Two-loop Design for Dual-rate Cascade System

Sho Ito * **Takao Sato** * **Nozomu Araki** * **Yasuo Konishi** *

* *Department of Mechanical Engineering, Graduate School of Engineering, University of Hyogo, 2167 Shosha, Himeji, Hyogo 671-2280, Japan (e-mail: {tsato,araki,konishi}@eng.u-hyogo.ac.jp).*

Abstract:
In this study, we evaluated a new data-driven approach for designing a dual-rate cascade control system. A dual-rate cascade control system consists of inner and outer loops, where the update interval of the inner loop controller is shorter than that of the outer controller. In the proposed method, fictitious reference iterative tuning is used to optimize the controller parameters in the inner and outer loops. Hence, the controller parameters are designed using only the control data, and no modeling procedure is needed. The controller parameter for the inner loop is optimized first, and then the outer loop parameter is optimized. Because the inner loop is updated faster than the outer loop in the dual-rate system, the control performance of the proposed dual-rate design is superior to that of conventional single-rate systems. The effectiveness of the proposed method was demonstrated using numerical examples.

Keywords: Cascade system, data-driven control, dual-rate system, PID control, iterative tuning methods, transient responses, sampled-data system

1. INTRODUCTION

Data-driven control has advantages relative to the model-based approach because the control system is designed using the control data directly and modeling is not needed. The leading data-driven methods are iterative feedback tuning (IFT) (Bruyne, 2003; Hjalmarsson, 2002; Hjalmarsson et al., 1998), virtual reference feedback tuning (VRFT) (Campi et al., 2002), noniterative correlation-based tuning (NCbT) (Karimi et al., 2007), and fictitious reference iterative tuning (FRIT) (Souma et al., 2004; Kaneko et al., 2012). Because of its usefulness, the data-driven approach has been widely used in various situations. One of the challenges in systems control is to design the cascade control system shown in Fig. 1, which has inner and outer control loops. Although the control structure of the cascade control system is more complex than that of a single-measurement system, it has many industrial applications, such as control of processes (Hyl and Wagnerova, 2016; Hu et al., 2017) and mechanical systems (Gama et al., 2013).

Conventional data-driven methods for the cascade control system were designed as a single-rate system (Nguyen et al., 2016; Kinoshita et al., 2017; Nguyen and Kaneko, 2017), in which all data signals are updated synchronously. Therefore, when conventional single-rate data-driven approaches are used, the update intervals of the inner and outer loops must be equivalent even if the update interval of the inner loop can be shorter than that of the outer loop Ling et al. (2004). Although the FRIT design method was proposed for a dual-rate system (Ito et al., 2018), the study did not address the design of a cascade control system with dual-rate sampling. Therefore, this study evaluated a dual-rate data-driven approach for a cascade control system

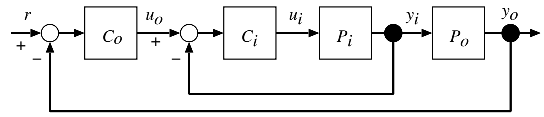

Fig. 1. Cascade control system

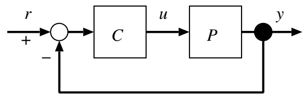

Fig. 2. Conventional feedback control system

with an inner loop update interval that is shorter than that of the outer loop.

This paper is organized as follows. An overview of the standard FRIT method is given in Section 2, and Section 3 describes a cascade control system. The proposed data-driven approach for the dual-rate cascade control system is given in Section 4. The effectiveness of the proposed method is demonstrated in Section 5, and concluding remarks are given in Section 6.

2. SINGLE-RATE FRIT

The standard single-rate FRIT method (Souma et al., 2004; Kaneko et al., 2012) was designed using a fictitious reference input (Safonov and Tsao, 1997) for the single-loop feedback control system shown in Fig. 2, where the plant P is unknown and the controller C is governed by the controller parameter θ. In this section, the procedure for control design with FRIT is introduced briefly.

The design objective of FRIT is to have the plant output follow the reference model T_d during a finite interval, and hence the objective function is given as follows:

$$J(\theta) = ||T_d r - y(\theta)||_N^2 \tag{1}$$

where r is the reference input and $||x||_N^2 := \frac{1}{N}\sum_{k=1}^{N} x(k)^2$ for a discrete time signal x. Then the following problem is solved to obtain the optimal parameter θ^*:

$$\arg\min_{\theta} J(\theta) \tag{2}$$

The controller $C(\theta^0)$ with the initial parameter θ^0 is assumed to be designed so that the closed-loop system is stable. Using the initial controller parameter, the control loop is executed for the reference input, and the controlled input data $u(\theta^0)$ and output data $y(\theta^0)$ are collected. Based on the initial controlled data value, a fictitious reference input is calculated as follows:

$$\tilde{r}(\theta) = C(\theta)^{-1}u(\theta^0) + y(\theta^0), \tag{3}$$

where the closed-loop plant output is always $y(\theta^0)$ for any θ when $\tilde{r}(\theta)$ is used as the reference input. Using $\tilde{r}(\theta)$, a new objective function is defined as follows:

$$J_F(\theta) = ||T_d\tilde{r}(\theta) - y(\theta^0)||_N^2 \tag{4}$$

Eq. (4) is minimized offline since it consists of only the initial input and output data. The input and output relationship $y(\theta^0) = Pu(\theta^0)$ and $\tilde{r}(\theta)$ are substituted into Eq. (4), and $J_F(\theta)$ is rearranged as follows:

$$J_F(\theta) = \left|\left|\left(\frac{T_d}{G_{cl}} - 1\right)y(\theta^0)\right|\right|_N^2 \tag{5}$$

$$G_{cl} = \frac{PC}{1+PC},$$

where G_{cl} denotes the closed-loop transfer function from the reference input r to the plant output y. Therefore, Eq. (5) is interpreted as minimizing the error between the closed-loop system and the desired reference model based on the initial output data. The use of nonlinear optimization gives the optimal controller parameter.

3. DUAL-RATE CASCADE CONTROL SYSTEM

This study addresses a control system design for a dual-rate cascade control system where the update intervals of the inner and outer loops are different. The cascade control system is implemented using a digital computer. The proposed method is designed using the following assumption:

Assumption 1. The update interval of the outer loop is twice the update interval of the inner loop.

From Assumption 1, The input and output intervals are as illustrated in Fig. 3. The block diagram of the dual-rate cascade control system is shown in Fig. 4, where P_i and P_o are unknown plants in the inner and outer loops, respectively, and C_i and C_o are the inner and outer loop controllers, respectively. In the cascade control system, r is the reference input; u_i and y_i are the input and output of the inner loop, respectively; and u_o and y_o are the input and output of the outer loop, respectively. Moreover, because of Assumption 1, the down-sampler S_d and up-sampler S_u are used to interpolate the different signal intervals.

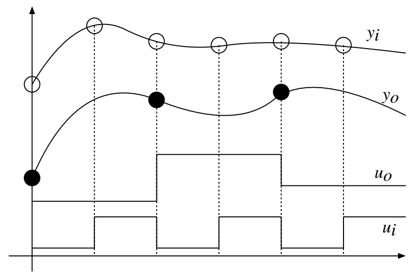

Fig. 3. Dual-rate input and output intervals

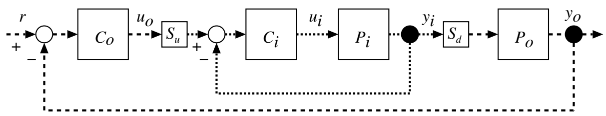

Fig. 4. Dual-rate cascade control system

The inner and outer controllers are assumed to be governed by controller parameters θ_i and θ_o, respectively, and the control inputs are determined by the following control laws:

$$u_i = C_i(\theta_i)(u_o - y_i) \tag{6}$$

$$u_o = C_o(\theta_o)(r - y_o) \tag{7}$$

Thus, the control input $u_o(k)$ in the outer loop is the reference input for the inner controller $C_i(\theta_i)$. This study proposes a method for determining the controller parameters θ_i and θ_o using only input and output data in the dual-rate cascade control system.

4. TWO-LOOP DESIGN FOR DUAL-RATE CASCADE CONTROL SYSTEM

The objective function of the dual-rate cascade control system is defined as follows:

$$J(\theta_i, \theta_o) = ||T_d r - y_o(\theta_i, \theta_o)||_N^2 \tag{8}$$

Because the update intervals of the inner and outer loops are different, the controller parameters are optimized in series. In the proposed method, firstly the controller parameter in the inner loop is optimized based on the first control result, and next the controller parameter in the outer loop is optimized based on the second control result using the optimized inner loop controller parameter instead of the initial value. This study proposes a new design method in which the controller parameter in the inner loop is optimized independently of the outer loop parameter, even though the inner and outer loops influence each other. This section describes why the controller parameter in the inner loop is designed independently of the outer loop controller parameter.

The objective functions of the inner and outer loops are defined as follows:

$$J_i(\theta_i) = ||T_{d_i} u_o - y_i(\theta_i)||_{N_i}^2 \tag{9}$$

$$J_o(\theta_o) = ||T_{d_o} r - y_o(\theta_o)||_{N_o}^2, \tag{10}$$

where $N_o = 2N_i$ because of Assumption 1.

The initial controller parameters θ_i^{ini} and θ_o^{ini} are assumed to be designed to stabilize both closed loops, and the input data $u_i^{1st}(\theta_i^{ini})$ and output data $y_i^{1st}(\theta_i^{ini})$ in the inner loop are obtained from the first execution of the cascade control system. The outer loop input data $u_o^{1st}(\theta_o^{ini})$ and output data $y_o^{1st}(\theta_o^{ini})$ are also obtained but not used for tuning the inner loop. Using the data obtained from the first execution, the fictitious reference input for the inner loop is defined as follows:

$$\tilde{u}_o(\theta_i) = C_i(\theta_i)^{-1} u_i^{1st}(\theta_i^{ini}) + y_i^{1st}(\theta_i^{ini}) \tag{11}$$

In Eq. (11), $\tilde{u}_o(\theta_i)$ may have to be expressed as $\tilde{u}_o(\theta_o)$ because it is determined as the controller parameter in the outer loop. However, since it is used as the fictitious signal, it is expressed as a function of θ_i, which is the only parameter tuned in the first optimization. Using Eq. (11), a new objective function for the inner loop to replace Eq. (9) is defined as follows:

$$J_{i_F}(\theta_i) = ||T_{d_i}\tilde{u}_o(\theta_i) - y_i^{1st}(\theta_i^{ini})||^2_{N_i}. \tag{12}$$

By optimizing Eq. (12) for θ_i, we obtain the optimal controller parameter for the inner loop, θ_i^*. Because Eq. (12) is optimized using a fictitious reference input, the optimization in the inner loop is independent of the controller parameter in the outer loop.

Next, using both θ_i^* and θ_o^{ini}, the second cascade control instruction is executed for the outer loop, and we obtain $u_o^{2nd}(\theta_o^{ini})$ and $y_o^{2nd}(\theta_o^{ini})$. Using the obtained data, the fictitious reference input and a new objective function for the outer loop are defined as follows:

$$\tilde{r}(\theta_o) = C_o(\theta_o)^{-1} u_o^{2nd}(\theta_o^{ini}) + y_o^{2nd}(\theta_o^{ini}) \tag{13}$$

$$J_{o_F}(\theta_o) = ||T_{d_o}\tilde{r}(\theta_o) - y_o^{2nd}(\theta_o^{ini})||^2_{N_o}. \tag{14}$$

Based on the optimization of Eq. (14), we obtain the optimal controller parameter in the outer loop, θ_o^*. Because the control result of the outer loop is influenced by the controller in the inner loop, the controller parameter of the outer loop is determined using the optimized controller parameter of the inner loop. However, the controller parameter of the inner loop is obtained independently of that in the outer loop because the control input from the outer loop is the reference input for the inner loop.

The proposed control design algorithm is summarized as follows:

(1) Select initial controller parameters θ_i^{ini} and θ_o^{ini} so that both closed loops are stabilized.
(2) Execute the first control instruction for the inner loop using the initial controller parameters to obtain control data u_i^{1st} and y_i^{1st} in the inner loop.
(3) Calculate θ_i^* in the inner loop based on the optimization of Eq. (12).
(4) Execute the second control instruction for the outer loop after replacing θ_i^* with θ_i^{ini} to obtain control data u_o^{2nd} and y_o^{2nd} in the outer loop.
(5) Calculate θ_o^* in the outer loop based on the optimization of Eq. (14).

5. NUMERICAL EXAMPLE

The controlled plants in the inner and outer loops are given as follows:

$$P_i = \frac{4}{s^2 + 1.15s + 4} \tag{15}$$

$$P_o = \frac{1}{s^2 + 12s + 1} \tag{16}$$

To control the inner and outer loops, the following discrete-time proportional–integral–derivative (PID) controllers are used:

Table 1. Initial PID parameters

	K_p	K_I	K_D
C_o	1.00	0.200	1.00
C_i	0.100	0.0100	0

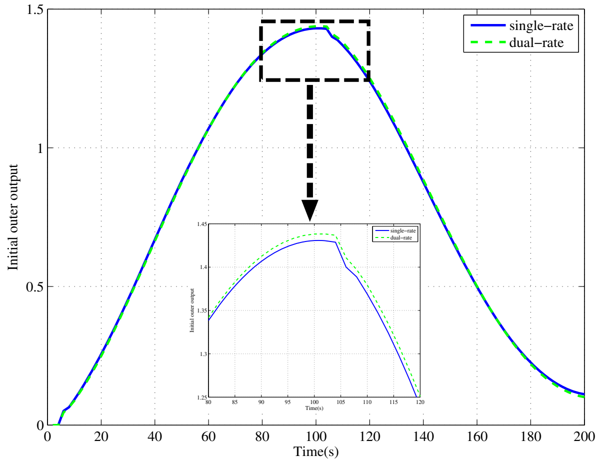

Fig. 5. Initial outer loop output

$$u_i = C_i(\theta_i)(u_o - y_i) \tag{17}$$

$$C_i(\theta_i) = \left[1 \quad \frac{1}{1 - z^{-1}} \quad 1 - z^{-1}\right]\theta_i \tag{18}$$

$$\theta_i = [K_{P_i} \quad K_{I_i} \quad K_{D_i}]^T \tag{19}$$

$$u_o = C_o(\theta_o)(r - y_o) \tag{20}$$

$$C_o(\theta_o) = \left[1 \quad \frac{1}{1 - z^{-1}} \quad 1 - z^{-1}\right]\theta_o \tag{21}$$

$$\theta_o = [K_{P_o} \quad K_{I_o} \quad K_{D_o}]^T, \tag{22}$$

where z^{-1} denotes the backward shift operator. The update intervals of the inner and outer loops are 1 s and 2 s, respectively. For comparison with the proposed method, a single-rate system was also designed in which the update intervals of both the inner and outer loops are 2 s. The reference input was 1.0 from 0 s to 100 s and 0.5 from 100 s to 200 s.

To tune the controller parameters based on the input and output data, the initial parameter values must be selected so that the closed-loop system is stable. The initial PID parameters selected by trial and error are shown in Table 1. These parameters were used in both the single-rate and dual-rate systems. The output data of the outer loop plant using the initial PID parameters are shown in Fig. 5, where the solid and dashed lines are the outer loop output trajectories of the single-rate and dual-rate systems, respectively.

Based on the initial controlled data, the PID parameters were tuned to have the output of the outer loop follow two reference model outputs. The reference models were established with slow and fast transfer functions, $T_{d_{slow}}$ and $T_{d_{fast}}$, respectively, and their control results were compared.

$$T_{d_{slow}} = \frac{1}{(10s + 1)^2} \tag{23}$$

$$T_{d_{fast}} = \frac{1}{(0.8s + 1)^2}. \tag{24}$$

Table 2. Single-rate PID parameters for slow model

	K_p	K_I	K_D
C_o	1.16	0.0872	2.62
C_i	−0.0906	0.0962	0.0228

Table 3. Dual-rate PID parameters for slow model

	K_p	K_I	K_D
C_o	1.19	0.0875	1.97
C_i	−0.0124	0.0495	−0.00627

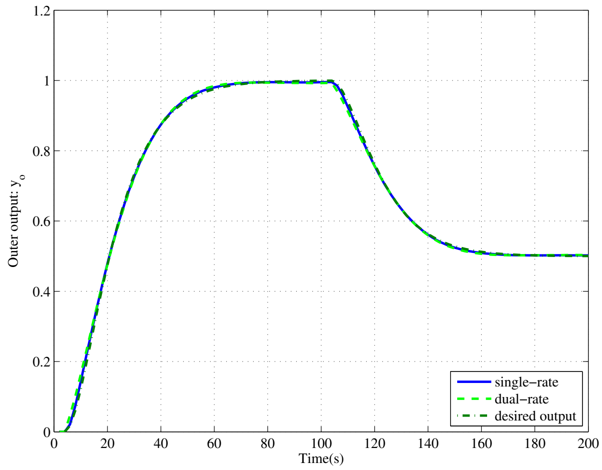

Fig. 6. Outer loop output for slow model

Table 4. Single-rate PID parameters for fast model

	K_p	K_I	K_D
C_o	2.59	0.422	1.56
C_i	−0.214	0.750	0.0151

The same reference model was used in both the inner and outer loops. Because the control system was implemented using a digital computer, dead-time for the sampling was appended to the reference models.

The tuned single-rate and dual-rate PID parameters for the slow reference model are shown in Table 2 and Table 3, respectively. The control results obtained using the tuned PID parameters are shown in Fig. 6 and Fig. 7, where the solid and the dashed lines are the trajectories of the single-rate and dual-rate systems, respectively. Fig. 6 also shows the desired output trajectory of the slow reference model (dashed-dotted line). The figure shows that both the single-rate and dual-rate outer outputs followed the reference model output without error.

Next, the PID parameters were tuned for the fast reference model, and these are shown in Table 4 and Table 5. The control results are shown in Fig. 8 and Fig. 9. Fig. 8 shows that the output of the dual-rate system followed the reference model, but the output of the single-rate system deviated from the desired output when the reference input was changed.

These results show that the outer loop outputs followed the slow reference model in both the single-rate and dual-rate systems. However, the tracking performance of the

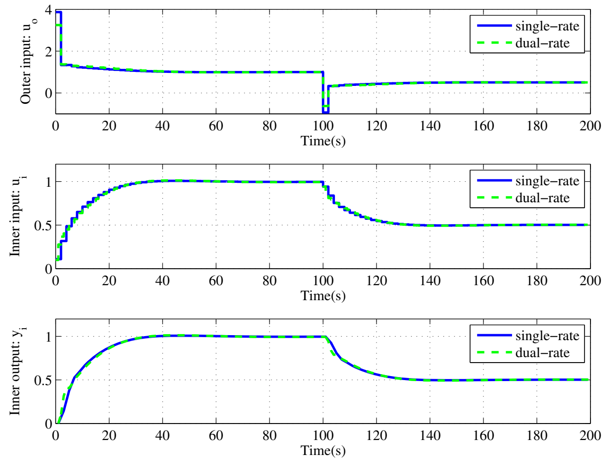

Fig. 7. Outer loop input and inner loop input/output for slow model

Table 5. Dual-rate PID parameters for fast model

	K_p	K_I	K_D
C_o	2.70	0.432	1.85
C_i	0.0988	0.481	−0.00794

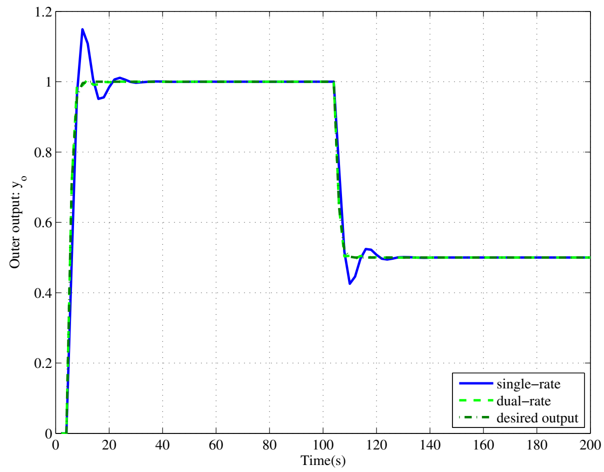

Fig. 8. Outer loop output for fast model

dual-rate system was superior to that of the single-rate system for the fast reference model.

6. CONCLUSION

This study proposes a two-loop data-driven design method for a dual-rate cascade control system, where the update intervals of the inner and outer loops are different. Conventional cascade control systems are designed as single-rate systems, where all the update intervals in the inner and outer loops must be equal. In contrast, the proposed method can be used to design a dual-rate system as well as a single-rate system, and the inner and outer controller parameters are tuned in series. In the dual-rate system, the inner control loop is updated faster than the outer loop, so the control performance of the proposed dual-rate method is improved with respect to conventional single-rate methods. The effectiveness of the proposed method was demonstrated with numerical examples.

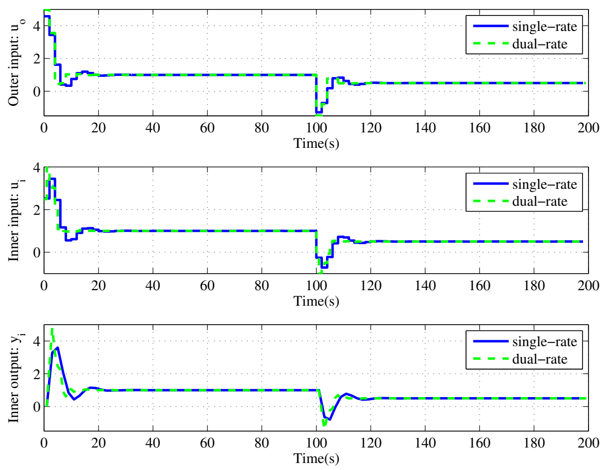

Fig. 9. Outer loop input and inner loop input/output for fast model

ACKNOWLEDGEMENTS

This study was supported by the Japan Society for the Promotion of Science, Kakenhi Grant No. JP16K06425.

REFERENCES

Bruyne, F. (2003). Iterative feedback tuning for internal model controllers. *Control Engineering Practice*, 11, 1043–1048.

Campi, M., Lecchini, A., and Savaresi, S. (2002). Virtual reference feedback tuning (VRFT): a direct method for the design of feedback controllers. *Automatica*, 38, 1337–1346.

Gama, F., Martins, J.C., Miranda, T., Tome, W.F., and Fernandes, M. (2013). Speed and current control of a permanent-magnet dc servo motor using a real-time microcontroller. In *10th International Conference on Electrical Engineering, Computing Science and Automatic Control (CCE)*.

Hjalmarsson, H. (2002). Iterative feedback tuning -an review. *International Journal of Adaptive Control and Signal Processing*, 16, 373–395.

Hjalmarsson, H., Gevers, M., Gunnarsson, S., and Lequin, O. (1998). Iterative feedback tuning: Theory and applications. *IEEE Control Systems Magazine*, 26–41.

Hu, H., Zhang, J., Yang, Q., and Cai, Y. (2017). Feedforward dmc-pid cascade strategy for main steam temperature control system in fossil-fired power plant. In *29th Chinese Control And Decision Conference (CCDC)*.

Hyl, R. and Wagnerova, R. (2016). Design and implementation of cascade control structure for superheated steam temperature control. In *17th International Carpathian Control Conference (ICCC)*.

Ito, S., Sato, T., Araki, N., and Konishi, Y. (2018). Data-driven control system design for a multi-input single-output dual-rate system based on input-multiple closed-loop data. *IEEJ Transactions on Electrical and Electronic Engineering*, 13(4), 654–655.

Kaneko, O., Wadagaki, Y., and Yamamoto, S. (2012). FRIT based PID parameter tuning for linear time delay systems -simultaneous attainment of models and controllers-. In *IFAC Conference on Advances in PID Control*, 86–91.

Karimi, A., Heusden, K.V., and Bonvin, D. (2007). Noniterative data-driven controller tuning using the correlation approach. In *Proc. of European Control Conference*, 5189–5195.

Kinoshita, T., Yamamoto, T., and Samavedham, L. (2017). Design of a data-oriented cascade control system. In *6th International Symposium on Advanced Control of Industrial Processes*, 365–370. Taipei,Taiwan.

Ling, K., Bingfang, W., Minghua, H., and Yu, Z. (2004). A model predictive controller for multirate cascade systems. In *Proc. of the 2004 American Control Conference*, 1575–1579. Boston.

Nguyen, H. and Kaneko, O. (2017). Fictitious reference iterative tuning of cascade controllers for nonminimum phase systems. In *6th International Symposium on Advanced Control of Industrial Processes*, 517–522. Taipei,Taiwan.

Nguyen, H., Kaneko, O., and Kitazaki, Y. (2016). Virtual reference feedback tuning for cascade control systems. *Journal of Robotics and Mechatronics*, 28(5), 739–744.

Safonov, M. and Tsao, T. (1997). The unfalsified control concept and learning. *IEEE Trans. on Automatic Control*, 42, 843–847.

Souma, S., Kaneko, O., and Fujii, T. (2004). A new method of controller parameter tuning based on input-output data, -fictitious reference iterative tuning-. In *Proceedings of IFAC Workshop on Adaptation and Learning in Control and Signal Processing*, 789–794.