# Control of Refrigeration Systems based on Vapour Compression using Multi-objective Optimization Techniques \*

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**Abstract:** In this work a tuning procedure by means of multi-objective optimization techniques is used for a refrigeration system based on vapour compression, stated as the benchmark process control challenge organized by the IFAC Conference on Advances in Proportional-Integral-Derivative (PID) Control. The advantage of such a procedure lies in the capacity to perform an analysis on the trade-off among conflicting design objectives. The resulting controller fulfills the requirements of the contest, and gets and overall performance index of 0.4028 outperforming the base line controller.

Keywords: Refrigeration system, PID controller, multi-objective optimization.

# 1. INTRODUCTION

PID controller is by far the most used control structure, due to its simplicity, robustness, efficiency and implementability (Åström and Hägglund, 2005; Visioli, 2006). They represent a common solution for several industrial applications; for this reason there is a continued interest in new tuning design methodologies in order to improve their overall performance guaranteeing reasonable stability margins for a wide variety of processes (Åström and Hägglund, 2001; Stewart and Samad, 2011; Garpinger et al., 2014).

Since Ziegler and Nichols (1942) presented their wellknown tuning rules, several works have been developed for PID controllers and similar structures. Some examples are autotuning methods (Åström and Hägglund, 2001; Skogestad, 2003; Kristiansson and Lennartson, 2006), tuning rules based on the control system performance (setpoint or load-disturbance) (Rovira et al., 1969; Chien and Fruehauf, 1990; Tavakoli and Tavakoli, 2003) or robustness-based (Panagopoulos et al., 2002; Kristiansson and Lennartson, 2006; Alfaro et al., 2010). Recently, alternative methods for tuning PID controllers based on multi-objective optimization techniques have been suggested (Reynoso-Meza et al., 2014a, 2016; Sánchez et al., 2017). With such procedures it is possible to handle design objectives simultaneously; therefore, the controller tuning can be seen as a multi-objective problem (MOP), where the designer seeks for a set of pareto optimal solutions to approximate the Pareto Front during the optimization process (Miettinen, 1999; Marler and Arora, 2004). From here according to his/her preferences, the designer needs to choose the best solution, which takes place in a multi-criteria decision making (MCDM) step.

In this paper, we present a tuning methodology based on a multi-objective optimization design (MOOD) procedure to adjust the parameters of a multi-variable PID controller. The process under consideration is the refrigeration system based on vapour compression described as the benchmark challenge in Bejarano et al. (2017). The main advantage of the proposed approach is to give to the designer the possibility to analyze, at the end of the optimization process, a set of solutions with different trade-offs and select a solution with the desired balance between competing design objectives. The paper is structured as follows: in Section 2, the description of the contest and the refrigeration system are introduced. Section 3 defines the MOOD methodology and its properties. Section 4 presents the optimization and control results. Finally, Section 5 draws some conclusions of this approach.

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Input variable		Mathematical Symbol	Range	Units
Manipulated Variables	Expansion valve opening	$A_v$	[10-100]	%
	Compressor speed	N	[30-50]	Hz
Disturbances	Inlet temperature of the condenser secondary flux	$T_{c, sec, in}$	[27-33]	°C
	Mass flow of the condenser secondary flux	$\dot{m}_{c, sec}$	[125 - 175]	$g s^{-1}$
	Inlet pressure of the condenser secondary flux	$P_{c, sec, in}$	-	bar
	Inlet temperature of the evaporator secondary flux	$T_{e, sec, in}$	[-2218]	°C
	Mass flow of the evaporator secondary flux	$\dot{m}_{e, \ sec}$	-	$g s^{-1}$
	Inlet pressure of the evaporator secondary flux	$P_{e, sec, in}$	-	bar
	Compressor surrounding temperature	$T_{surr}$	[20-30]	°C

Table 1. Input variables ranges.

# 2. BENCHMARK DESCRIPTION

The process under consideration is the refrigeration system based on vapour compression described by Bejarano et al. (2017). A refrigeration system is made up by a closed cycle, whose components are connected through various pipes and valves, which causes a non-linear multivariable systems, where all the variables involved are highly coupled (Sarabia et al., 2009). The cycle is to remove heat at the evaporator from its secondary flux and eject heat into the condenser by transferring it to the secondary flux. The compressor provides the required pressure increase supplied to the refrigerant, whereas the expansion valve just holds up the pressure difference at the liquid line.



Fig. 1. Vapour compression system.

Figure 1 shows a canonical one-compression-stage, oneload demand refrigeration cycle, where the main components are comprised of a variable-speed compressor, an electronic expansion valve and two heat exchangers (an evaporator and a condenser). One of the control objectives is to provide the desired cooling power  $\dot{Q}_e$  while reducing the degree of super heating  $T_{SH}$ . This is done by means of the so-called Coefficient of Performance (COP), which is defined as:

$$COP = \frac{\dot{Q}_e}{\dot{W}_{comp}} = \frac{\dot{m}(h_e, out - h_e, in)}{\dot{m}(h_c, in - h_e, out)} = \frac{h_e, out - h_e, in}{h_c, in - h_e, out}$$
(1)

where  $\dot{W}_{comp}$  denotes the compression power. The cycle, working with R404a as refrigerant, is expected to provide a certain cooling power  $\dot{Q}_e$  to a continuous flow entering the evaporator as secondary flux. The evaporator secondary fluid is a 60% propylene glycol aqueaous solution, whereas the condenser secondary fluid is air. Then, the cooling demand can be expressed as a reference on the outlet temperature of the evaporator secondary flux  $T_{e, sec, out}$ , where the mass flow and inlet temperature act as measurable disturbances.

The manipulated variables and the disturbances are described in Table 1, while the controlled variables are  $T_{e, sec, out}$  and  $T_{SH}$ , respectively. It is worth to mention that the manipulated variables are saturated within the system, in such way that if a value is out of the ranges, as indicated in Table 1, it will be saturated to the closest value within the corresponding range. The model is ready to be controlled with a sampling period equal or greater than 1 second, starting always at the same operating points given by Table 2.

Table 2. Initial operating point.

Input variabl	Range	Units	
Manipulated Variables	$A_v$	$\cong 48.79$	%
Manipulated Vallables	N	$\cong 36.45$	Hz
	$T_{c, sec, in}$	30	°C
	$\dot{m}_{c, sec}$	150	$g s^{-1}$
	$P_{c, sec, in}$	1	bar
Disturbances	$T_{e, sec, in}$	-20	°C
	$\dot{m}_{e, sec}$	64.503	$g s^{-1}$
	$P_{e, sec, in}$	1	bar
	$T_{surr}$	25	C
Output variables	$T_{e, sec, out}$	$\simeq -22.15$	°C
Curput variables	$T_{SH}$	$\cong 14.65$	°C

The Benchmark PID 2018 provides the Simulink model presented in Bejarano et al. (2017) to test a multivariable discrete controller with or without feedforward. Neverthless, any type of controller could be tested using this model. The multivariable controller needs to be a 11x2 simulink block, and there is a total freedom to decide the structure of the block. In particular,  $T_{e, sec, out}$  is controlled by mean of  $A_v$  though the tranfer function:

$$C_{1R}(z) = \frac{-1.0136 - 0.0627 \, z^{-1} + 0.9988 \, z^{-2}}{1 - 1.9853 \, z^{-1} + 0.9853 \, z^{-2}} \tag{2}$$

while N controls  $T_{SH}$  though the transfer function:

$$C_{2R}(z) = \frac{0.42 - 0.02 \, z^{-1}}{1 - z^{-1}} \tag{3}$$

The disturbance information is not used, thus it is a MIMO controller without feedforward compensation. Finally, it is important to mention that all fluid thermodynamic properties are computed in the Benchmark PID 2018 using the *CoolProp tool* (Bell et al., 2014).

#### 3. TOOLS AND METHODOLOGY

The MOOD procedure is based on Pareto optimality (Figure 2) and the so-called multi-objective optimization (MOO), to handle MOPs. This procedure is used in order to tune a controller for the benchmark.

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Given a MOP, a MOO procedure consists in optimizing simultaneously all design objectives (and not an aggregation of them). As consequence, a set of solutions is calculated, where none is better than others in all design objectives. That is, a set of solutions with different trade-off.

A MOP, with m objectives, can be stated as follows (Miettinen, 1999):

$$\min_{\boldsymbol{x}} \boldsymbol{J}(\boldsymbol{x}) = [J_1(\boldsymbol{x}), \dots, J_m(\boldsymbol{x})]$$
(4)

subject to:

$$\boldsymbol{K}(\boldsymbol{x}) \le 0 \tag{5}$$

$$\boldsymbol{L}(\boldsymbol{x}) = 0 \tag{6}$$

$$\underline{x_i} \le x_i \le \overline{x_i}, i = [1, \dots, n] \tag{7}$$

where  $\boldsymbol{x} = [x_1, x_2, \dots, x_n]$  is defined as the decision vector with dim $(\boldsymbol{x}) = n$ ;  $\boldsymbol{J}(\boldsymbol{x})$  as the objective vector and  $\boldsymbol{K}(\boldsymbol{x})$ ,  $\boldsymbol{L}(\boldsymbol{x})$  as the inequality and equality constraint vectors respectively;  $\underline{x}_i, \overline{x}_i$  are the lower and the upper bounds in the decision space.



Fig. 2. Pareto optimality and dominance concepts for a min-min problem. Dark solutions is the subset of non-dominated solutions which approximates a Pareto front (right) and a Pareto set (left). Remainder solutions are dominated, because it is possible to find at least one solution with better values in all design objectives (Source: Carrau et al. (2017)).

For the successful implementation of the MOOD procedure, three main steps are required: the MOP statement, the MOO process, and a MCDM stage. Next, they are explained within the context of the benchmark.

#### 3.1 MOP statement

In the first step, design objectives, decision variables and constraints are defined. It is assumed that a model is available in order to evaluate the performance (objective vector) of a given solution (design vector) fulfilling all requirements (constraints).

In order to evaluate design vectors, a linear model has been identified via simple step response tests on the non-linear model provided by Bejarano et al. (2017). Four models identified have the following structure:

$$P(s) = K_p \frac{1+as}{1+bs} \tag{8}$$

Such model has been implemented in simulink to perform a simulation in order to test the control structure. In this case, it will be used the same control structure proposed for the reference controller  $\theta_{\mathbf{R}}$ . Equation 2 is a PID controller with gains  $k_{p1}, k_{i1}, k_d$  and a filter  $\frac{1}{s+f}$ ; Equation 3 is a PI controller with gains  $k_{p2}, k_{i2}$ . This means that a given decision vector  $\boldsymbol{\theta}$  has 6 decision variables.

Design objectives selected are those proposed in Meza et al. (2017): the integral of the absolute error (IAE) as a performance measure, and the total variation of the control action (TV) as a robustness measure.

$$J_{IAE_{(T_e,T_{SH})}}(\boldsymbol{\theta}) \ [^{\circ}C, ^{\circ}C] \tag{9}$$

$$J_{TV_{(Av,N)}}(\boldsymbol{\theta}) \ [\%, Hz] \tag{10}$$

For interpretability purposes, design objectives are normalized using the base line controller in the identified model with the selected simulation test. Therefore, the MOP under consideration is:

$$\min_{\boldsymbol{\theta}} \boldsymbol{J}(\boldsymbol{\theta}) = [\hat{J}_{IAE_1}(\boldsymbol{\theta}), \hat{J}_{IAE_2}(\boldsymbol{\theta}), \hat{J}_{TV_1}(\boldsymbol{\theta}), \\ \hat{J}_{TV_2}(\boldsymbol{\theta}), Lcm(\boldsymbol{\theta})]$$
(11)

where

$$\boldsymbol{\theta} = [-k_{p1}, -k_{i1}, -k_d, f, k_{p2}, k_{i2}]$$
(12)

subject to:

$$0 \le k_{p1,p2} \le 10$$
$$0 \le k_{i1,i2} \le 3$$
$$0 \le k_d \le 1$$
$$0 \le f \le 1$$

and

$$\hat{J}_{IAE_1}(\boldsymbol{\theta}) = \frac{IAE_{Te}(\boldsymbol{\theta})}{IAE_{Te}(\boldsymbol{\theta}_R)}$$
(13)

$$\hat{J}_{IAE_2}(\boldsymbol{\theta}) = \frac{IAE_{T_{SH}}(\boldsymbol{\theta})}{IAE_{T_{SH}}(\boldsymbol{\theta}_R)}$$
(14)

$$\hat{J}_{TV_1}(\boldsymbol{\theta}) = \frac{TV_{Av}(\boldsymbol{\theta})}{TV_{Av}(\boldsymbol{\theta}_R)}$$
(15)

$$\hat{J}_{TV_2}(\boldsymbol{\theta}) = \frac{TV_N(\boldsymbol{\theta})}{TV_N(\boldsymbol{\theta}_R)}$$
(16)

Basically, a step reference test for each input is performed, and IAE and TV values are recorded and normalized using the base line controller. Please note that such test is different from the one that will be used in the final evaluation of the controller (in the provided benchmark). Design objective  $Lcm(\theta)$  is the maximum value of the closed loop log modulus, used in the BLT (biggest log modulus tuning) criterion for multivariable PI tuning (Luyben, 1986). Such design objective is incorporated to include an overall measure of robustness in the MOO process.

3.2 MOO

In the second step, the Pareto front and Pareto set are approximated via some ad-hoc algorithm. In this case, the

Table 3. Preferences Set for multivariable PI controller tuning. Five preference ranges have been defined: highly desirable (HD), desirable (D), tolerable (T) undesirable (U) and highly undesirable (HU).



Fig. 3. Pareto front approximated. Selected controller  $\theta_S$  is depicted with a  $\Box$ .

sp-MODEx algorithm  $^1\,$  is used. Main characteristics of interest for this benchmark are:

- It uses Differential Evolution (Storn and Price, 1997; Das et al., 2016) as evolutionary process to produce its offspring at each generation and evolve towards the Pareto front.
- It uses a spherical grid in order to improve diversity of solutions (Reynoso-Meza et al., 2010). Basically, inside each spherical sector only one solution is allowed to be archived trough the evolutionary process.
- It uses a mechanism for pertinency improvement based on Physical Programming (Reynoso-Meza et al., 2014b). Basically, it uses preferences stated by the designer in the form of highly desirable and undesirable values for each design objective, in order to favour solutions closest to the requirements of the designer

in the final approximated set. The preference matrix used here is shown in Table 3.

• It belongs to the spMODE family of algorithms, which have shown good performance in controller tuning applications (Reynoso-Meza et al., 2012, 2014b; Carrau et al., 2017).

#### 3.3 MCDM stage

In the third step, the Pareto front approximated is analyzed in order to select a solution from the Pareto set to be implemented. In order to visualize calculated approximations, Level diagrams (Blasco et al., 2008) are used<sup>2</sup>. Level diagrams present the following characteristics:

• All the information is available with m + n subplots, one for each design objective and one for each decision variable.

<sup>1</sup> https://www.mathworks.com/matlabcentral/fileexchange/ 65145

 $<sup>^2\</sup> https://www.mathworks.com/matlabcentral/fileexchange/62224$ 

• Multidimensional entities are synchronized in the vertical axis, using some *p*-norm. This norm is a (normalized) distance of a given solution to the utopian solution within the Pareto front approximation.

# 4. RESULTS

#### 4.1 Optimization stage

In Figure 3 the approximated Pareto front is shown. After some analysis, a controller  $\theta_s$  has been selected (depicted with a  $\Box$ ). The digital implementation of such controller corresponds to:

$$C_{1S}(z) = \frac{-0.1890 - 9.7481 \, z^{-1} + 9.3521 \, z^{-2}}{1 - 1.9666 \, z^{-1} + 0.9666 \, z^{-2}} \tag{17}$$

$$C_{2S}(z) = \frac{2.602 - 0.864 \, z^{-1}}{1 - z^{-1}} \tag{18}$$

where the minimal sampling time of 1 second specified by the benchmark has been used.

#### 4.2 Further control tests

The selected controller from the approximated Pareto front will be tested with the original scripts of the benchmark challenge, where a qualitative and a quantitative comparison with the reference controller proposed by Bejarano et al. (2017) is presented. Qualitative comparison is depicted in Figures 4, 5, 6, 7, and 8 (Controller  $1 = \theta_R$ and Controller  $2 = \theta_S$ ). In the quantitative comparison, the controller attain the following performance:

$$R_{indices} = \begin{bmatrix} 0.2892, 0.3569, 0.6148, 0.1705, \\ 0.2291, 0.0967, 1.1470, 1.1531 \end{bmatrix}$$
(19)  
$$J(C_R(z), C_S(z)) = 0.4028$$
(20)

As the overall  $J(C_R(z), C_S(z))$  index is below 1, the selected controller  $C_S(z)$  outperforms the base line controller  $C_R(z)$ . This measure is an aggregation index using  $R_{indices}$  provided within the benchmark.



Fig. 4. Performance of control variables (benchmark refrigeration system).

# 5. CONCLUSIONS

In this paper, a MOOD procedure has been proposed in order to tune a decentralized controller for a control problem. Such a problem consists in designing a controller for a refrigeration systems based on vapour compression.



Fig. 5. Performance of manipulated variables (benchmark refrigeration system).



Fig. 6. Compressor and Evaporator pressures (benchmark refrigeration system).



Fig. 7. Thermal performance (benchmark refrigeration system).



Fig. 8. Compressor efficiency and performance coefficient (benchmark refrigeration system).

It was possible to tune a controller (with the same structure as the base line controller) getting an overall performance index of 0.4028. Nevertheless, the selection of different structures or additional design objectives would be incorporated into the MOP statement and MOO process, in order to improve the performance of the feedback loop overall.

It is also important to notice that a different MOP was used in the optimization stage, different from the aggregate objective function used to evaluate the overall performance of a given controller. This was done in order to test a general MOP for tuning purposes; a re-statement of the MOP closer to the performance index used might lead to more preferable controllers; this is also true if, instead using linear models, more accurate (and perhaps complex) models are used.

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