# Tuning and performance assessment of complex fractional-order PI controllers

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**Abstract:** In this paper, we propose an optimization-based tuning methodology for real and complex Fractional-Order Proportional-Integral (FOPI) controllers. The proposed approach hinges on a modified version of the Integral Absolute Error (IAE) sensitivity-constrained optimization problem, which is suitably adapted to the design of fractional controllers. As such, it allows the exploitation of the potentiality of the (possibly complex) fractional integrator. We also propose a method, based on the well-known CRONE approximation, which delivers a band-limited real-rational approximation of the real part of the complex-order integrator. Finally, based on a First-Order-Plus-Dead-Time (FOPDT) model of the process, we use our design and approximation techniques to find an optimal tuning for real, complex fractional-order, and integer PI controllers and we provide a quantitative performance assessment.

*Keywords:* Fractional control, complex fractional-order, PI control, tuning, optimization.

## 1. INTRODUCTION

The Proportional-Integral-Derivative (PID) control algorithm is ubiquitous in industry because of its capability to provide an excellent cost/benefit ratio and because of the clear physical interpretation of its parameters (Visioli, 2006; Arrieta and Vilanova, 2007; Vilanova et al., 2012). The PID algorithm is based on two fundamental operations: integration and differentiation. In a classical framework, these operations can only be repeated a finite (integer) number of times. While this renders the classical differential operators rather intuitive, it constrains the derivative and integral orders to be integer, often resulting in an underexploitation of the potentiality of the controller. Real fractional calculus hinges on the removal of this limitation, thus allowing the differential order to assume any real value (Podlubny, 1999a; Padula et al., 2013; Padula and Visioli, 2014). The Fractional-Order PID (FOPID) controller is obtained by exploiting fractional calculus to generalize the derivative and integral parts of the classical PID controller (Podlubny, 1999b). FOPID controllers have attracted a great deal of attention in recent years because they provide better performance and improved robustness (Padula and Visioli, 2015; Vinagre et al., 2007; Caponetto et al., 2013; Muresan et al., 2016; Tepljakov et al., 2014; Tejado et al., 2014; Beschi et al., 2016). The main challenge in the tuning of a fractional PID controller lies in the large number of parameters to be tuned (five against the three of a classical PID controller, the two additional ones being the differential orders). To address this problem, different methods for the design of a FOPID controller have been proposed in the literature, where different objective functions have been taken into account, see e.g. (Monje et al., 2008, 2004; Padula and Visioli, 2012) and various tuning rules have been suggested (Li et al., 2010; Valério and Sá da Costa, 2010; Padula and Visioli, 2011).

The positive results of FOPID controllers obtained in the aforementioned references suggest that a worthwhile research direction may involve the generalization of their structure along different directions. In this context, a natural generalization of the FOPID controller appears to be the Complex FOPID controller (CFOPID) obtained by allowing the derivative and integral actions to assume any *complex* value (Valério and Sá da Costa, 2013). The complex-order differentiation has been firstly exploited for control purposes in the so-called third generation CRONE controller (Lanusse et al., 2015). In (Tenreiro Machado, 2013), new controller architectures using the complex-order derivatives concept are proposed, and in (Shahiri et al., 2015, 2016) a Complex Fractional-Order Proportional Integral (CFOPI) controller design methodology is proposed. However, in contrast with the wealth of literature in the area of FOPID control, CFOPID controllers are still largely uncharted territory (Shah and Agashe, 2016). In particular, a systematic quantitative evaluation of the improvement of the performance/robustness that can be achieved by using a CFOPID controller is missing, and this prevents the user from being able to evaluate the advantages/disadvantages of CFOPID controllers against their increased complexity. The reason for the lack of results in complex-order fractional control and, more specifically, in CFOPID control, is that dealing with complex fractional operators is considerably more challenging than dealing with their real counterparts. Indeed, when using a complex-order integrator, the controller itself results in a complex differential equation. For control purposes, however, we are interested in designing a controller that produces a real-valued signal. Taking the real part of the output of the controller may, however, result in an undesired frequency behavior. Moreover, because of their non-local nature, fractional operators require to be approximated. To this end, the well-known Oustaloup method, hereafter referred to as CRONE approximation, can be conveniently used also in the complex case (Oustaloup et al., 2000). This, however, results in a complex-rational transfer function, i.e., in a classical complex linear differential equation, from which we need to extract a real signal.

In this paper, we restrict our attention to CFOPI controllers, and we provide a quantitative assessment of the performance achievable with CFOPI, FOPI and PI controllers. Firstly, we present some preliminary results for a generalization of the CRONE approximation that results in a *real-rational* transfer function which approximates the real part of the fractional complex-order integrator. Secondly, we define an optimization problem based on a modified version of the sensitivity-constrained IAE minimization problem, which is suitable for (possibly complex) fractional integrators. Indeed, when considering the IAE minimization as the objective, the obtained optimal integrator is always of integer order (Padula and Visioli, 2011, 2012, 2015; Sánchez et al., 2017) for first-order processes. To overcome this limitation, we introduce a dead-band around the steady-state which allows the exploitation of the potentiality of the (complex) fractional integrator. We find an optimal tuning for CFOPI, FOPI and PI regulators by numerically solving the modified optimization problem for FOPDT processes with different normalized dead times. We also consider different control tasks, different levels of sensitivity and different dead-bands. Finally, we compare the performance achieved with the three regulators in all the cases mentioned above. The obtained results show that the use of a CFOPI is a viable solution in practice and that, in particular when robustness is of main concern, it can deliver much better performance while maintaining the same level of robustness.

The paper is organized as follows. Complex fractional-order PI controllers are described in Section 2. The approximation of $\Re(s^\nu)$ is obtained in Section 3. In Section 4, the optimization problem is formulated, and the performance assessment is provided in Section 5, along with a detailed discussion. Simulation results are then shown in Section 6. Finally, conclusions are drawn in Section 7.

## 2. CFOPI CONTROLLER

In this section, we briefly present, for the reader's convenience, the structure of the CFOPI. It is well-known that the FOPI controller has the following transfer function (Padula and Visioli, 2015)

$$C(s) = K_p\left(1+\frac{1}{T_i s^\nu}\right), \quad \nu \in \mathbb{R}, \tag{1}$$

where $K_p$ is the proportional gain, $T_i$ is the integral time constant and $\nu$ is the fractional integral order. Note that the FOPI controller represents a genuine generalization of the PI controller which can be obtained as a special case of the former by setting $\nu = 1$. The main advantage of the FOPI controller with respect to the standard PI is the possibility of continuously regulating the low-frequency phase delay introduced by the integrator. As a consequence, however, the slope of the magnitude of the Bode plot changes accordingly.

A further generalization can be obtained by allowing the integral order $\nu$ to span over the complex set $\mathbb{C}$. Indeed, this operation enables, in principle, to regulate both the magnitude slope and the phase slope independently, (Valério and Sá da Costa, 2013). This, however, results in a controller whose dynamics is represented by a complex fractional differential equation. This is clearly unsuitable for control purposes where the controller input, i.e., the control error, is a real (physical) signal and the controller output, i.e., the control variable, must be a real signal. To overcome this limitation, along the lines of (Shahiri et al., 2015, 2016), we consider the following controller

$$C(s) = \Re\left(K_p\big(1+\frac{1}{T_i s^\nu}\big)\right) = K_p\left(1+\frac{1}{T_i \Re(s^\nu)}\right), \quad \nu \in \mathbb{C}, \tag{2}$$

where $\nu = \lambda + \mathrm{i}\mu$ and

$$\begin{aligned}\Re(s^\nu) &= \Re(s^\lambda\, \cos(\mu\, \log s) + \mathrm{i} s^\lambda\, \sin(\mu\, \log s)) \\ &= \begin{cases} s^\lambda\, \cos(\mu\, \log s), & \text{if } \mu \le 0 \\ s^\lambda\, \sec(\mu\, \log s), & \text{if } \mu > 0. \end{cases}\end{aligned} \tag{3}$$

Note that when $\mu = 0$ we re-obtain a FOPI; furthermore, when also $\lambda = 1$, a PI controller is obtained.

A complete discussion on the frequency behavior of $\Re(s^\nu)$ compared to the one of $s^\nu$ is out of the scope of this paper; the reader may refer to (Valério and Sá da Costa, 2013) for a comprehensive discussion. We stress, however, that $\Re(s^\nu)$ provides a good approximation of $s^\nu$ and we can still independently regulate phase and magnitude slopes of the Bode diagram through the parameters $\lambda$ and $\mu$.

## 3. APPROXIMATION OF $\Re(S^\nu)$

In order to implement the CFOPI controller (2), we need the controller algorithm to require a computational effort that remains constant over time. A viable solution is the approximation of the controller transfer function in the frequency domain with a system of finite order. To this end, the frequency behavior of $s^\nu$ can be approximated in a given frequency range $[\omega_l,\ \omega_h]$ by using the CRONE continuous approximation (Oustaloup et al., 2000), which consists of the following recursive distribution of zeros and poles:

$$s^\nu \approx S(s)_\nu = k\prod_{n=1}^{N}\frac{1+\frac{s}{\omega_{z,n}}}{1+\frac{s}{\omega_{p,n}}}, \tag{4}$$

where $\omega_{z,n}$ and $\omega_{p,n}$ are, respectively, the frequencies of the zeros and the poles, and the gain $k$ is adjusted so that the right side of (4) has unity gain at the gain crossover frequency of $s^\nu$. When $\nu \in \mathbb{C}\setminus\mathbb{R}$, however, the CRONE approximation provides a complex-rational transfer function, i.e., a transfer function which is the ratio of polynomials with complex coefficients. Since we need a controller that produces a real signal, we need to extract the real part from the CRONE approximation. Given $\nu = \lambda + \mathrm{i}\mu$, consider the transfer function

$$\frac{N(s)}{D(s)} = \begin{cases} S(s)_\nu, & \text{if } \mu < 0 \\ S(s)_{-\nu}, & \text{if } \mu > 0 \end{cases} \tag{5}$$

and the polynomial $\overline{D}(s)$, whose coefficients are the complex-conjugate of the coefficients of $D(s)$. Now build two polynomials $H(s) = D(s) * \overline{D}(s)$ and $L(s) = N(s) * \overline{D}(s)$, where the symbol $*$ denotes the convolution operator. The polynomial $H(s)$ has, by construction, real coefficients. Finally, we obtain a real-rational approximation of the real part of $s^\nu$ as

$$\Re(s^\nu) \approx S(s)_\nu^{\mathbb{R}} = \begin{cases} \frac{\Re\big(L(s)\big)}{H(s)}, & \text{if } \mu < 0 \\ \frac{H(s)}{\Re\big(L(s)\big)}, & \text{if } \mu > 0, \end{cases} \tag{6}$$

where $\Re\big(L(s)\big)$ is the polynomial obtained by taking the real part of the coefficients of $L(s)$. Given a CRONE approximation $S(s)_\nu$ of order $N$, the proposed approach delivers a real-rational approximation of order $2N$ of the real-part $\Re(s^\nu)$. It is worth stressing that the obtained approximation might be nonminimum-phase, unstable or both. Indeed, the reason for choosing a complex-order fractional operator is that we can regulate phase and magnitude slopes *independently*. However, it is well-known that (using the Bode formula) it is possible to entirely reconstruct the magnitude of a system from the phase, and vice-versa, if and only if the system is stable and minimum phase. Nevertheless, considering that we want to approximate the fractional complex-order behavior in a *finite* frequency range $[\omega_l,\ \omega_h]$ and that the Bode formula is integral in nature, the approximation of $\Re(s^\nu)$ can still be stable and minimum phase, mainly depending on the width of the frequency range and on the imaginary part $\mu$ of the fractional order. A full characterization of the relationship between all the parameters involved in the approximation and the stability/minimum-phase property of the approximation is currently under investigation. However, we can say that in general the approximation becomes unstable and minimum-phase when $\mu >> 0$, and stable and nonminimum-phase when $\mu << 0$.

It is worth stressing that, when $\mu < 0$, it is possible to simply use the CRONE approximation $S(s)_\nu$ and then take the real part of the output signal. This, however, does not prevent the systems from becoming nonminimum phase. Indeed, the reader may easily check via numerical simulation that the transfer function of this approximation (which is well-defined because "taking the real part" is a linear operation) is the same of $S(s)_\nu^{\mathbb{R}}$. On the contrary, when $\mu > 0$ this simple approach does not produce any good approximation of $\Re(s^\nu)$ in general. This is intuitive considering that $S(s)_\nu$ is always stable and minimum-phase, and taking the real part of the output signal cannot destabilize the approximation, which should however be unstable when $\mu >> 0$.

## 4. PROBLEM FORMULATION

Consider the unity-feedback control scheme of Figure 1. The process is assumed to have a FOPDT dynamics of the form

$$P(s) = \frac{K}{Ts+1} e^{-Ls}, \tag{7}$$

where $K$ is the gain, $T$ is the time constant and $L$ is the dead time. The process dynamics can be characterized by means of the so-called normalized dead time, which is defined as

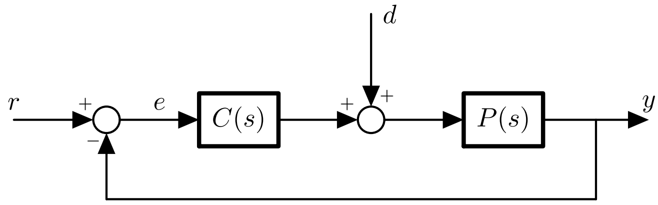


Fig. 1. The considered control scheme.

$$\tau = \frac{L}{L+T}, \tag{8}$$

and provides a measure of the difficulty of controlling the process. In this paper, we consider normalized dead times in the range $0.05 \leq \tau \leq 0.85$. Note that when $\tau < 0.05$ the control problem becomes trivial, and when $\tau > 0.85$, other control paradigms, such as the Smith predictor, are more appropriate.

For the purpose of tuning the controller, a widespread approach in process control is to find the set of parameters that minimizes a given functional. Among the possible optimization functions, the IAE is often chosen, despite the difficulties that a non-convex optimization problem involves, because it yields, in general, a low overshoot and a low settling time at the same time. When considering IAE minimization as the objective, however, the obtained optimal integrator is always of integer order, as pointed out in (Padula and Visioli, 2011, 2012, 2015; Sánchez et al., 2017). This phenomenon is related to the asymptotic rate of convergence of the closed-loop systems with a fractional integrator and prevents the use of IAE as an objective function in the tuning of (C)FOPI controllers. In general, the integer order integrator is mainly used to eliminate the steady-state error. However, regarding the speed and the quality of the transient response, limiting the controller to a PI one may have a negative impact. To overcome this limitation, we propose in this paper the following optimization function

$$IAE^\delta := \int_0^{t^\star} |e(t)|dt = \int_0^{t^\star} |r(t) - y(t)|dt, \tag{9}$$

where $\delta \in \mathbb{R}^+$ and $t^\star := \min_{t\in(0,\,+\infty)}$ such that for all $t > t^\star$ we have $|e(t)| < \delta$. Roughly speaking, we optimize the controller disregarding the shape of the response when the transient response is already very close the desired value. This is sensible in most practical applications, and in particular in process control. In any case, since the minimization of $IAE^\delta$ obviously implies stability, any fractional integrator guarantees $\lim_{t\to\infty} e(t) = 0$. Finally, to explicitly consider the robustness issue of the closed-loop system, we optimize (9) subject to the following inequality constraint on the maximum sensitivity

$$M_s = \max_{\omega\in[0,+\infty)} \frac{1}{1 + C(s)P(s)} \leq \overline{M}_s. \tag{10}$$

In this paper, we consider two significant cases, namely $\overline{M}_s = 1.4$ and $\overline{M}_s = 2.0$, as they represent the extrema of the range of suitable values, the former referring to a case where the robustness issue is of primary concern, the latter to a case where the aggressiveness is more important. Finally, we consider two different control tasks: the unit step disturbance rejection and the unit step set-point tracking. For each task, we minimize $IAE^\delta$, with $\delta = 0.02$ and $\delta = 0.1$. The first case refers to a control

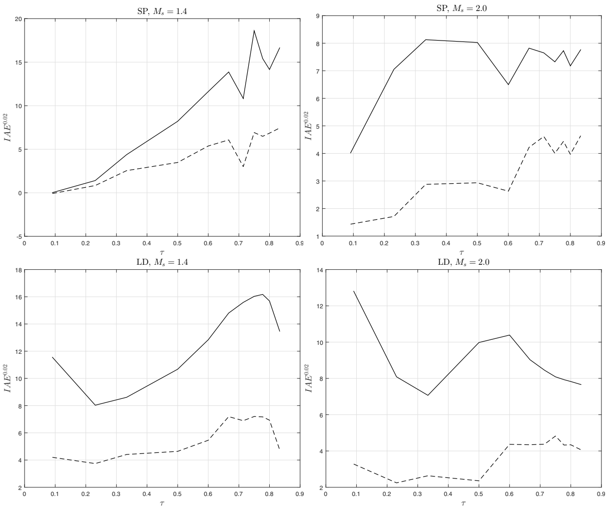


Fig. 2. $IAE^{0.02}$ decrement achieved by using the CFOPI with respect to integer-order PI controller (solid line) or with respect to FOPI controller (dashed line).

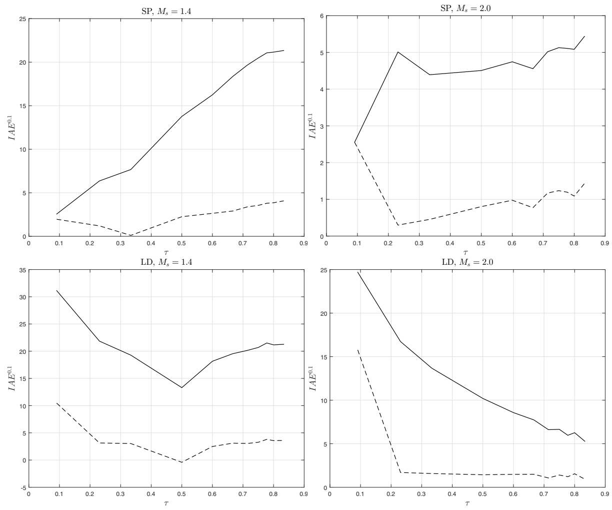


Fig. 3. $IAE^{0.1}$ decrement achieved by using the CFOPI with respect to integer-order PI controller (solid line) or with respect to FOPI controller (dashed line).

problem where a tight control close to the steady-state value is required, while in the second case the first part of the transient response is more important.

## 5. PERFORMANCE ASSESSMENT

To evaluate the performance that can be achieved by using a CFOPI controller, the set-point tracking and the load disturbance rejection tasks have been examined individually, and different normalized processes with several values of the normalized dead time have been considered. For any of them, the values of the parameters of the CFOPI controller have been obtained by solving the constrained optimization problem presented in Section 4 via genetic algorithm, which is known to produce a global optimum of a problem in a stochastic framework. Further, four sub-cases have been considered, namely $M_s = 1.4$ and $M_s = 2$, and $\delta = 2\%$ and $\delta = 10\%$. The (possibly complex) fractional integrator has been approximated by using the results proposed in Section 3 with $N = 8$, $\omega_l \approx 0.001\omega_c$, and $\omega_h \approx 1000\omega_c$, where $\omega_c$ is the gain crossover frequency.

It is essential to quantitatively assess the performance that can be obtained using the fractional complex-order controller. However, assessing the performance in terms of $IAE^\delta$ values is difficult and unnatural for the user. It is more meaningful to evaluate the performance in terms of the percentage improvement that can be achieved with respect to the integer- and real-order controllers. To this end, the same optimization procedure has been applied to PI and FOPI controllers. The results are shown in Figures 2 and 3, for $\delta = 0.02$ and $\delta = 0.1$, respectively, and where SP stands for set-point and LD stands for load disturbance.

As expected, we observe that an optimally tuned CFOPI controller always leads to an improvement of the performance. It is interesting to note how the performance improvement increases with the normalized dead time when the set-point tracking task is considered. On the contrary, when the load disturbance rejection is of concern, the optimal performance improvement exhibits a variety of behaviors and each different case requires and *ad hoc* evaluation. Note, however, that the trend of the $IAE^\delta$ reduction in each case remains the same, independently of the controller considered. Another interesting point is that the performance improvement is greater when the maximum sensitivity is bounded below 1.4. This is a major advantage of the CFOPI controllers over the FOPI and the PI ones: they considerably improve the control performance when the robustness is the main concern. Indeed, when constraining $M_s \leq 1.4$, the optimally tuned controllers result in a closed-loop sensitivity which is $M_s \approx 1.4$, as shown in Figure 4, where the set-point tracking case with $\delta = 0.02$ is presented (the other cases are similar, and they are omitted for the sake of brevity). Consequently, the level of robustness is fixed and a fair comparison between the controllers is obtained. Vice-versa, constraining the maximum sensitivity to be smaller than or equal to 2.0 normally results in $M_s << 2.0$ (Sánchez et al., 2017). In other words, contrarily to what is commonly understood, reducing the robustness beyond a certain threshold does not provide any improvement in the performance. This, however, results in optimal CFOPI, FOPI and PI controllers that, for the same process, exhibit different levels of robustness (see Figure 4). Therefore, the optimizer can adjust the robustness to improve the performance, as long as the former remains below 2.0, and this reduces the gap between the $IAE^\delta$ achieved with PI, FOPI and CFOPI controllers, when $\overline{M}(s) = 2.0$. Finally, note that the performance is always higher when the threshold $\delta = 0.1$ is considered. This is in line with the results in (Padula and Visioli, 2011, 2012; Sánchez et al., 2017), where it is shown that the use of a fractional integrator is not beneficial for the minimization of IAE without a threshold, i.e., when $\delta = 0$. In other words, if we include the steady-state performance in the optimization function, there is no advantage in using a (possibly complex) fractional integrator. Conversely, the more we focus on the transient performance (i.e., the greater the threshold $\delta$) the higher is the benefit of using a (possibly complex) fractional integrator.

In Figure 5 the optimal values of the parameters of the controller with respect to different normalized dead times $\tau$ are reported for the set-point tracking task with $\overline{M}_s =$

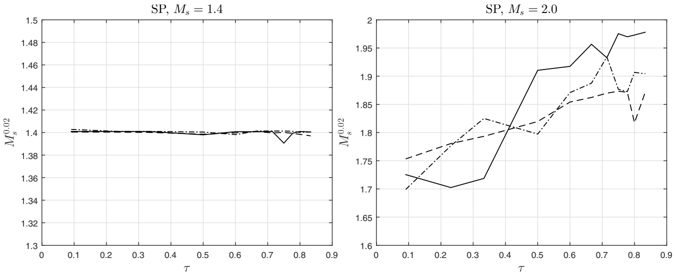


Fig. 4. $M_s$ achieved by using the CFOPI (dotted line), FOPI (dashed line) and PI (solid line) controller.

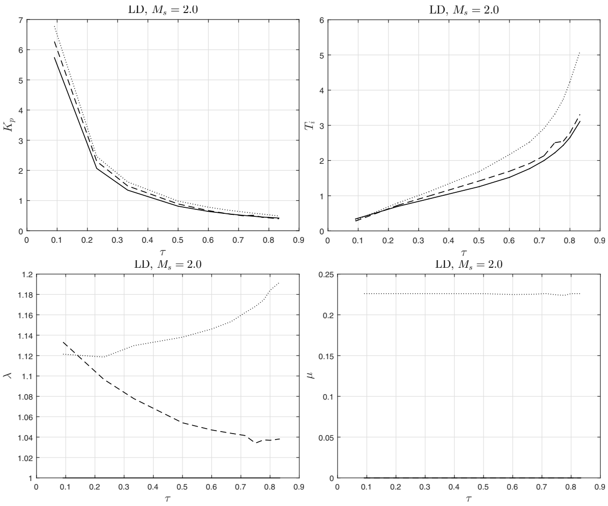


Fig. 5. Optimal parameters for the load disturbance rejection task with $M_s = 2.0$ and $\delta = 0.02\%$. Solid line: PI controller. Dashed line: FOPI controller. Dotted line: CFOPI controller.

2.0 and $\delta = 0.02$. Note that similar results are obtained in all other cases. The parameters $K_p$ and $T_i$ exhibit the same trend for all the considered controllers, even though the use of a complex-order integrator allows to noticeably increase the integral time constant. The interesting point, however, is the integrator order. When a real fractional-order integrator is considered, the real part (which is also the order of the integrator since, in this case, $\nu = \lambda$) shows a decreasing behavior. On the contrary, with the complex-order integrator, the real part $\lambda$ increases with the normalized dead time, while the complex part $\mu$ remains virtually constant. The reason of this phenomenon is that the optimizer converges to the highest possible $\mu$ which guarantees that the obtained approximated controller is both stable and minimum-phase. A complete analysis of this behavior would require a full characterization of the adopted approximation method, and will the subject of future investigations. Nevertheless, we note here that this behavior depends on the approximation bandwidth, while it is mostly insensible to the number of poles and zeros of the approximation.

## 6. SIMULATION RESULTS

The aim of this section is to verify the effectiveness of the achieved results. We consider the following process with $K = 1$, $T = 1$ and $L = 1.5$:

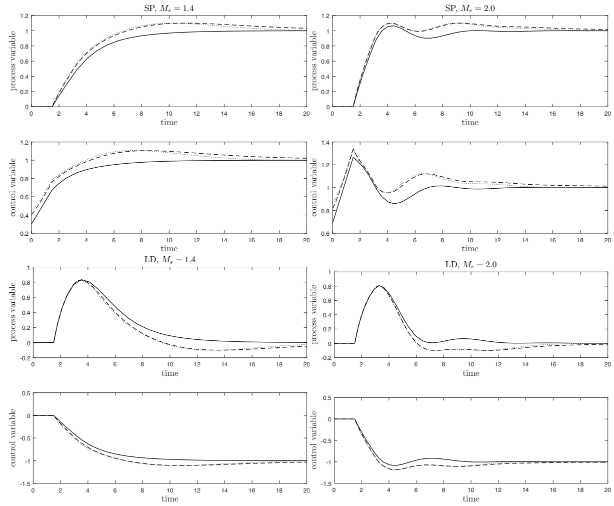


Fig. 6. Simulation results for process $P_1(s)$ with $\delta = 0.1\%$. Top-left: set-point step response with $M_s = 1.4$. Top-right: set-point step response with $M_s = 2.0$. Bottom-left: load disturbance step response with $M_s = 1.4$. Top-left: load disturbance step response with $M_s = 2.0$. Solid line: PI controller. Dashed line: FOPI controller. Dotted line: CFOPI controller.

$$P_1(s) = \frac{1}{s+1} e^{-1.5s}, \quad \tau = 0.6. \tag{11}$$

In Table 1, the $IAE^{\delta}$ values are reported for all the possible cases for the process $P_1(s)$. The simulation results of the set-point and load disturbance step responses for process $P_1(s)$ with the integer-, the real- and the complex-order PI controller, subject to $\overline{M}_s = 1.4$ and $\overline{M}_s = 2.0$ are shown in Figure 6. For the sake of brevity, we only show the case $\delta = 0.1$.

| Tuning | $IAE_{sp}$ $\delta = 0.02$ | $IAE_{ld}$ $\delta = 0.02$ | $IAE_{sp}$ $\delta = 0.1$ | $IAE_{ld}$ $\delta = 0.1$ |
|---|---|---|---|---|
| SP 1.4 I | 3.95 | 3.95 | 3.76 | 3.76 |
| SP 1.4 F | 3.73 | 3.72 | 3.32 | 3.28 |
| SP 1.4 CF | 3.54 | 3.52 | 3.23 | 3.19 |
| SP 2.0 I | 2.70 | 2.54 | 2.38 | 2.26 |
| SP 2.0 F | 2.61 | 2.49 | 2.29 | 2.14 |
| SP 2.0 CF | 2.54 | 2.47 | 2.27 | 2.12 |
| LD 1.4 I | 3.95 | 3.95 | 3.76 | 3.76 |
| LD 1.4 F | 3.81 | 3.69 | 3.79 | 3.26 |
| LD 1.4 CF | 3.60 | 3.50 | 3.68 | 3.18 |
| LD 2.0 I | 2.74 | 2.42 | 2.39 | 2.25 |
| LD 2.0 F | 2.68 | 2.29 | 3.02 | 2.11 |
| LD 2.0 CF | 2.61 | 2.20 | 2.96 | 2.08 |

Table 1. Results achieved with the different tuning rules for $P_1(s)$.

The results confirm that the proposed approach is effective and the value of $IAE^{\delta}$ obtained with a CFOPI controller is always lower than the one obtained with the other controllers, for both the load disturbance rejection task and the set-point tracking task. Figure 6 also shows that the responses maintain an acceptable shape in all the considered cases and that the control variable remains always within an acceptable range. Interestingly, the use of a (complex) fractional controller generally speeds-up the process response when the process output is far from the steady-state, and in particular when $\overline{M}_s = 1.4$. Also note that the responses of real- and complex-order fractional controllers appear very similar, in particular when $\overline{M}_s = 2.0$. However, an integral-type performance index such as IAE is capable to capture the differences.

## 7. CONCLUSIONS

In this paper, an optimization-based tuning methodology for real- and complex-order fractional PI controllers has been proposed by considering a modified version of the integral absolute error sensitivity-constrained optimization problem, which is suitably adapted to the design of fractional controllers. A method to approximate the complex-order fractional operator with a real-rational transfer function has also been proposed. The performance improvement achievable by using the CFOPI controller has been quantitatively assessed against the performance obtainable with FOPI and PI controllers. Simulation results validate the effectiveness of the devised tuning method.

## 8. ACKNOWLEDGMENTS

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