

Fractional-order PI Controller Design for Integrating Processes Based on Gain and Phase Margin Specifications

Erdal Cokmez*, Serdal Atiç**, Fuat Peker* and Ibrahim Kaya*

**Dicle University, Engineering Faculty, Electrical-Electronics Engineering Department, Diyarbakır, TURKEY
(Tel: 0412 248 84 03; e-mail: erdal.cokmez@dicle.edu.tr, fuat.peker@dicle.edu.tr, ikaya@dicle.edu.tr)*

***Electricity and Energy Department, Vocational High School, Batman University,
Batman, Turkey, (e-mail: serdal.atic@batman.edu.tr)*

Abstract: Fractional-order PID controllers have been introduced as a general form of conventional PID controllers and gained considerable attention latterly due to the flexibility of two extra parameters (fractional integral order λ and fractional derivative order μ) provided. Designing fractional controllers in the time domain has still difficulties. Moreover, it has been observed that the techniques based on gain and phase margins existing in the literature for integer-order systems are not completely applicable to the fractional-order systems. In this study, stability regions based on specified gain and phase margins for a fractional-order PI controller to control integrating processes with time delay have been obtained and visualized in the plane. Fractional integral order λ is assumed to vary in a range between 0.1 and 1.7. Depending on the values of the order λ , and phase and gain margins, different stability regions have been obtained. To obtain stability regions, two stability boundaries have been used; RRB (Real Root Boundary) and CRB (Complex Root Boundary). Obtained stability regions can be used to design all stabilizing fractional-order PI controllers.

Keywords: Fractional-order systems, integrating process, time delay, stability region

1. INTRODUCTION

Fractional calculus is defined as a branch of mathematics that analyses the orders of integral or differential as real or complex numbers. In other words, it is a generalization form of traditional calculus. With the Laplace transform of these equations, fractional-order transfer functions (FOTF) are obtained. Riemann-Liouville, Grunwald-Letnikov, and Caputo made acceptable definitions for fractional derivative and integral (Samko, Kilbas, and Marichev, 1993). In the wake of these studies, the numerical methods and implementation of fractional calculus made significant progress and the popularity of fractional calculus increased. In the control engineering area, fractional calculus received a big deal of attention in recent years because of the fact that fractional-order differential equations identify real systems better than integer-order differential equations (Valério & da Costa 2006).

Fractional calculus has found a wide application area in control engineering (Monje et al. 2008; Wang et al. 2013; Valério & da Costa 2006), due to the reason that real systems can be characterized better than integer calculus (Hartley & Lorenzo 2003; Gabano & Poinot 2011). Fractional-order PID controllers can result in more successful performances than integer-order PID controllers as they have two more adjustable parameters (Zheng et al. 2017; Podlubny 1999). On the other hand, two extra tuning parameters make the controller design more complex (Podlubny 1999). Studies have been made both in time domain and frequency domain to obtain a better system performance (Zheng et al. 2017).

Up to now, many design methods have been proposed for fractional-order PID controller tuning and, the big majority of these methods are in the frequency domain (Luo & Chen 2009). The rest of the methods are concentrated on optimizations in the time domain (Valentim & David 2015). Gain and phase margins are among the most used control loop specifications to find frequency response of systems. As the phase margin is related to damping of the system, it represents the performance of the system and gain and phase margins together represent the stability of a control system. That is why these frequency response specifications are extensively used for controller design. A solution to stabilization based on frequency domain for unstable systems can be seen in (Cheng & Hwang 2006). Another important frequency domain design method cooperating gain and phase margins was proposed in (Hamamci 2007). Ruzewski (Ruzewski 2008) used the approach given by Hamamci (Hamamci 2007) to get stability boundary locus of a fractional-order PI controller for controlling the first order fractional processes with time delay. Hamamci and Koksall (Hamamci & Koksall 2010) proposed a method for controlling integrating processes with time delay being controlled by fractional-order PD controllers. Luo and Chen (Luo & Chen 2012) suggested a design method for first order processes based on phase and gain margins. Recently, Sondhi and Hote (Sondhi & Hote 2015) gave a method for achieving stability regions for specified gain and phase margins for first order plus dead time processes controlled by a fractional-order PI controller.

This paper represents a method obtaining stability boundary loci of all stabilizing fractional-order PI controllers for controlling integrating processes with time delay. Achieved stability boundary loci depend on assumed gain and phase margins. The effect of changing integral order (λ), gain and phase margins and dead time on the stability boundary loci has also been investigated.

The rest of the paper is organized as follows. Section 2 presents the fundamentals of fractional calculus and fractional PI and PID controllers. Section 3 shows the approach of achieving stability regions for integrating processes under the fractional PI controller, Section 4 represents the results and finally in Section 5 conclusions are given.

2. FRACTIONAL ORDER CONTROL SYSTEMS AND FRACTIONAL ORDER PI/PID CONTROLLERS

In the literature, it is seen that several approaches for fractional-order differentials have been presented since 1695. From these methods, three of them gained considerable attention throughout the history of fractional calculus. Riemann-Liouville, Grunwald-Letnikov, and Caputo made important definitions for fractional derivative and integral, and researchers made lots of studies using these definitions (Monje et al. 2010).

Riemann-Liouville's fractional derivative definition is given as follows for $n-1 < \alpha < n$ (Chen, Petráš, and Xue, 2009; Monje et al., 2010):

$${}_a D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau) d\tau}{(t-\tau)^{\alpha-n+1}} \quad (1)$$

In equation (1), $\Gamma(\cdot)$ is defined as Gama function. Riemann-Liouville's fractional integral definition is also defined as follows for $0 < \alpha < 1$ and $t < 0$ (Chen, Petráš, and Xue, 2009; Monje et al., 2010):

$${}_a D_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{\alpha-1} f(\tau) d\tau \quad (2)$$

Grunwald-Letnikov fractional derivative definition is given below:

$${}_a D_t^\alpha f(t) = \lim_{h \rightarrow 0} h^{-\alpha} \sum_{j=0}^{\frac{t-\alpha}{h}} (-1)^j \binom{\alpha}{j} f(t-jh) \quad (3)$$

Caputo fractional derivative definition is:

$${}_a D_t^\alpha f(t) = \frac{1}{\Gamma(\alpha-n)} \int_a^t \frac{f^n(\tau) d\tau}{(t-\tau)^{\alpha-n+1}} \quad (4)$$

Fractional-order PID controllers are known as the generalization of conventional PID controllers. The history of fractional PID controllers started with Podlubny (Podlubny 1999) when he used fractional-order PID controller with fractional integral order of λ and fractional derivative order of μ in place of classical PID controllers. In course of time,

researchers have shown that with fractional-order controllers it is possible to get better closed-loop responses because of two extra new tuning parameters λ and μ .

One of the biggest advantages of $PI^\lambda D^\mu$ controllers is that thanks to two extra tuning parameters, $PI^\lambda D^\mu$ controllers provide better control than conventional PID controllers for fractional-order dynamical systems. Moreover, $PI^\lambda D^\mu$ controllers are less sensitive to change in parameters of the system.

Designing fractional-order PID controllers in the time domain still has difficulties, so the majority of studies are in the frequency domain using gain crossover frequency, gain margin, phase crossover frequency and phase margin of the open-loop system (De Keyser, Muresan, and Ionescu, 2015). A researcher can simply find phase and gain margin of a system using one of the frequency domain approaches, such as Nyquist, Bode or Ziegler-Nichols methods.

3. STABILITY REGIONS FOR INTEGRATING PROCESSES WITH TIME DELAY

In this section, a procedure for all stabilizing fractional-order PI controllers based on specified gain and phase margins to control integrating processes plus dead time will be given. It is assumed that integrating plus dead time process can be modeled by:

$$G_p(s) = \frac{K e^{-\theta s}}{s(Ts+1)} \quad (5)$$

Fractional-order PI controller transfer function is given by:

$$G_c(s) = K_p + \frac{K_i}{s^\lambda} = \frac{K_p s^\lambda + K_i}{s^\lambda} \quad (6)$$

In order to design fractional-order PI controller for specified phase and gain margins, the method proposed in (Hamamci 2007) has been adopted. To apply the method, a gain-phase margin tester (GPMT) $C_i(A, \phi)$ is used before the controller as shown in Fig. 1.

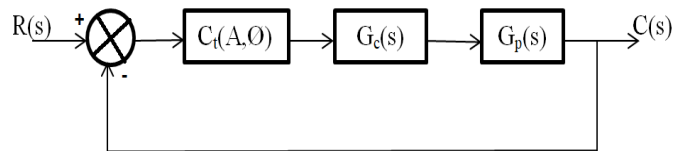


Fig. 1. Control system with gain-phase margin tester

$C_i(A, \phi)$ is defined as virtual compensator that helps to get information about constant gain and phase margin of the system. The form of $C_i(A, \phi)$ is as the following:

$$C_i(A, \phi) = A e^{-j\phi} \quad (7)$$

For the closed-loop system shown in Fig. 1, the open-loop and closed-loop transfer functions are, respectively, given by:

$$C_i(A, \phi) G_c(s) G_p(s) \quad (8)$$

$$\frac{C_i(A, \phi)G_c(s)G_p(s)}{1 + C_i(A, \phi)G_c(s)G_p(s)} \quad (9)$$

From the closed-loop transfer function of the system it is seen that the characteristic equation of the system is:

$$P(s; K_p, K_i, \lambda) = 1 + C_i(A, \phi)G_c(s)G_p(s) = 0 \quad (10)$$

Substituting (5), (6), and (7) into (10), the following can be obtained:

$$P(s; K_p, K_i, \lambda) = 1 + A e^{-j\phi} \frac{K_p s^\lambda + K_i}{s^\lambda} \frac{K e^{-\theta s}}{s(Ts + 1)} = 0 \quad (11)$$

Rearranging (11), one can easily obtain the following:

$$P(s; K_p, K_i, \lambda) = A K e^{-j(w\theta + \phi)} (K_p s^\lambda + K_i) + T s^{\lambda+2} + s^{\lambda+1} = 0 \quad (12)$$

A closed-loop control system having no the right-half plane poles means that the system is stable, otherwise, the system is unstable. The stability domain is defined as the region for which, when K_p , K_i and λ are the members of the domain, equation (12) has no roots in the right half of the s-plane. The stability boundary of the system for fractional-order PI controller is obtained by Real Root Boundary (RRB) and Complex Root Boundary (CRB). To obtain these boundaries following equations can be used.

$$RRB : P(0; K_p, K_i, \lambda) = 0, \text{ for } w \in (0, \infty),$$

$$CRB : P(jw; K_p, K_i, \lambda) = 0, \text{ for } w \in (0, \infty),$$

Then, to obtain RRB, substituting $s = 0$ in (12), $K_i = 0$ is obtained.

The next step is to find Complex Root Boundary (CRB). Substituting $s = jw$ in (12), one can get the following:

$$P(jw; K_p, K_i, \lambda) = A K e^{-j(w\theta + \phi)} (K_p (jw)^\lambda + K_i) + T (jw)^{\lambda+2} + (jw)^{\lambda+1} = 0 \quad (13)$$

Solving (13), requires the use of identities below:

$$j^\lambda = \cos\left(\frac{\lambda\pi}{2}\right) + j \sin\left(\frac{\lambda\pi}{2}\right) \quad (14)$$

$$e^{j\alpha} = \cos(\alpha) + j \sin(\alpha) \quad (15)$$

Replacing (14) and (15) in (13):

$$\begin{aligned} & T w^{\lambda+2} [\cos((\lambda+2)\frac{\pi}{2}) + j \sin((\lambda+2)\frac{\pi}{2})] + \\ & w^{\lambda+1} [\cos((\lambda+1)\frac{\pi}{2}) + j \sin((\lambda+1)\frac{\pi}{2})] + \\ & A K [\cos(w\theta + \phi) - j \sin(w\theta + \phi)] [K_p w^\lambda (\cos(\frac{\lambda\pi}{2}) + \\ & j \sin(\frac{\lambda\pi}{2})) + K_i] = 0 \end{aligned} \quad (16)$$

can easily be achieved. Separating (16) into its real and imaginary parts, respectively, the following two equations have been calculated:

$$\begin{aligned} & T w^{\lambda+2} \cos((\lambda+2)\frac{\pi}{2}) + w^{\lambda+1} \cos((\lambda+1)\frac{\pi}{2}) + A K K_p w^\lambda \\ & \cos(w\theta + \phi) \cos(\frac{\lambda\pi}{2}) + A K K_p w^\lambda \sin(w\theta + \phi) \sin(\frac{\lambda\pi}{2}) \\ & + A K K_i \cos(w\theta + \phi) = 0 \end{aligned} \quad (17)$$

$$\begin{aligned} & T w^{\lambda+2} \sin((\lambda+2)\frac{\pi}{2}) + w^{\lambda+1} \sin((\lambda+1)\frac{\pi}{2}) + A K K_p w^\lambda \\ & \cos(w\theta + \phi) \sin(\frac{\lambda\pi}{2}) - A K K_p w^\lambda \sin(w\theta + \phi) \cos(\frac{\lambda\pi}{2}) \\ & - A K K_i \sin(w\theta + \phi) = 0 \end{aligned} \quad (18)$$

The next step is to solve obtained equations simultaneously to get the expressions for K_p and K_i . After solution using the equations (19) and (20) are obtained for K_p and these equations draw a curve for a fixed value of λ while ω varies in a range of $(0, \infty)$.

$$\begin{aligned} K_p = & \frac{T w^{\lambda+2} \cos((\lambda+2)\frac{\pi}{2}) + w^{\lambda+1} \cos((\lambda+1)\frac{\pi}{2})}{-A K w^\lambda \cos(w\theta + \phi - \frac{\lambda\pi}{2})} \dots \\ & \dots + \frac{A K K_i \cos(w\theta + \phi)}{-A K w^\lambda \cos(w\theta + \phi - \frac{\lambda\pi}{2})} \end{aligned} \quad (19)$$

$$K_i = \frac{T w^{\lambda+2} \sin(w\theta + \phi + \pi) + w^{\lambda+1} \sin(w\theta + \phi + \frac{\pi}{2})}{A K \sin(\frac{\lambda\pi}{2})} \quad (20)$$

Finally, after getting equations for K_p and K_i , stability boundary locus can be plotted in $K_p - K_i$ plane for a specified gain and phase margin. It is worth to point out the following notes:

- To find global stability region for a fixed value of λ , parameters should be set as $A = 1$ And $\phi = 0$
- To find the stability region of a system having X dB gain margin, parameters should be set as $A = X$ and $\phi = 0$; X any real number
- To find stability region for a system having X° phase margin, parameters should be set as $A = 1$ and $\phi = X^\circ$; X any positive natural number.

To find stability boundary locus, following steps should be performed;

- From (12), find the RRB result.
- To obtain K_p and K_i expressions in terms of λ and ω , using (17) and (18).

- For selected values of λ , plot RRB and CRB lines in the same $K_p - K_i$ plane.
- Use test points to determine general stability region in the $K_p - K_i$ plane.
- Plot the global stability region.

4. RESULTS

In this section, stability regions are found for integrating processes with time delay under the control of a fractional-order PI controller. The section also represents how the stability region changes as the integral order λ , time delay θ , gain margin A and phase margin ϕ change. To trace stability regions in $K_p - K_i$ plane, RRB and CRB equations are used.

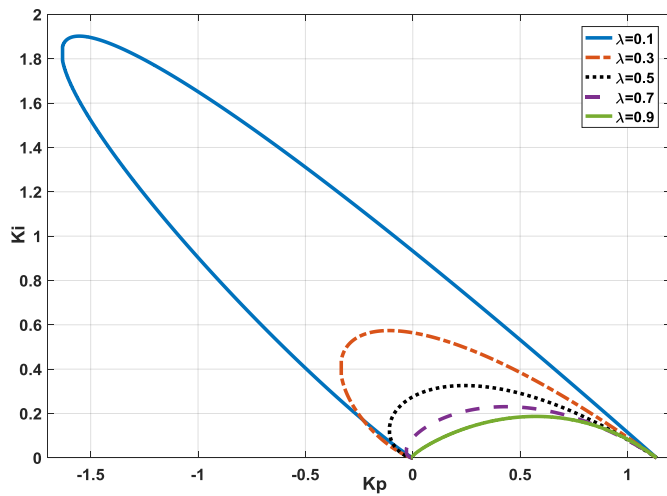


Fig. 2. Stability regions for different λ values in a range of [0.1-0.9].

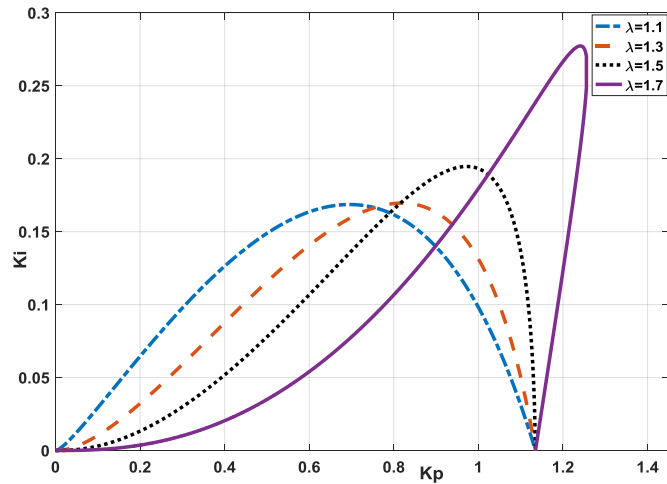


Fig. 3. Stability regions for different λ values in a range of [1.1-1.7].

Stability regions of the process transfer function given by (5) under fractional-order PI controller given by (6) for different fractional integral order λ are shown in Fig. 2 and 3. Plots are obtained from (19) and (20), assuming that $K = 1$, $T = 1$,

$\theta = 1$, $A = 1$ and $\phi = 0$. For fractional integral order $\lambda < 1$, it is observed from Fig.2 that stability region gets larger as the fractional integral order λ gets smaller. On the other hand, for fractional integral order $\lambda > 1$, it is not possible to make such a simple comment, and the stability regions take quite different shapes for different fractional integral order.

Effect of varying time delay value is illustrated in Fig. 4. In this case, the fractional integral order λ is considered to be 0.1, since a larger stability region can be obtained for a lower λ value. System parameters are assumed as $K = 1$, $T = 1$, $\lambda = 0.1$, $A = 1$ and $\phi = 0$. As seen from the Fig. 4, large time delays results in a narrower stability region.

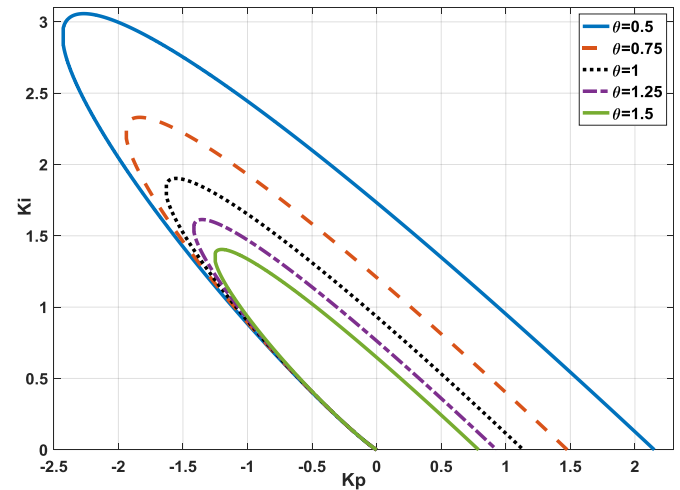


Fig. 4. Global stability regions for different time delays.

Afterward, stability regions of the integrating process with time delay controlled by a fractional-order PI controller are depicted in Fig. 5 for varying gain margin value. The system parameters are assumed as $K = 1$, $T = 1$, $\theta = 1$, $\lambda = 0.1$ and $\phi = 0$. It is seen from Fig. 5 that larger gain margin causes narrower stability boundary locus.

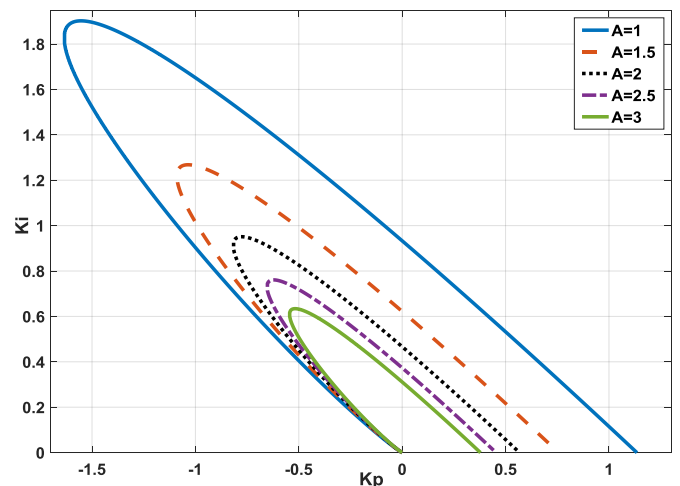


Fig. 5. Global stability regions for different gain margin values.

The effect of varying phase margin on the stability boundary locus is illustrated in Fig. 6. The system parameters are assumed as $K = 1$, $T = 1$, $\theta = 1$, $\lambda = 0.1$ and $A = 1$ to plot

the stability boundary locus. From the figure, it is clearly seen that there is an inverse proportion between stability boundary locus and phase margin.

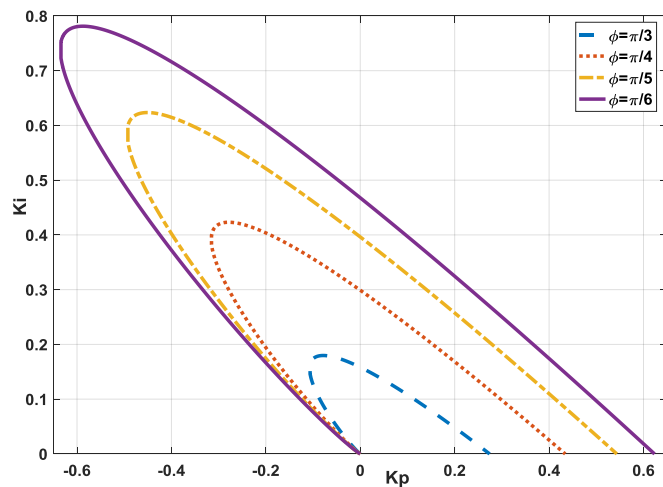


Fig. 6. Global stability regions for different phase margins.

4.1 Example

This example is given to show the use of the proposed approach for higher order integrating processes. Let us consider a plant transfer function given by $G(s) = e^{-0.5s} / s(s+1)(0.5s+1)(0.25s+1)(0.1s+1)$. Using relay feedback identification method suggested by Kaya (Kaya 2006) the IFOPDT model was obtained as $G(s) = e^{-1.123s} / s(1.756s+1)$. Using the procedure given in section 3, stability regions for different integral orders and phase and gain margins have been obtained. In Fig. 7, different stability regions are depicted. Stability region for $\lambda = 0.1$ has also been obtained, but the stability region for this case is much larger than others, hence in order to be seen the results clearly, it has not been added to the Fig. 7. Fractional-order PI controller parameters corresponding to mid-point of those stability regions are summarized in Table 1. Closed-loop responses to a step input change are illustrated in Figs 8 and 9.

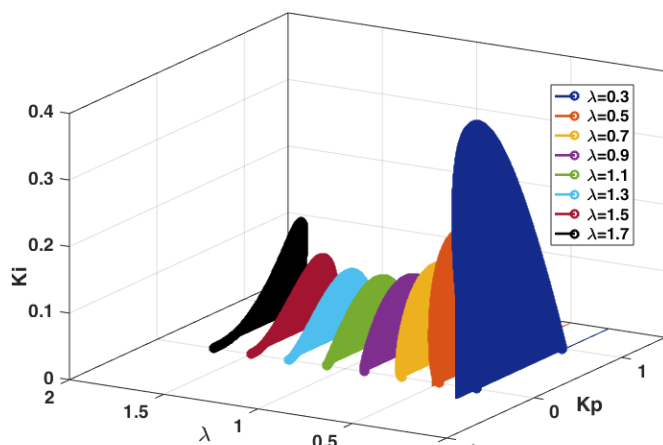


Fig. 7. Global stability regions for different fractional orders

Table 1. Controller parameters for different gain and phase margins and fractional orders

	$\lambda = 0.5$		$\lambda = 1$		$\lambda = 1.5$	
	K_p	K_i	K_p	K_i	K_p	K_i
$A=2$	0.15	0.05	0.275	0.02	0.375	0.02
$A=3$	0.1	0.03	0.175	0.015	0.25	0.015
$\phi = 45^\circ$	0.15	0.015	0.2	0.04	0.25	0.0025
$\phi = 60^\circ$	0.1	0.005	0.125	0.001	0.15	0.0005

From Fig. 7, it is observed that for a fixed value of gain margin, the smaller the integral order λ the better the closed-loop performance in the sense of maximum overshoot can be attained. For a fixed value of the integral order $\lambda < 1$, larger gain margin yields a slightly slower closed-loop response with slightly less overshoot. For a fixed value of the integral order $\lambda > 1$, larger gain margin makes the closed-loop response worse.

From Fig. 8, the following conclusion can be made. For a fixed value of phase margin, the integral order $\lambda < 1$ results in better closed-loop responses. For $\lambda > 1$, overshoots in step responses gets smaller but the settling time becomes longer.

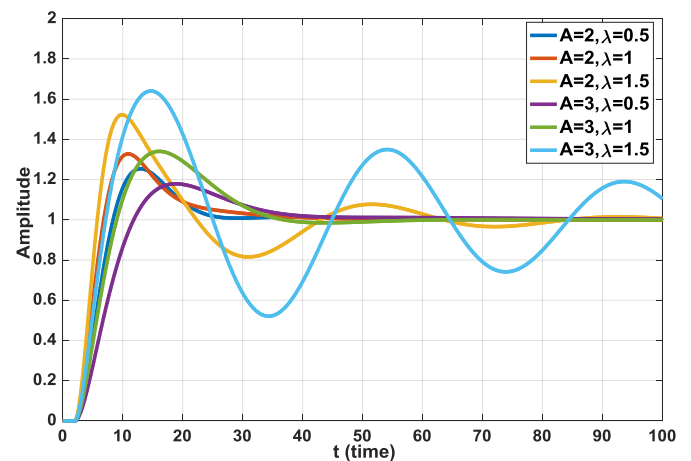


Fig. 8. Step responses for different gain margins and fractional integral orders

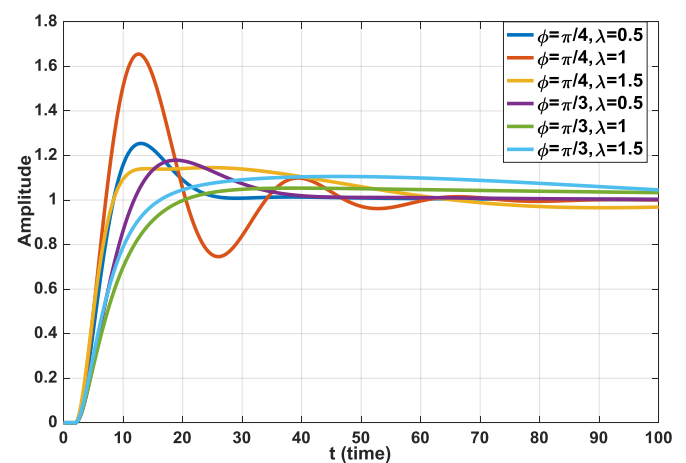


Fig. 9. Step responses for different phase margins and fractional integral orders

5. CONCLUSIONS

In this paper, stability regions satisfying specified gain and phase margins for varying integral order λ for integrating processes with fractional-order PI controller have been obtained. It has been shown that smaller integral orders result in larger stability regions when integral order $\lambda < 1$. Again, it has been shown that, for a fixed value of the integral order λ , smaller gain and phase margin values results in larger stability regions. The simulation example revealed that, generally, integral order $\lambda < 1$ yields improved closed-loop responses and integral order $\lambda > 1$ makes the closed-loop response worse when compared to integer-order PI controller.

REFERENCES

- Chen, Y.Q., Petráš, I. and Xue, D., (2009). Fractional-order control - A tutorial. *Proceedings of the American Control Conference*, (May 2014), pp.1397–1411.
- Cheng, Y.C. & Hwang, C., 2006. Stabilization of unstable first-order time-delay systems using fractional-order pd controllers. *Journal of the Chinese Institute of Engineers, Transactions of the Chinese Institute of Engineers, Series A/Chung-kuo Kung Ch'eng Hsueh K'an*, 29(2), pp.241–249.
- Gabano, J.D. and Poinot, T., (2011). Fractional modelling and identification of thermal systems. *Signal Processing*, 91(3), pp.531–541.
- Hamamci, S.E., (2007). An algorithm for stabilization of fractional-order time delay systems using fractional-order PID controllers. *IEEE Transactions on Automatic Control*, 52(10), pp.1964–1969.
- Hamamci, S.E. and Koksali, M., (2010). Calculation of all stabilizing fractional-order PD controllers for integrating time delay systems. *Computers and Mathematics with Applications*, 59(5), pp.1621–1629.
- Hartley, T.T. and Lorenzo, C.F., (2003). Fractional-order system identification based on continuous order-distributions. *Signal Processing*, 83(11), pp.2287–2300.
- Kaya, I., (2006). Parameter Estimation for integrating processes using relay feedback control under static load disturbances. *Industrial and Engineering Chemistry Research*, 45(13), pp.4726–4731.
- De Keyser, R., Muresan, C.I. and Ionescu, C.M., (2015). A novel auto-tuning method for fractional-order PI/PD controllers. *ISA Transactions*, 62, pp.268–275.
- Luo, Y. and Chen, Y., (2012). Stabilizing and robust fractional-order PI controller synthesis for first order plus time delay systems. *Automatica*, 48(9), pp.2159–2167.
- Luo, Y. and Chen, Y.Q., (2009). Fractional-order [proportional derivative] controller for a class of fractional-order systems. *Automatica*, 45(10), pp.2446–2450.
- Monje, C.A. et al., (2010). *Fractional-Order Systems And Control Fundamentals And Applications*, London: Springer-Verlag.
- Monje, C.A. et al., (2008). Tuning and auto-tuning of fractional-order controllers for industry applications. *Control Engineering Practice*, 16(7), pp.798–812.
- Podlubny, I., (1999). Fractional-order Systems And PID Controllers. *IEEE Transactions on Automatic Control*, pp.208–214.
- Ruszewski, A., (2008). Stability regions of closed-loop system with time delay inertial plant of fractional-order and fractional-order PI controller. *Bulletin of the Polish Academy of Sciences-Technical Sciences*, 56(4), pp.329–332.
- Samko, S.G., Kilbas, A.A. and Marichev, O.I., (1993). *Fractional integrals and derivatives*,
- Sondhi, S. and Hote, Y. V., (2015). Fractional-order PI controller with specific gain-phase margin for MABP control. *IETE Journal of Research*, 61(2), pp.142–153.
- Valentim, C.A. and David, S.A., (2015). A fractional PID controller of an oscillatory system. *AIP Conference Proceedings*, 1648(March), pp.1–6.
- Valério, D. and da Costa, J.S., (2006). Tuning of fractional PID controllers with Ziegler-Nichols-type rules. *Signal Processing*, 86(10), pp.2771–2784.
- Wang, D.-J., Li, W. and Guo, M.-L., (2013). Tuning of $PI^\lambda D^\mu$ controllers based on sensitivity constraint. *Journal of Process Control*, 23(6), pp.861–867.
- Zheng, W.J. et al., (2017). Fractional-order $PI^\lambda D^\mu$ controller design for satisfying time and frequency domain specifications simultaneously. *ISA Transactions*, 68(March), pp.212–222.