IEEE Transaction Automatic Control*, 36, 1163–1176.

Ho, W., Hang, C., and Zhou, J. (1996). Performance and gain and phase margins of well-known PID tuning formulas. *IEEE Transaction on control systems technology*, 3(2).

Maamri, N. and Trigeassou, J.C. (1993). PID design for time delayed systems by the method of moments. In *European Control Conference, Groningen Holland*.

Milanese, M., Norton, J., Piet-Lahavier, H., Walter, E., and Eds. (1996). *Bounding approaches to system identification*. Plenum, New York.

Neimark, Y. (1948). Structure of D-partition of the space of polynomials and the diagram of Vishnegradskii and Nyquist. *Akad Nauk SSSR*, 59, 853.

Rotstein, H., Galperin, N., and Gutman, P. (1998). Set membership approach for reducing value sets in the frequency domain. *IEEE Transactions on Automatic Control*, 43, 1346–1350.

Schlegel, M. (2000). *New approach for robust design of industrial controllers*. Habilitation thesis, University of West Bohemia, Pilsen.

Schlegel, M. (2002). Exact revision of the Ziegler-Nichols frequency response method. In *Proc. of IASTED Int. Conf. on control and applications*. Cancun, Mexico.

Schlegel, M., Balda, P., and Štětina, M. (2003). Robust PID autotuner – method of moments. *Automatizace*, 46(4).

Schlegel, M. and Čech, M. (2005). Computing value sets from one point of frequency response with applications. In *Proceedings of IFAC World Congress 2005*. Praque, Czech Republic.

Ziegler, J. and Nichols, N. (1942). Optimum settings for automatic controllers. *Trans. ASME*, 12, 759.