Composite PID Control with Unknown Dynamics Estimator for Rotomagnet Plant

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Abstract: Although PID control has been widely used in practical engineering, its ability to reject external disturbance and to handle severe nonlinearities should be further enhanced. In this paper, we present a simple robust unknown dynamics estimation, which can be easily incorporated into PID control to achieve satisfactory control performance for a rotomagnet plant subject to period disturbance. The use of this estimator together with PID control leads to a feedforward like composite control framework. Unlike other estimators, only low-pass filter operations on the input and output and simple algebraic operations are needed to construct our estimator, while exponential convergence can be guaranteed. Numerical simulations are given to show the validity of the proposed estimator and composite PID control.

Keywords: PID Control, Unknown Dynamics Estimator, Feedforward Compensation

1. INTRODUCTION

In most of control practice and applications, PID control (Aström and Hägglund, 1995; Moliner and Tanda, 2016) has always been used due to its simple structure and fair robustness, which are preferred by engineers. For some specific systems, many modified or enhanced PID control structures have further been reported, e.g. cascaded PID control, anti-windup PID control (Åström and Hägglund, 2006). However, a major well-documented drawback of PID control lies in its limited ability to handle external disturbances and severe nonlinearities involved in the systems (Oviedo et al., 2006). Another issue is that apart from its wide application, rigorous proof of the closedloop control system stability with PID control is not a trivial task. In fact, only very recent, a rigorous proof of stability of PID control has been reported for some specific plant (Zhao and Guo, 2017). This has left huge gap and unbalance between the application and theoretical studies.

To enhance the ability of disturbance rejection, other alternative control schemes have also been proposed. Among those schemes, the internal model control (IMC) (Garcia and Morari, 1982) has been proved to be an effective solution. In the IMC framework, the feedback signal is the difference between the plant output and model output, which allows proving the closed-loop stability. Moreover, for some specific period disturbance, repetitive control (RC) (Hillerström and Walgama, 1996) has been developed based on the internal model principle (IMP) (Francis and Wonham, 1975). The idea of RC has been applied to the specific application of a rotomagnet plant that was built as an educational system (Costa-Castelló et al., 2005). Also resonance based control is offering interesting performance for this plant (Costa-Castelló et al., 2012). On the other hand, to further address the modeling of uncertainties, a new idea named disturbance observer (DOB) has been reported (Oh and Chung, 1999). Chen (2004) designed a generic nonlinear disturbance observer (NDO) to estimate and compensate for the lumped disturbances of nonlinear systems. In parallel, an alternative observer called extended state observer (ESO) was also suggested by Han Han (1995), where the lumped unknown dynamics with bounded derivative can be taken as an augmented system state and thus estimated by using observers (Guo and Zhao, 2011). The idea of ESO has subsequently incorporated into an integrated nonlinear enhanced PID control framework, active disturbance rejection controller (ADRC) (Han, 1998), which has recently attracted significant attention in both academic and engineering field. However, it is noted that in all above advanced estimation methods, the plant model should be known precisely since the model should be used in the design and implementation of these estimators. Consequently, if there are nontrivial modeling uncertainties, the overall control response based on these estimators could be severely deteriorated.

Very recently, inspired by the principle of unknown input observer (UIO) (Stotsky and Kolmanovsky, 2002), we have investigated a new simple yet robust UIO for engine torque estimation (Na et al., 2017). The salient feature of this approach lies in its simplicity in implementation, while its convergence and robustness can be proved as the same as the sliding model observer (Edwards and Christopher, 1988). Hence, the aim of this paper is to further exploit the application of this new idea to enhance the performance of PID control.

In order to further improve the control performance of PID control while keeping its simplify in the implementation and allowing to prove the stability (or even convergence), this paper will present an enhanced PID control structure. In this composite PID control design, a simple yet robust unknown dynamics estimation is first introduced to estimate the lumped unknown dynamics that include the external disturbances and modeling uncertainties. The design of this estimator is very simple and straightforward since only low-pass filter operations and simple algebraic calculations are required, and only one constant (determining the bandwidth of filter) needs to be selected by the designer. Hence, the proposed estimator is to some extend a data-driven based approach, which relaxes the stringent assumptions on the plant model compared with DOB or ESO. Then the estimator output can be easily incorporated into classical PID control (proportional control in our case) as a feedforward compensator, such that perfect tracking response can be retained. Rigorous theoretical studies are carried out to prove the convergence of the proposed estimator, and also the tracking error of the closedloop system. Numerical simulations are provided to show the efficacy of the proposed composite PID control, and the improved control response over classical PID control.

The paper is organized as follows: Section 2 presents the modeling of the studied rotor plant and problem formulations. Section 3 introduces the estimator designs and the composite control with rigorous stability analysis. Section 4 gives the numerical simulations, and Section 5 draws conclusions.



Figure 1. Rotomagnet device picture.



Figure 2. Rotomagnet conceptual scheme.

2. PROBLEM FORMULATION

This paper will address the disturbance rejection and tracking control of a rotomagnet device built as a laboratory control plant for educational purposes (Costa-Castelló et al., 2005). The control plant is composed of a pulse width modulation (PWM) electronic amplifier, a small DC motor, two permanent magnets, and two fixed electromagnets. More specifically, a bar holds a permanent magnet in each end, with each magnet magnetically oriented in the

opposite way, and attached to a DC motor and two fixed electromagnets. The rotor plant is shown in Figure 1.

The rotation of the DC motor causes a pulsating load torque that depends on the mechanical angle θ of the motor axis. Additionally, the interaction between the fixed and mobile magnets creates a magnetic field that causes disturbances, d, on the movement of the bar. As the magnetic field depends on the relative position between the different magnets, the torque acting over the bar is 2π periodic on the position variable. Under constant mechanical speed operation ($\dot{w} = 0$), the pulsating load torque is a periodic signal, where a fundamental period directly related to the axis speed (Figure 2). At a fixed angular speed, any friction, unbalance or asymmetry appearing on the system generates a periodic disturbance that affects its dynamical behaviour.

Hence, the purpose of this paper is to design a composite PID control such that the disturbance perturbing the system and unknown dynamics could be handled effectively, and satisfactory tracking control can be obtained.

To facilitate the control design, a preliminary system identification process has been conducted as reported in (Ramos et al., 2013), and the following model is obtained

$$Y(s) = \frac{k}{as+1}U(s) + D(s) \tag{1}$$

where D(s) is the disturbance, U(s) is the control input (voltage applied on the motor), and Y(s) is the output (rotation velocity of motor), the constants k, a denote the amplification of DC motor and the friction coefficient, respectively.

3. COMPOSITE CONTROLLER ARCHITECTURE

In order to facilitate the control design and performance analysis, the above transfer function model (1) can be represented in the time-domain as

$$\dot{y} = -a_1 y + a_2 u + d \tag{2}$$

$$\triangleq F(y,d) + a_2 u \tag{3}$$

where y is the measured output variable, u is the control action, d is an unknown disturbance, and $a_1 = 1/a$ and $a_2 = k/a$ are two constants.

Clearly, for the studied plant, the above equations (1) and (2) cannot cover all dynamics (e.g. pulsating load, frictions) and the disturbances. All of these dynamics can be considered as the disturbance d, which will be addressed in the following control design. Without loss of generality, we only assume that a_2 is known in *a priori*, which is also commonly used in other literature (Han, 1995; Oh and Chung, 1999). In this case, all unknown dynamics can be lumped in F(y, d) as shown in (3).

Hence, we will first present a new estimator for the unknown dynamics F(y, d) and compensate its effect over the system by incorporating the estimator into the PID control. The proposed composite control structure can be found in Figure 3.1.

3.1 Unknown Dynamics Estimator

A difficulty in designing control for system (3) lies in the unknown lumped uncertainties F(y, d). Different to most available methods using function approximators (NNs and FSs) or DOB, with large computational costs and sluggish response, we will develop a simple and fast unknown dynamics estimator to estimate F(y, d). Without loss of generality, in the following analysis it is assumed that the derivative of F(y, d), $\dot{F}(y, d)$, is bounded by \hbar , i.e. $\sup_{t\geq 0} |\dot{F}(y, d)| \leq \hbar$.

Then to avoid using the derivative of output y, we impose a low-pass filter on the variable u, y, which can be defined as:

$$\tau \dot{y}_f + y_f = y, \, y_f(0) = 0 \tag{4}$$

$$\tau \dot{u}_f + u_f = u, u_f(0) = 0, \tag{5}$$

where $\tau > 0$ is the filter time constant (the filter bandwidth is defined by $\frac{1}{\tau}$).

Then, an implicit relationship between the variables u, y, and the unknown dynamics F(y, d) can be derived.

Lemma 1. : Consider system (3) and the filter operation (4), the following auxiliary variable

$$\beta = \frac{y - y_f}{\tau} - a_2 u_f - F \tag{6}$$

is bounded and will decrease to a small set around zero in an exponential sense for $\tau > 0$. Hence, the manifold $\beta = 0$ is an invariant manifold for $\tau \to 0$.

Proof. The derivative of β can be derived along (4)-(29) as

$$\dot{\beta} = \frac{\dot{y} - \dot{y}_f}{\tau} - a_2 \dot{u}_f - \dot{F} = -\frac{\beta}{\tau} - \dot{F}.$$
 (7)

We select a Lyapunov candidate as $V_{\beta} = \frac{\beta^2}{2}$, then based on Young's inequality $\pm ab \leq a^2\tau + b^2/\tau$ for $\tau > 0$, its derivative can be given as

$$\dot{V}_{\beta} = \beta \dot{\beta} = -\frac{\beta^2}{\tau} - \beta \dot{F} \le -\frac{\beta^2}{2\tau} + \frac{\tau \hbar^2}{2} \le -\frac{V_{\beta}}{\tau} + \frac{\tau \hbar^2}{2}.$$
 (8)

Hence, we can derive that

$$V_{\beta} \le \sqrt{e^{-t/\tau} V_{\beta}(0) + \tau^2 \hbar^2/2}$$

and thus β will exponentially converge to a small compact set bounded by

$$|\beta(t)| \le \sqrt{2V_{\beta}(t)} \le \sqrt{\beta^2(0)e^{-t/\tau} + \tau^2\hbar^2}.$$

This implies the boundedness of β for any finite $\tau > 0$. Moreover, it can be verified that

$$\lim_{t \to \infty} \lim_{\tau \to 0} \beta(t) = 0$$

holds for $\tau \to 0$ and/or $\hbar \to 0$. Thus, $\beta = 0$ is an invariant manifold. This completes the proof.

The above invariant manifold provides a mapping from the variables y, u, to the unknown dynamics F(y, d). Hence, an unknown dynamics estimator can be designed based on this manifold. From y_f and u_f , the value of F(y, d) can be estimated as:

$$\hat{F} = \frac{y - y_f}{\tau} - a_2 u_f. \tag{9}$$

Under the hypothesis that \dot{F} is bounded, it can be shown that $\hat{F} \to F$ as $\tau \to 0$ and/or $\hbar \to 0$. This can be summarized as the following thereom:

Theorem 2. The estimation error, $e_F \triangleq F - \hat{F}$, is bounded by:

$$|e_F(t)| \le \sqrt{e_F^2(0)e^{-t/\tau} + \tau^2\hbar^2}$$

and thus $F \to \hat{F}$, i.e. $e_F(t) \to 0$ for $\tau \to 0$ and $t \to \infty$.

Proof. Firstly, both sides of equation (3) are filtered by using a low-pass filter $(\cdot)_f = [\cdot]/(\tau s + 1)$ given in (4), so that

$$\frac{s}{\tau s+1}[y] = \frac{1}{\tau s+1}[F] + a_2 \cdot \frac{1}{\tau s+1}[u].$$
(10)

From (4) it can be obtained the following expression

$$\dot{y}_f = \frac{y - y_f}{\tau} = a_2 u_f - F_f$$
 (11)

where $F_f = \frac{1}{\tau s + 1} [F]$ is the filtered version of F.



Figure 3. Composite control structure.

Then it follows from (9) and (11) that $\hat{F} = F_f$, that is, the estimator gives the filtered version of the unknown dynamics. In this case, we can prove that the estimation error can be small by using sufficiently small τ . For this purpose, we derive the estimation error as

$$e_F = F - \hat{F} = \left(1 - \frac{1}{\tau s + 1}\right)[F] = \frac{\tau s}{\tau s + 1}[F].$$
 (12)

To facilitate the convergence proof, we further represent the estimation error (12) in the time-domain as

$$\dot{e}_F = \dot{F} - \dot{\hat{F}} = \dot{F} - \frac{1}{\tau} \left(F - F_f \right) = -\frac{1}{\tau} e_F + \dot{F}.$$
 (13)

Select a Lyapunov function as $V = \frac{1}{2}e_F^2$, then similar to the proof of Lemma 1, the derivative \dot{V} can be given as

$$\dot{V} = e_F \dot{e}_F = -\frac{1}{\tau} e_F^2 + e_F \dot{F} \le -\frac{1}{\tau} V + \frac{\tau}{2} \hbar^2.$$
(14)

Integrating both sides of (14) it is obtained:

$$V(t) \le e^{-t/\tau} V(0) + \tau^2 \hbar^2/2,$$

so that we can further obtain that

$$|e_F(t)| \le \sqrt{e_F^2(0)e^{-t/\tau} + \tau^2\hbar^2}.$$

In this case, one can verify that $e_F(t) \to 0$ as $t \to \infty$ for any $\tau \to 0$. It is noted that the convergence is faster for $\hbar \to 0$. This completes the proof.

It is shown in the above Theorem 2 that the estimation \hat{F} can exponentially converge to a small set around the true value of the unknown lumped uncertainties F, where the ultimate bound of the residual error depends on the

upper bound of \dot{F} and the filter coefficient τ . Hence, one may verify that precision estimation (with zero error) can be achieved for constant dynamics (e.g. F = const.). Moreover, we could set sufficiently small τ to retain satisfactory estimation performance. However, it is noted that the constant τ also determines the bandwidth of the low-pass filter in (4)-(29), and thus affects robustness of the proposed estimator. Hence, a trade-off should be made when select filter coefficient τ . In practice, we could set τ initially small and then increase this value by observing the smoothness of the estimator output.

Remark 3. : Compared with other estimator, (Chen, 2004; Han, 1995), one can find that the design, analysis and implementation of the proposed estimator is obviously easier, i.e. only low-pass filter operations (4) and algebraic calculation (9) are required, and it is almost input-output data driven. This property allows to easily incorporate the estimator output \hat{F} into any classical control designs that could retain the system stability. Then, this newly added feedforward compensation can enhance the overall control response without triggering instability. For the purpose of demonstration, a simple proportional control will be used for the studied rotor plant (1).

3.2 Composite PID control

In this subsection, a feedback control is designed to achieve tracking by using the estimated dynamics \hat{F} . The effects of both the estimation error and the tracking control error are considered in the stability analysis of the whole closed-loop system.

Denote the desired trajectory to be tracked as y_r , and the tracking error as

$$e = y_r - y. \tag{15}$$

Then by using the estimation of F as a feedforward compensator, a simple controller can be formatted. The composite controller has the following form:

$$u = k_p e - \frac{1}{a_2} (\hat{F} - \dot{y}_r).$$
(16)

where $e = y_r - y$ is the tracking error, \hat{F} is the estimation of lumped unknown dynamics F, which can be online obtained based on (4) and (29).

Clearly, this composite controller takes the form of a proportional control $k_p e$ with a proportional gain $k_p > 0$, and a feedforward term $\frac{\hat{F} - \dot{y}_r}{a_2}$ with the estimated dynamics \hat{F} and trajectory \dot{y}_r .

Remark 4. : In the above control (16), the proposed estimaiton \hat{F} is integrated into a simple proportional control, which is easy to implement. However, it should be that the proposed estimator \hat{F} can be incorporated into other advanced control methods (e.g. adaptive control) in a similar way, provided that the adopted feedback control could retain the stability of the nominal closed-loop system without disturbance F. Moreover, since the rotomagnet plant, (1), is with low-order, the proportional control (16) could be used. For high order systems, we can use $s = \lambda e + \dot{e}$ as the feedback signal, which leads to sliding mode type (or PD like) control as shown in Slotine and Li (2004). Using the proposed controller, (16), the closed-loop tracking error becomes:

$$\dot{e} = \dot{y}_r - \dot{y} = \dot{y}_r - F(y, d) - a_2 u \tag{17}$$

$$= -F + k_p a_2 e + \hat{F} \tag{18}$$

$$=-k_p a_2 e - e_F. \tag{19}$$

The following theorem proves the stability and convergence of the closed-loop system, composed of the controller (16), the estimator (29) and the plant (3).

Theorem 5. The closed-loop system consisting of system (3), estimator (29) and controller (16) is uniformly ultimately stable for any bounded unknown dynamics F with $\sup_{t\geq 0} \left| \dot{F} \right| \leq \hbar$. Moreover, the estimation error e_F and the tracking error e will exponentially converge to a small compact set around zero.

Proof. Select a Lyapunov function defined as

$$V = \frac{1}{2}e^2 + \frac{1}{2}e_F^2.$$

 $\frac{dV}{dt}$ can be calculated along (13) and (17) as:

$$\dot{V} = e\dot{e} + e_F \dot{e}_F = -k_p a_2 e^2 - ee_F - \frac{1}{\tau} e_F^2 + e_F \dot{F}.$$
 (20)

Then by applying Young's inequality on the terms ee_F and $e_F \dot{F}$, we can further have:

$$\dot{V} \leq -(k_p a_2 - \frac{\eta}{2})e^2 + \frac{1}{2\eta}e_F^2 - \frac{1}{\tau}e_F^2 + \frac{1}{2\eta}e_F^2 + \frac{\eta}{2}\dot{F}^2$$

$$\leq -(k_p a_2 - \frac{\eta}{2})e^2 - (\frac{1}{\tau} - \frac{1}{\eta})e^2 + \frac{\eta}{2}\hbar^2$$

$$\leq -\alpha V + \frac{\eta}{2}\hbar^2$$
(21)

where $\alpha = \min\{2(k_pa_2 - \eta/2), 2(1/\tau - 1/\eta)\}$ is a positive constant for any appropriately chosen parameters $k_pa_2 > \eta/2 > \tau/2, \tau > 0$.

Thus, by integrating both sides of (21), we can obtain that: $V(t) \leq e^{-\alpha t} V(0) + \eta \hbar^2 / (2\alpha)$

holds and this implies that e and e_F will exponentially converge to a compact set defined by

$$\Omega := \left\{ e, e_F | |p| \le \sqrt{\eta \hbar^2 / \alpha}, |e_F| \le \sqrt{\eta \hbar^2 / \alpha} \right\}.$$

whose size depends on the upper bounds of \hbar , the filter coefficients τ and the feedback gain k_p . This completes the proof.

Remark 6. : In the above control design, we only assume that the input gain a_2 of the rotomagnet plant is known (this condition is also required in the design of DOB (Chen, 2004) and ESO (Han, 1995)), while the friction dynamics with a_1 are not necessarily known in a *priori*. However, if precision knowledge of friction can be obtained, i.e. a_1 is known, the above control design could be easily reformulated as given below.

In case precision information of a_1 is known, it is possible to change the estimator (29) with (4) to estimate the exact disturbance d. In this case, the system can be rewritten as

$$\dot{y} = -a_1 y + a_2 u + d \tag{22}$$

$$=F(y,d) - a_1y + a_2u \tag{23}$$

In this case, we denote F(y,d) = d as the unknown disturbance, and the dynamics $-a_1y$ are known.

Then, an appropriate estimation of unknown dynamics F(y, d) can be given as

$$\hat{F} = \frac{y - y_f}{\tau} - a_2 u_f + a_1 y_f.$$
(24)

Then, the control (16) can be changed as

$$u = k_p e - \frac{1}{a_2} (\hat{F} - a_1 y - \dot{y}_r).$$
(25)

With the above modifications, similar claims as Theorem 2 and Theorem 5 can be derived. Here, we do not repeat them again.

4. ALTERNATIVE ESTIMATION AND CONTROL

In the previous section, the trajectory to be tracked, \dot{y}_r , must exist and be used in the control (16), and the proposed estimator (29) is designed based on the original system (2). In this section, we will further modify the design of estimator and thus control based on the error dynamics. For this purpose, we rewrite the error system as

$$\dot{e} = \dot{y}_r - \dot{y} = \dot{y}_r + a_1 y - d - a_2 u \tag{26}$$

$$=F(y,y_r,d)-a_2u\tag{27}$$

where $F(y, y_r, d) = \dot{y}_r + a_1 y - d$ is the lumped dynamics to be estimated, which cover the friction $a_1 y$, external disturbance d and trajectory \dot{y}_r .

Hence, we can redefine the filter operations on the tracking error e and the input u as

$$\tau \dot{e}_f + e_f = e, e_f(0) = 0 \tag{28}$$

$$\tau \dot{u}_f + u_f = u, u_f(0) = 0, \tag{29}$$

where $\tau > 0$ is the filter constant.

Then the estimation of unknown dynamics $F(y, y_r, d)$ can be given by

$$\hat{F} = \frac{e - e_f}{\tau} + a_2 u_f. \tag{30}$$

With the obtained estimation \hat{F} given in (30), we can design the control for (26) as

$$u = k_p e + \frac{1}{a_2} \hat{F}.$$
(31)

where $k_p > 0$ is the feedback gain, and \hat{F} is the estimator given by (30).

Similar stability and convergence analysis as that presented in Section 3 can be carried out, which will not be repeated again. Compared to the method shown in Section 3, the estimator and control given in this section only depends on the error signal e and control signal u, and thus they are data-driven based method. Moreover, we do not require the accurate information of \dot{y}_r in the control implementation, which is more suitable in practice. Instead this has been lumped int unknown dynamics $F(y, y_r, d)$ and then estimated online by using the estimator (30).

5. NUMERICAL SIMULATIONS

Using system identification procedure appropriate parameters to describe the rotomagnet system behavior have



Figure 4. Disturbance signal, d(t), used in the simulations.



Figure 5. Evolution of the output, y(t), and the reference $y_r(t)$ in the closed-loop system.



Figure 6. Evolution of the tracking error, e, and unknown dynamics estimation error e_F in the closed-loop system.

been obtained (Ramos et al., 2013): k = 16.152 and a = 0.457.

To analyze the proposed controller performance a disturbance similar to that appearing in the real system has been used. The concrete disturbance is shown in Figure 4 and is composed a 2.5 rad/s sinusoidal and two additional higher harmonics. A filter with $\tau = 0.001$ and controller with $k_p = 1$ have been selected.

Proposed control scheme (Figure 3.1) have been implemented to the rotomagne plant subject to previously described disturbance. The output and the reference are Preprints of the 3rd IFAC Conference on Advances in Proportional-Integral-Derivative Control, Ghent, Belgium, May 9-11, 2018



Figure 7. Uncertainty dynamics, F, and its estimation F in the closed-loop system.

shown in Figure 5, as it can be seen after a small transient, the output is tracking nicely the reference. As it can be seen in Figure 6 the error is always bound and small. Finally, Figure 7 show the evolution of the unmodeled dynamics, F, and its estimation \hat{F} , as it can be seen both are almost the same from the beginning. It can be concluded that propose controller is working very well in the case under study.

6. CONCLUSIONS

This paper has proposed a simple control scheme which is composed of a regular PID controller plus a disturbance observer. The paper has shown a formulation of the proposed architecture and formal proof of the closed-loop stability. Also bounds on the estimation error are provided.

Currently, the authors are working to experimentally validate the proposed mechanisms, extend the results to more generic system and improve the robustness against uncertainty in a_1 and a_2 .

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REFERENCES

- Åström, K.J. and Hägglund, T. (1995). *PID Controllers: Theory, Design, and Tuning.* Instrument Society of America, Research Triangle Park, NC, 2 edition.
- Åström, K.J. and Hägglund, T. (2006). Advanced PID control. ISA, Advanced PID control.
- Chen, W.H. (2004). Disturbance observer based control for nonlinear systems. *IEEE/ASME Transactions on Mechatronics*, 9(4), 706–710. doi: 10.1109/TMECH.2004.839034.

- Costa-Castelló, R., Nebot, J., and Grinó, R. (2005). Demonstration of the internal model principle by digital repetitive control of an educational laboratory plant. *IEEE Transactions on Education*, 48(1), 73–80.
- Costa-Castelló, R., Olm, J.M., Vargas, H., and Ramos., G.A. (2012). An educational approach to the internal model principle for periodic signals. *International Jour*nal of Innovative Computing, Information and Control, 8(8), 5591–5606.
- Edwards and Christopher (1988). Sliding mode control : theory and applications. Taylor & Francis.
- Francis, B. and Wonham, W. (1975). The internal model principle of linear control theory. *IFAC Proceedings Volumes*, 8(1, Part 1), 331 – 336. 6th IFAC World Congress (IFAC 1975) - Part 1: Theory, Boston/Cambridge, MA, USA, August 24-30, 1975.
- Garcia, C.E. and Morari, M. (1982). Internal model control. a unifying review and some new results. *Industrial* & Engineering Chemistry Process Design and Development, 21(2), 308–323.
- Guo, B. and Zhao, Z. (2011). On the convergence of an extended state observer for nonlinear systems with uncertainty. *Systems & Control Letters*, 60(6), 420 – 430.
- Han, J. (1995). The "extended state observer" of a class of uncertain systems. *Control & Decision*.
- Han, J. (1998). Auto-disturbances-rejection controller and its applications. *Control & Decision*.
- Hillerström, G. and Walgama, K. (1996). Repetitive control theory and applications - a survey. *IFAC Proceedings Volumes*, 29(1), 1446 – 1451. 13th World Congress of IFAC, 1996, San Francisco USA, 30 June -5 July.
- Moliner, R. and Tanda, R. (2016). Tool for robust tuning of pi/pid controllers with two degree of freedom. *Revista Iberoamericana de Automática e Informática Industrial RIAI*, 13(1), 22 – 31.
- Na, J., Chen, A.S., Herrmann, G., Burke, R., and Brace, C. (2017). Vehicle engine torque estimation via unknown input observer and adaptive parameter estimation. *IEEE Transactions on Vehicular Technology*, PP(99), 1–1.
- Oh, Y. and Chung, W.K. (1999). Disturbance-observerbased motion control of redundant manipulators using inertially decoupled dynamics. *IEEE/ASME Transactions on Mechatronics*, 4(2), 133–146.
- Oviedo, J.J.E., Boelen, T., and van Overschee, P. (2006). Robust advanced pid control (rapid): Pid tuning based on engineering specifications. *IEEE Control Systems*, 26(1), 15–19.
- Ramos, G.A., Costa-Castelló, R., and Olm, J.M. (2013). Digital Repetitive Control under Varying Frequency Conditions, volume 446 of Lecture Notes in Control and Information Sciences. Springer.
- Slotine, J.J.E. and Li, W. (2004). Applied nonlinear control. China Machine Press.
- Stotsky, A. and Kolmanovsky, I. (2002). Application of input estimation techniques to charge estimation and control in automotive engines. *Control Engineering Practice*, 10(12), 1371 – 1383.
- Zhao, C. and Guo, L. (2017). Pid controller design for second order nonlinear uncertain systems. *Science China Information Sciences*, 60(2), 022201.