Robust QFT-based PI controller for a feedforward control scheme ⋆

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Abstract: Feedforward control schemes to compensate for disturbances are very well known in process control. In those control approaches, PID controllers are usually considered in the feedback loop, where nominal design for both feedback and feedforward controllers are commonly performed. This paper presents a robustness analysis to study how uncertainties can affect the classical feedforward control scheme. Afterwards, a robust PI controller is designed by using Quantitative Feedback Theory to account for these uncertainties and to fulfill robust specifications for the regulation control problem. Results based on frequency and time domains are presented. © *Copyright IFAC 2018.*

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1. INTRODUCTION

In process control, feedforward compensation helps the feedback control to attenuate the effects of measurable disturbances on the process. When the disturbance enters into the process, the feedforward controller helps the classic feedback controller, which only has reactive action, by supplying an extra corrective signal before the disturbance leads the system away from its setpoint (Guzmán et al., 2015).

The feedback with feedforward control strategy is commonly used in process industry. It is implemented in most distributed control systems to improve the control performance in applications such as distillation columns (Nisenfeld and Miyasaki, 1973), power plants (Weng and Ray, 1997) or microalgae cultures among many other examples (Adam and Marchetti, 2004).

The common feedforward compensator is formed as the dynamics between the load disturbance and the process output, divided by the dynamic between the control signal and the process output with reverse sign. However, the ideal compensator may not be realizable in many cases due to negative delay, having more zeros than poles, poles in the right-half plane, or in general non-minimum phase behaviours. In the last few years, several works have appeared proposing new tuning rules to account for these inversion problems (Hast and Hägglund, 2014),(Rodríguez et al., 2013), (Rodríguez et al., 2014),(Guzmán et al., 2015).

In all these works, nominal models were used and the robustness case was not studied. Notice that when uncertainties are considered, the feedforward control scheme is deteriorated even for the perfect cancellation case. In that case, the cancellation of the feedforward is not perfect and the closed-loop specifications may not be fulfilled when the system deviates from the nominal conditions (Guzmán and Hägglund, 2011). Thus, it is interesting to analyze this situation and to propose robust solutions for this problem.

There are only a few works in literature where the robustness for the feedforward control scheme has been studied. In (Guzmán and Hägglund, 2011) the robustness of the feedback control with feedforward compensator was analyzed with respect to uncertainties in the process gain and approximated high-order dynamics. It was demonstrated that the variability of the process model parameters affects the response of the PID controller with the feedforward compensation.

On the other hand, (Adam and Marchetti, 2004) presented a robust design solution for feedforward controllers when the available dynamic models include estimated limits for the uncertainties. A model-based design and tuning procedure is proposed to account for model uncertainties. As conclusion, it is derived that the feedforward and the feedback controllers should be tuned simultaneously for an efficient disturbance rejection. This idea has been used later in other works. For instance, in (Vilanova et al., 2009) a sequential tuning of feedforward controllers within an IMC control structure is proposed. In this work, the compensator is defined as the invertible part of the quotient plus a tunable filter which is chosen to minimise the interaction between both controllers. Furthermore, in (Rodríguez et al., 2016) a robust design methodology for simultaneously tuning both feedforward and feedback

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controllers is presented. Disturbance rejection performance condition is expressed as a degradation band above a desired shape. Finally, in (Elso et al., 2013), Quantitative Feedback Theory (QFT) was used to design robust feedback and feedofrward controllers. New QFT bounds were obtained for the design stage and it was the first time that the feedback was linked to the existence of a feedforward controller in QFT.

The aim of this paper is to propose a QFT-based robust solution for the feedforward control scheme presented in (Guzmán and Hägglund, 2011), where PI control is combined with feedforward compensators. The idea consists in moving the uncertainties effect to bounds in the QFT specifications and then designing a robust PI controller. The main contribution of this paper consists in modifying the original boundaries of the QFT methodology for the regulation problem, and to design a robust PI controller to account for the uncertainties. Notice that the feedforward compensator is not designed from a robust point view, since its effect is moved to the QFT specifications.

The paper is organized as follows. In section 2, the classic feedforward control scheme is described. Section 3 is focused on the robustness problem analysis and the proposed control design approach. In section 4, a numerical example is presented to demonstrate the contributions described in section 4. Finally, section 5 conducts the conclusion of the work.

2. PRELIMINARIES

The feedforward control scheme used in this work is shown in Figure 1. It consists of a feedback controller $C(s)$, a process $P_u(s)$, a setpoint signal r, a control signal u, and a process output y. The disturbance d, which is measurable, influences the feedback loop as shown in the figure. The transfer function between the load d and the output y is $P_d(s)$. The feedforward compensator $FF(s)$ feeds the disturbance d, and its output is added to the feedback control signal.

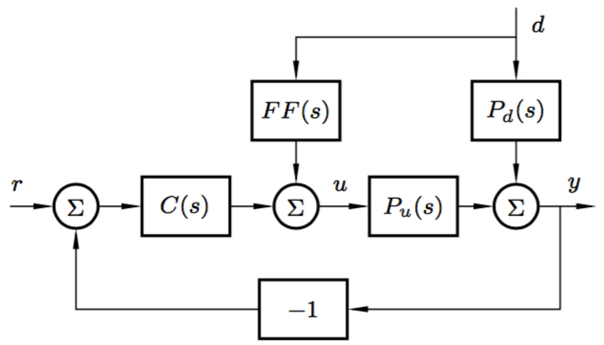

Fig. 1. Classical feedforward control scheme.

A PI controller is considered as feedback regulator $C(s)$ such as used in (Guzmán and Hägglund, 2011):

$$C(s) = K_p \left(1 + \frac{1}{T_i s}\right) \tag{1}$$

The models of the process $P_u(s)$ and the disturbance $P_d(s)$ are described by first-order transfer functions without time delay

$$P_u(s) = \frac{k_u}{\tau_u s + 1} \tag{2}$$

$$P_d(s) = \frac{k_d}{\tau_d s + 1} \tag{3}$$

where the gains and the time constants in both cases, k_u, τ_u for the process and k_d, τ_d for the disturbance, are the parameters that bring the uncertainty to the system as described below. Notice that in this work, the free-delay case is considered as a first approach to the robust problem. However, the solution can easily be extended to the time delay case.

From Figure 1, the closed loop transfer function between the output y and the disturbance signal d, $T_{dy}(s)$, is given by the following equation

$$T_{dy}(s) = \frac{-FF(s)P_u(s) + P_d(s)}{1 + C(s)P_u(s)} \tag{4}$$

The proposed feedforward compensator is given by the following transfer function

$$FF(s) = -K_{ff}\frac{T_z s + 1}{T_p s + 1} \tag{5}$$

where $T_z = \tau_u$, $T_p = \tau_d$ and K_{ff} is set as follows:

$$K_{ff} = \frac{k_d}{k_u} \tag{6}$$

in order to cancel the effect of the disturbance effect as suggested in (Guzmán and Hägglund, 2011).

3. ROBUSTNESS ANALYSIS AND DESIGN

Such as described in the previous section, the feedforward element is commonly used as a lead-lag compensator described by (5). Usually, it is calculated from models in Eq. (2) and (3), assuming that there is no uncertainty (Guzmán and Hägglund, 2011). Notice that for the nominal case and if there are no inversion problems, the disturbance effect can be totally cancelled.

In this paper, we assume uncertainties in gain and time constant in the process transfer functions P_u and P_d to perform a robustness analysis of the classical feedforward control design. Thus, now we have a set of plants given by the following equations

$$\mathcal{P}_u = \{P_u : k_u \in [k_{u,low}, k_{u,high}], \tau_u \in [\tau_{u,low}, \tau_{u,high}]\} \tag{7}$$

$$\mathcal{P}_d = \{P_d : k_d \in [k_{d,low}, k_{d,high}], \tau_d \in [\tau_{d,low}, \tau_{d,high}]\} \tag{8}$$

$$P_u^0(s) = \frac{k_u^0}{\tau_u^0 s + 1} \tag{9}$$

$$P_d^0(s) = \frac{k_d^0}{\tau_d^0 s + 1} \quad (10)$$

where $P_u^0(s) \in \mathcal{P}_u$ and $P_d^0(s) \in \mathcal{P}_d$ are the nominal plants.

The first issue to be analyzed is to observe how the use of the feedfoward control scheme and the presence of uncertainties affect the system specifications in the regulation control problem.

Lets rewrite equation (4) as follows

$$T_{dy}(s) = \frac{P_{dg}(s)}{1+C(s)P_u(s)}$$

where P_{dg} is given by the following equation

$$P_{dg}(s) = -FF(s)P_u(s) + P_d(s) \quad (11)$$

Notice that if $FF = 0$ we have the classical regulation problem.

When P_u and P_d are uncertain and QFT is used to design the feedback regulator, the specification for the robust regulation problem is given as

$$|T_{dy}(j\omega)|_{dB} = \left| \frac{P_{dg}(j\omega)}{1 + C(j\omega)P_u(j\omega)} \right|_{dB} \leq \delta_{dB} \quad (12)$$

or equivalently as

$$\left| \frac{1}{1 + C(j\omega)P_u(j\omega)} \right|_{dB} \leq \delta_{dB} - |P_{dg}(j\omega)|_{dB} = \gamma(\omega) \quad (13)$$

for all plants $P_u \in \mathcal{P}_u$, $P_d \in \mathcal{P}_d$ and where γ is the new specification bound.

Therefore, it can be observed how the feedforward compensator and the uncertainties affect the bound in the specification problem. Thus, according to (12) and (13), there are two different ways to account for the robust control problem:

- The first one would be to use (13) and follow the classical robust design with QFT. In this case, the specification bound γ depends on the uncertainties and the feedforward compensator presented in $P_{dg}(j\omega)$. Then, the minimum value of γ for all plants $P_u \in \mathcal{P}_u$ and $P_d \in \mathcal{P}_d$, and for all evaluated frequencies, must be considered as specification. Once this minimum bound is calculated, classical QFT is used to obtain the resulting controller. However, a very conservative solution will be obtained since the specification bound is computed for the worst possible case.
- On the other hand, a second solution to this problem is to consider the specification (12) explicitly and to modify the boundary calculation in QFT. So, new boundaries are obtained considering the presence of the feedforward compensator and then a robust controller is obtained based on these new limits. This is the new solution proposed in this paper and presented in the following.

First, the nominal plants $P_u^0(s)$ and $P_d^0(s)$ are selected, and the nominal feedforward compensator is calculated using the rules proposed in (Guzmán and Hägglund, 2011). That is, K_{ff} is set as shown in (6), $T_p = \tau_d^0$ and $T_z = \tau_u^0$

Then, the algorithm proposed in (Moreno et al., 2006) to compute classical boundaries in QFT is modified in order to include a new kind of boundary that assures the satisfaction of specification in Eq. (12) when $FF \neq 0$. It is important to remember the concept of crossection defined in (Moreno et al., 2006). Fixed a phase in Nichols plane (NP) (and the frequency ω for which the boundary is being computed), the crossection is a function of the magnitude of the nominal open loop transfer function $L_0 = CP_u^0$, that provides a value of interest, the value of $Max|T_{dy}(j\omega)|_{dB}$ in this paper. Obviously the location of $L_0(j\omega)$ in the NP depends on the value of $C(j\omega)$ because P_u^0 is fixed.

Figure 2 shows an example of crossection for the regulation problem for the cases with and without feedforward compensator and for $\omega = 1$ rad/s and phase$(L_0(j\omega)) = -100$ degrees. In this figure, two different specifications are shown, for $\delta_{dB} = -20$ and $\delta_{dB} = -10$, respectively. It can be observed how for specifications where $\delta_{dB} < -20$, both cases are equal for this frequency. See for instance the case where both solutions cut the specification of $\delta_{dB} = -20$ at the value of 5.66 dB. However, for specifications with $\delta_{dB} \geq -20$ both solutions are different. This means that different boundaries will be obtained for both cases and thus different control design must be done. For instance, for the specification of $\delta_{dB} = -10$, the case where $FF = 0$ does not cut the limit, while the case with $FF \neq 0$ cuts the specification bound at the value of -4.19 dB. Thus, a more restrictive solution is given for the case when the feedforward is included in the control scheme. This result indicates that the use of the feedforward compensator can affect the control problem negatively when modelling errors appear in the system. This fact can be better seen from Figure 3. This figure shows the boundaries for the specification of $\delta_{dB} = -10$. As observed, the boundary is open when $FF \neq 0$ and closed when $FF = 0$, thus being the first one much more restrictive. However, it is interesting to see how for the zone around (-180^o, 0 dB), the boundary for $FF \neq 0$ is smaller and thus less restric-

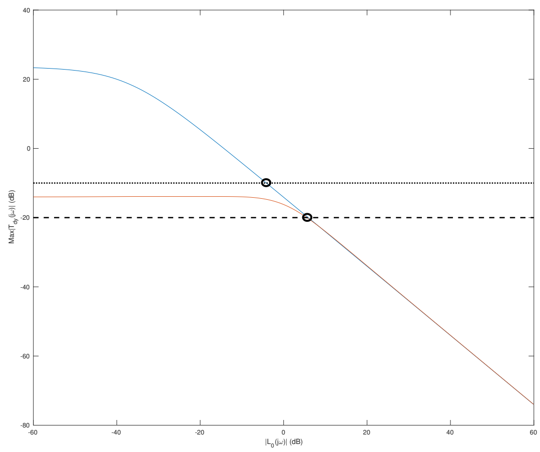

Fig. 2. Crossections for the regulation problem for $FF = 0$ (red) and for $FF \neq 0$ (blue) for $\omega = 1$ rad/s and $phase(L_0(j\omega)) = -100$ degrees. Two speficiations are shown, for $\delta_{dB} = -20$ (–) and $\delta_{dB} = -10$ (-).

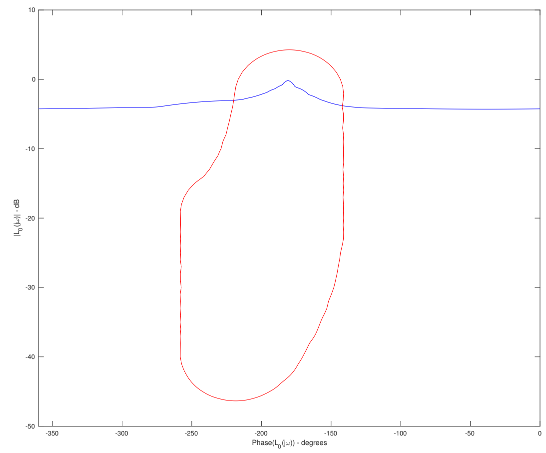

Fig. 3. Boundary comparisons for the regulation problem for $FF = 0$ (red) and for $FF \neq 0$ (blue) for $\omega = 1$ rad/s and $phase(L_0(j\omega)) = -100$ degrees and for the specification $\delta_{dB} = -10$.

tive. In any case, this is usually protected by the stability boundaries and thus it is not an important advantage.

Thus, it is concluded that QFT can be used as robust control design method when a feedforward control scheme is considered to account for the uncertainties in the process. However, the presence of the feedforward compensator affects to the calculation of the classical boundaries in QFT and new specifications must be fulfilled during the controller design stage such as shown in the following section with numerical examples.

4. NUMERICAL EXAMPLE

This section presents a numerical example to demonstrate the contributions described in the previous section. Lets assume the following models for the process:

$$\mathcal{P}_u(s) = \frac{k_u}{\tau_u s + 1},\ k_u \in [1, 10], \tau_u \in [1, 10] \tag{14}$$

and

$$\mathcal{P}_d(s) = \frac{k_d}{\tau_d s + 1},\ k_d \in [3, 7], \tau_d \in [11, 15] \tag{15}$$

with nominal models P_u^0 given by $k_u^0 = 1$ and $\tau_u^0 = 10$ and P_d^0 given by $k_d^0 = 3$ and $\tau_d^0 = 11$.

Such as commented in the previous section, there are two possible solutions to the problem based on considering the specifications as shown in (12) or (13). The following subsections show the analysis and results for both cases.

4.1 Classical solution

One solution for the problem would be to use the specification of the problem according to (13) for the worst case and then use the classical stages for QFT with classical boundaries.

Figure 4 shows different values for the γ function defined by Eq. (13) for all the plants in $\mathcal{P}_u$ and $\mathcal{P}_d$. The curve represented by asterisks shows the case when $FF = 0$

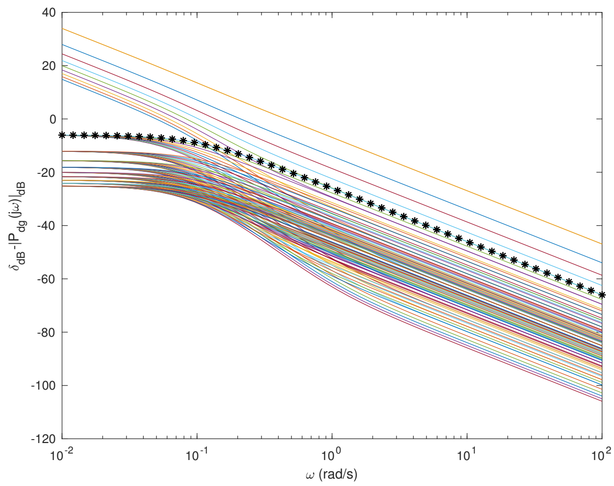

Fig. 4. Right side of Eq. (13) with $FF = 0$ (*) and with $FF \neq 0$ (-) for nominal models P_u^0 given by $k_u = 1$ and $\tau_u = 10$ and P_d^0 given by $k_d = 3$ and $\tau_d = 11$.

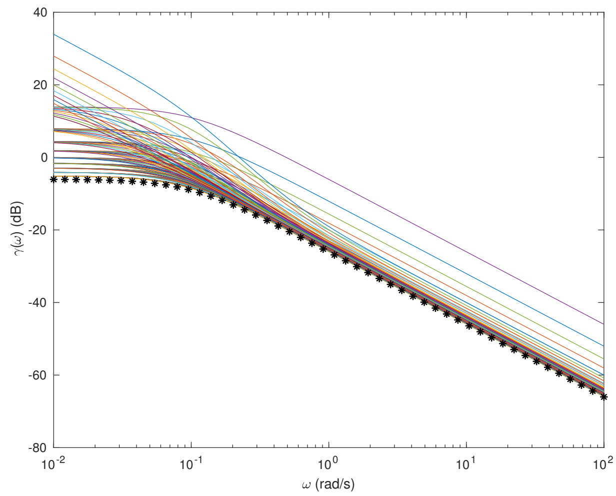

Fig. 5. Right side of Eq. (13) with $FF = 0$ (*) and with $FF \neq 0$ (-) for nominal models P_u^0 given by $k_u = 10$ and $\tau_u = 1$ and P_d^0 given by $k_d = 3$ and $\tau_d = 11$.

and the rest of the curves are all γ values when $FF \neq 0$. Thus, it is observed how for this nominal choice, there are many cases of the uncertainty where the presence of the feeforward compensator results in a more restrictive specification (a more aggressive controller will be required) since they are below the case when $FF = 0$. Therefore, this result indicates that in this case, the selection of the nominal plant cannot be done arbitrarily. It would be necessary to obtain the nominal plant that gives the maximum value of γ for all possible combinations.

If we perform this for this example, the results presented in Figure 5 are obtained for the best case. That is, it is the selection of the nominal models that give the less restrictive γ values. This solution has been obtained for the nominal models P_u^0 given by $k_u^0 = 10$ and $\tau_u^0 = 1$ and P_d^0 given by $k_d^0 = 3$ and $\tau_d^0 = 11$. It can be seen that when $FF \neq 0$ all γ functions are greater than or equal to the γ function corresponding to $FF = 0$. Then, the function that must be used as specification in Eq. (13) is given by $\gamma(\omega) = \delta_{dB} - |P_d(j\omega)|_{dB}$, which is the same specification as

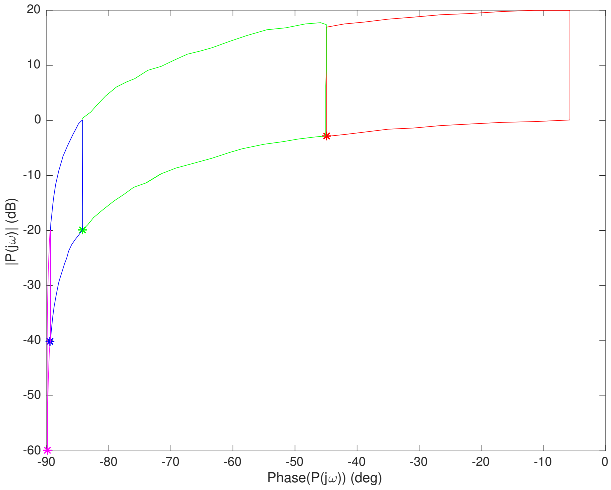

Fig. 6. Templates for $\omega \in \{0.1, 1, 10, 100\}$ rad/s

when the feedforward term is not considered. If any other value for nominal used to compute FF is chosen, some γ functions will be located below the line with asterisks in the Fig. 5 as shown in the case of Figure 4. Thus, the specification would be more restrictive and a more demanding feedback controller will be necessary in order to assure the specifications.

Hence, we can conclude that when the specification (13) is considered, the same specification as the case when $FF = 0$ can be used for the robust control problem.

4.2 New solution

In this case the specification (12) is considered and the new solution described in section 3 is used for this example.

Then, classical stability specifications in QFT and the new kind of disturbance rejection specifications including the feedforward element are taken into account. A phase margin greater than or equal to 45 degrees for the whole uncertainty set is used as stability specification. So, this specification on the closed loop transfer function is given by

$$|T(j\omega)| = \left| \frac{C(j\omega)P_u(j\omega)}{1 + C(j\omega)P_u(j\omega)} \right| \leq 2.32dB \qquad (16)$$

On the other hand, a disturbance rejection specification given by $\delta_{dB} = -40dB$. The set of design frequencies chosen is $W = \{0.1, 1, 10, 100\}$ rad/s.

Figure 6 shows the templates for the set of design frequencies and for the set of plants from Eq. (14). Figure 7 shows the stability bounds, all of them are closed boundaries, and the disturbance rejection bounds, all open boundaries, for the same set of frequencies. A nominal open loop transfer function shaped fulfilling all the boundaries is drawn, given by a PI controller with $K_p = 400$ and $T_i = 100$. The parameters used for the FF element are $T_z = 10$, $T_p = 11$, and $K_{ff} = 3$.

Figures 8 and 9 show that the control system satisfies all the specifications from a frequency domain point of view.

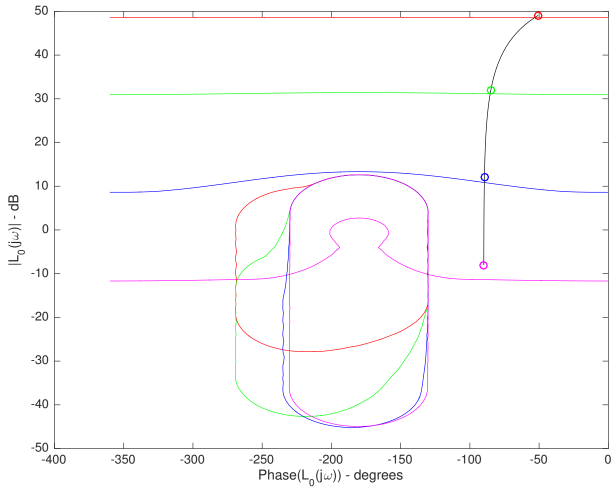

Fig. 7. Nominal open-loop shaping and stability and disturbance on output rejection bounds taking the FF element into account.

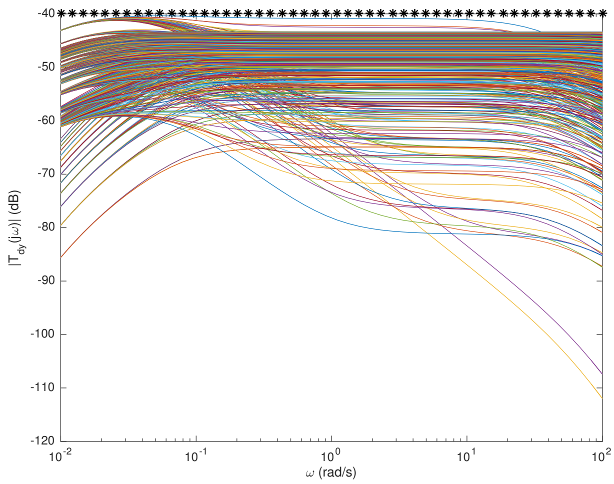

Fig. 8. $T_{dy}(j\omega)$ transfer functions and specification

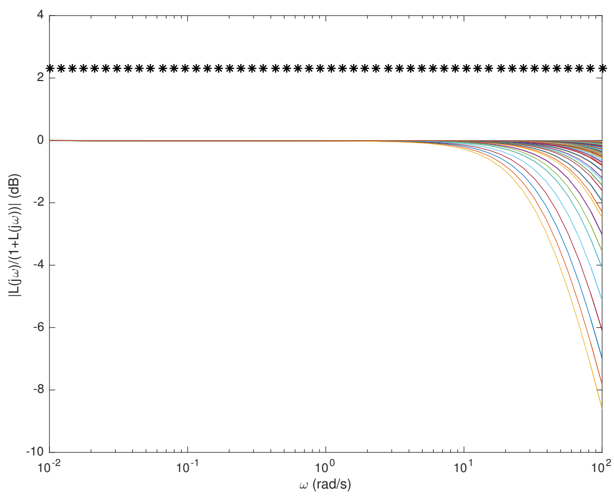

Fig. 9. $T(j\omega)$ transfer functions and specification

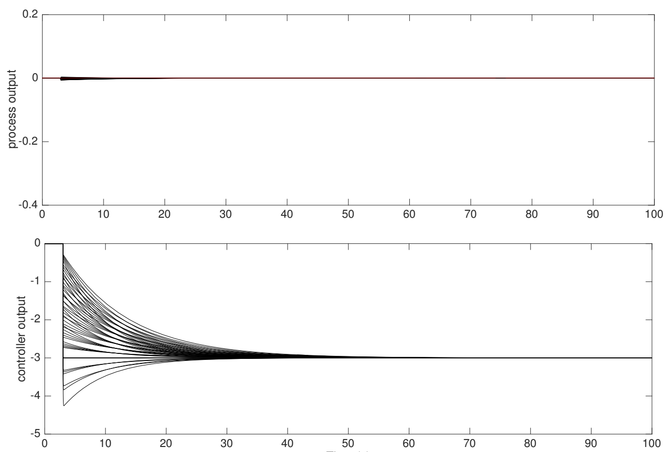

Fig. 10. Time domain simulations for the proposed robust control design. A unitary step disturbance was included at time $t = 3$ seconds

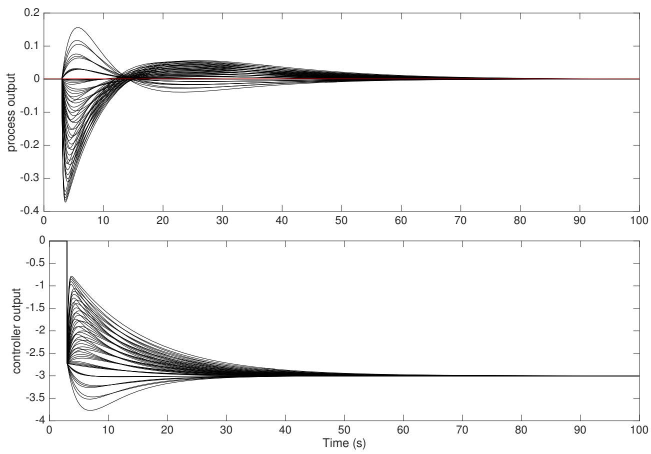

Fig. 11. Time domain simulations for nominal control design (Guzmán and Hägglund, 2011). A unitary step disturbance was included at time $t = 3$ seconds

Finally, Figures 10 and 11 show the results in time domain for the proposed robust control approach presented in this paper and for the nominal control design presented in (Guzmán and Hägglund, 2011), respectively. It can be observed how for the proposed case, the disturbance is almost rejected beside the uncertainties with an important performance improvement with respect to the nominal case in Figure 11. Moreover, this result is achieved with very similar control effort as shown in the controller output signal of both figures.

5. CONCLUSIONS

This paper has analyzed the classical feedforward control scheme for measured disturbances for the case when uncertainties are considered. QFT has been used as robust control design method. It was shown that the presence of the feedforward compensator changes the classical QFT specification for the regulation problem. This modification leads to two different solutions. The first one consists in using the same specification as the case when the feedforward is not considered, and classical QFT boundaries for the control design process are calculated. This approach would result in very conservative results and the presence of the feedforward compensator would not give remarkable advantages. The second solution is based on modifying the boundaries of the regulation problem with QFT to include the presence of the feedforward controller. In this case, new boundaries were obtained and the QFT method was used to design a robust PI controller to account for uncertainties obtaining promising results.

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