# Multivariable Fractional Order PI Autotuning Method for Heterogeneous Dynamic Systems

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**Abstract:** In this paper the application of robust Fractional Order Proportional-Integral (FO-PI) autotuning control strategy is presented and applied to heterogeneous dynamic systems using decentralized control. The automatic tuning of controller gains is based on a single sine test, with user-defined robustness margins guaranteed. Its performance is compared against two other fractional order controllers based on PI gain-crossover autotuning method and Internal Model Control (IMC). The closed loop control simulations applied on the heterogeneous dynamic systems indicate that the proposed method performs properly.

*Keywords:* Multi-Input Multi-Output, KC autotuner, Fractional order PI controller, Fractional order IMC controller, Decentralized control, Heterogeneous Dynamic System.

### 1. INTRODUCTION

In the last decade, fractional calculation techniques in control theory have achieved considerable popularity and importance due to their valuable advantages (Monje CA., 2010). Actually most industrial applications use PID controllers extensively and fractional order PID controllers represent a generalization of the classic PID controllers, therefore there is a main interest in the development of algorithms for the adjustment of PID controller parameters of fractional order.

The interest, auto-tuning control algorithms, due to the fact that methodologies allow to handle the variations of the process, which can arise for different reasons, from infrastructure problems of the control system, instrumentation problems to non-critical failures. The relay auto-tuning process is widely used in industrial applications because its simplicity and reliability (Åström et al., 2006).

Some methods to tunning fractional order PID controllers have been proposed: the method proposed by (Dulf E. H. et al., 2015) based on vector geometry algorithm needs to imposed the modulus and phase of the controller at two critical frequencies and use an autotuning control structure. Therefore, this method requires to know the process model to adjust the parameter of the fractional controller in real time. On the other hand, the method developed by (Monje CA et al, 2008) used the relay test for auto-tuning of fractional order controllers, to take advantage of the introduction of two additional parameters and additional design specifications. Due to the benefits of robustness presented by fractional controllers, many researchers have motivated on the design and implementation problem of fractional order controllers (Chevalier A. et al. 2014; Copot C. et al. 2013; Muresan et al., 2013). Therefore, the present work proposes the application of a robust fractional order PI based KC autotuning method (De Keyser et al, 2017), for heterogeneous dynamic systems. This method consists in defining a 'forbidden region' in the Nyquist plane based on user-defined specifications, which will guarantee the system margin requirements. Hence, an adequate fractional order PI controller is designed, where the loop frequency response is tangent to this forbidden region (to avoid violating the robustness limits). The performance of the KC autotuning method is compared against two fractional order controllers: PI gain-crossover autotuning method (De Keyser et al., 2016) and Internal Model Control (IMC) method presented in (Muresan et al., 2016).

This paper is structured as follows. In section 2, the detailed theory of fractional order PI based on KC autotuning method, fractional order PI gain-crossover autotuning method and fractional order IMC controller is shown. Numerical examples with two heterogeneous dynamic systems are exposed in section 3. Finally, the analysis of results and conclusions are given in section 4 and section 5 respectively.

### 2. CONTROL STRATEGIES

The transfer function of the fractional order PI (FO-PI) controller that will be used in the different control strategies is indicated below:

$$C_{FO-PI}(s) = k_p \left(1 + \frac{k_i}{s^{\mu}}\right) \tag{1}$$

with the controller parameters defined as follows:  $\mu \in (0-2)$  is the fractional order and  $k_p$ ,  $k_i$  are the proportional and integrative gains, respectively. Furthermore, for all tuning methods, it is assumed to apply a decentralized approach for heterogeneous dynamic systems, with the input-output pairing selected according to the Relative Gain Array.

# 2.1 Fractional Order PI based on KC autotuning method

Fig. 1 illustrates the main idea of this autotuner as to move a point B on the Nyquist curve of process  $P(j\omega)$  to another point A on the Nyquist curve of the loop  $L(j\omega)=P(j\omega)C_{FO-PI}(j\omega)$  through the FO-PI controller indicated by  $C_{FO-PI}(j\omega)$ . The tuning procedure can be summarized as follows (De Keyser et al, 2017).

- 1) Select a frequency  $\varpi$  ( $\varpi$  is usually critical frequency, but might be different).
- 2) Perform sine tests on the system.
- 3) Define a 'forbidden region' in the Nyquist plane according to gain and phase margins (*GM* and *PM*) in this case defined as a circle.
- 4) For each point on the region border, calculate FO-PI controller.
- 5) Find the point where the loop  $L(j\omega)$  is tangent to the 'forbidden region'.
- 6) The controller from step 5) corresponding at the final FO-PI controller.

In order to have the loop  $L(j\omega)$  frequency response tangent to the 'forbidden region', the slope of 'forbidden region' and slope of loop  $L(j\omega)$  should be the same. In Fig. 1, the point D and point E are obtained according to gain and phase margins. D is the intersection of gain margin (GM) with negative real axis. E is the intersection of phase margin (PM) with unit circle. According to points D and E in Fig. 1, the circle can be calculated as:

Forbidden region: 
$$(\text{Re+C})^2 + \text{Im}^2 = R^2$$
  
 $D \Rightarrow (-1/GM + C)^2 = R^2$   
 $E \Rightarrow (-\cos PM + C)^2 + (-\sin PM)^2 = R^2$ 
(2)

and the center and radius of the 'forbidden region' are calculated as follows:

$$C = \frac{GM^2 - 1}{2GM(GM\cos PM - 1)}; R = C - \frac{1}{GM}$$
(3)

The slope of the 'forbidden region' border on the point A defined by the radius R and the angle  $\alpha$  is:

$$\left. \frac{d \operatorname{Im}}{d \operatorname{Re}} \right|_{\alpha} = \frac{-\operatorname{Re}+C}{\operatorname{Im}} = \frac{\cos \alpha}{\sin \alpha}$$
(4)

In order to get the slope of process  $L(j\omega)$ , the derivative is used.

$$\frac{dL(j\omega)}{d\omega} = \frac{dP(j\omega)C_{FO-PI}(j\omega)}{d\omega}$$

$$= P(j\omega)\frac{dC_{FO-PI}(j\omega)}{d\omega} + C_{FO-PI}(j\omega)\frac{dP(j\omega)}{d\omega}$$

$$= \frac{d\operatorname{Re}_{PC_{FO-PI}}}{d\omega} + j\frac{d\operatorname{Im}_{PC_{FO-PI}}}{d\omega}$$

$$\Rightarrow \Rightarrow \frac{d\operatorname{Im}_{PC_{FO-PI}}}{d\operatorname{Re}_{PC_{FO-PI}}}\Big|_{\omega=\varpi}$$
(5)

with  $\varpi$  test frequency. At the point A, the following equation is given.

$$M_A e^{j\varphi_A} = M_{PC_{FO-Pl}}(j\overline{\varpi}) e^{j\varphi_{PCFO-Pl}(j\overline{\varpi})}$$
(6)

It can be rewritten as:

$$\begin{cases} M_{A} = M_{PC_{FO-Pl}}(j\varpi) = M_{P}(j\varpi)M_{C_{FO-Pl}}(j\varpi) \\ \varphi_{A} = \varphi_{PC_{FO-Pl}}(j\varpi) = \varphi_{P}(j\varpi)\varphi_{C_{FO-Pl}}(j\varpi) \end{cases}$$
(7)

The following equation for the fractional order PI controller in (1) is obtained taking  $s \rightarrow j\omega$ .

$$C_{FO-PI}(j\omega) = k_p [1 + k_i \omega^{-\mu} (\cos \frac{\pi\mu}{2} - j \sin \frac{\pi\mu}{2})]$$
(8)

The modulus and phase of the controller are as follows.

$$M_{C_{FO-FI}}(j\varpi) = \left| k_p \left[ 1 + k_i \varpi^{-\mu} \left( \cos \frac{\pi\mu}{2} - j \sin \frac{\pi\mu}{2} \right) \right] \right|$$

$$\varphi_{C_{FO-FI}}(j\varpi) = a \tan \left( -\frac{k_i \varpi^{-\mu} \sin \frac{\pi\mu}{2}}{1 + k_i \varpi^{-\mu} \cos \frac{\pi\mu}{2}} \right)$$
(9)

From the point A on circle 2, the modulus and phase can be calculated as follows.

$$M_A = \sqrt{C^2 + R^2 - 2CR\cos\alpha} \tag{10}$$

$$\tan(\varphi_{C_{FO-PI}} + \varphi_P) = \frac{R\sin\alpha}{C - R\cos\alpha} = \frac{\tan\varphi_{C_{FO-PI}} + \tan\varphi_P}{1 - \tan\varphi_{C_{FO-PI}} \tan\varphi_P} \quad (11)$$

Hence we have:

$$\tan \varphi_{C_{FO-PI}} = \frac{R \sin \alpha - \tan \varphi_P (C - R \cos \alpha)}{\tan \varphi_P R \sin \alpha + (C - R \cos \alpha)}$$
(12)

Therefore  $\left. \frac{dC_{FO-PI}(j\omega)}{d\omega} \right|_{\omega=\omega}$  can be calculated as follows:

$$\frac{dC_{FO-PI}(j\omega)}{d\omega} = k_p k_i \mu \omega^{-\mu - 1} \left[ -\cos\frac{\pi\mu}{2} + j\sin\frac{\pi\mu}{2} \right]$$
(13)

The parameters  $P(j\varpi)$ ,  $\varphi_P(j\varpi)$  and  $\frac{dP(j\omega)}{d\omega}\Big|_{\omega=\omega}$  can be obtained according to a sine test (De Keyser et al., 2016).

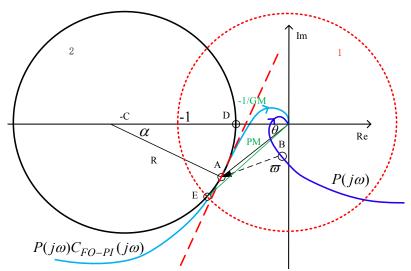


Fig. 1. Graphic illustration of autotuning principle. See text for description.

Hence, the 
$$\frac{d \operatorname{Im}_{PC_{FO-PI}}}{d \operatorname{Re}_{PC_{FO-PI}}}\Big|_{\omega=\overline{\omega}}$$
 can be calculated.

For the KC autotuning method, the design is defined as a minimization problem:

$$\min_{\alpha} \left\| \frac{\mathrm{d}\,\mathrm{Im}}{\mathrm{d}\,\mathrm{Re}} \right|_{\alpha} - \frac{\mathrm{d}\,\mathrm{Im}_{PC_{FO-FI}}}{\mathrm{d}\,\mathrm{Re}_{PC_{FO-FI}}} \right\|_{\omega = \sigma} \left\|, \ 0 \le \alpha \le \alpha_{\max} \right\|$$
(14)

Referring to Fig. 1, it is obvious that the touching point of the Nyquist curve must be in the range  $0 \le \alpha \le 90^{\circ}$ , making  $\alpha_{max}=90^{\circ}$ . The minimization problem can be simply solved with a single for-loop where  $\alpha$  varies from 0 to  $\alpha_{max}$  in 1° steps.

## 2.2 Fractional Order PI gain-crossover autotuning method

This method for FO-PI controllers is based on three performance specifications (Monje CA, 2010) a gain crossover frequency  $\omega_{gc}$ , phase margin  $\phi_m$  and the iso-damping property. In order for the system to ensure the (a) imposed gain crossover frequency, (b) certain phase margin and (c) iso-damping property, the following conditions must hold:

(a) 
$$|C_{open-loop}(j\omega_{gc})| = 1$$
  
(b)  $\angle C_{open-loop}(j\omega_{gc}) = -\pi + \phi_m$   
(c)  $\frac{d(\angle C_{open-loop}(j\omega))}{d\omega}\Big|_{\omega=\omega_m} = 0$ 

where  $C_{open-loop}(s)$  is the loop transfer function defined as:  $C_{open-loop}(s) = P(s).C_{FO-PI}(s)$ , where P(s) is the transfer function of the process to be controlled and  $C_{FO-PI}(s)$  is the FO-PI controller defined in (1). Also, the following equation for the fractional order PI controller in (1) is obtained taking  $s \to j\omega_{gc}$ .

$$C_{FO-PI}(j\omega_{gc}) = k_p [1 + k_i \omega_{gc}^{-\mu} (\cos\frac{\pi\mu}{2} - j\sin\frac{\pi\mu}{2})] \quad (16)$$

The phase of the open loop transfer function is computed as:

$$\angle C_{open-loop}(j\omega_{gc}) = a \tan\left(-\frac{k_i \omega_{gc}^{-\mu} \sin\frac{\pi\mu}{2}}{1 + k_i \omega_{gc}^{-\mu} \cos\frac{\pi\mu}{2}}\right) + \varphi_p(j\omega_{gc}) \quad (17)$$

Using (16) and (17), the performance specifications in (15) become:

(a) 
$$\left| k_p [1 + k_i \omega_{gc}^{-\mu} (\cos \frac{\pi \mu}{2} - j \sin \frac{\pi \mu}{2})] \right| = \frac{1}{|P(j\omega_{gc})|}$$
  
(b)  $\frac{k_i \omega_{gc}^{-\mu} \sin \frac{\pi \mu}{2}}{1 + k_i \omega_{gc}^{-\mu} \cos \frac{\pi \mu}{2}} = tg \left( \pi - \phi_m + \phi_p(j\omega_{gc}) \right)$  (18)  
 $\mu k_i \omega_{gc}^{-\mu - 1} \sin \frac{\pi \mu}{2}$ 

(c) 
$$\frac{\mu k_i \omega_{gc} + \sin \frac{\pi}{2}}{1 + 2k_i \omega_{gc}^{-\mu} \cos \frac{\pi \mu}{2} + k_i^2 \omega_{gc}^{-2\mu}} + \frac{d \angle P(j\omega)}{d\omega} \bigg|_{\omega = \omega_{gc}} = 0$$

To tune the FO-PI controller, the system of nonlinear equations (18) need to be solved using either optimization techniques or graphical methods. Nevertheless, regardless of which approach will be taken to determine the controller parameters, to completely tune the FO-PI controller, the modulus  $|P(j\omega_{\rm gc})|$ , phase  $\varphi_p(j\omega_{gc})$  and phase slope of the process at the gain cross over frequency  $\frac{d \angle P(j\omega)}{d\omega}\Big|_{\omega=\omega_{gc}}$  have to

be known, for which a novel methodology described in (De Keyser et al, 2016) based on sine-test will be used.

#### 2.3 Fractional Order IMC Controller

The basic structure of the IMC is shown in Fig.2, where P(s) is the process transfer function,  $H_m(s)$  is the model of the process,  $H_{FO-IMC}(s)$  is the fractional order IMC controller transfer function and  $H_c(s)$  is the equivalent fractional order controller for a traditional closed loop system.

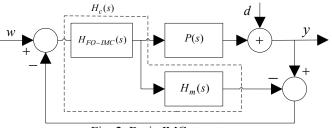


Fig. 2. Basic IMC structure

The process model can be approximated as a first order minimum-phase system without time delay.

$$H_m(s) = \frac{K}{\tau s + 1} \tag{19}$$

The equivalent controller for a traditional closed loop system  $H_c(s)$  may then be computed as:

$$H_c(s) = \frac{H_{FO-IMC}(s)}{1 - H_{FO-IMC}(s)H_m(s)}$$
(20)

where,

$$H_{FO-IMC}(s) = H_m^{-1}(s)F(s); \quad F(s) = \frac{1}{\lambda s^{\mu} + 1}$$
 (21)

The autotuning of the FO-IMC controller is based on two performance specifications (Muresan, 2016) a gain crossover frequency  $\omega_{gc}$  and phase margin  $\phi_m$ . In order for the system to ensure the imposed (a) gain crossover frequency and (b) certain phase margin for the system, the following conditions must hold:

(a) 
$$|H_{open-loop}(j\omega_{gc})| = 1$$
  
(b)  $\angle H_{open-loop}(j\omega_{gc}) = -\pi + \phi_m$ 
(22)

where  $H_{open-loop}(s)$  is the loop transfer function defined as:  $H_{open-loop}(s) = H_c(s)$ . The resulting controller has the transfer function:

$$H_c(s) = \frac{\tau s + 1}{K \lambda s^{\mu}} \tag{23}$$

Also, the following equation for the FO-IMC controller in (23) is obtained taking  $s \rightarrow j\omega_{gc}$ .

$$H_c(j\omega_{gc}) = \frac{\tau j\omega_{gc} + 1}{K\lambda} \omega_{gc}^{-\mu} \left[\cos\frac{\pi\mu}{2} - j\sin\frac{\pi\mu}{2}\right]$$
(24)

The phase of the open loop transfer function is computed as:

$$\angle H_{open-loop}(j\omega_{gc}) = a \tan\left(\frac{\tau\omega_{gc}\cos\frac{\pi\mu}{2} - \sin\frac{\pi\mu}{2}}{\cos\frac{\pi\mu}{2} + \tau\omega_{gc}\sin\frac{\pi\mu}{2}}\right) + \varphi_p(j\omega_{gc}) \quad (25)$$

Using (24) and (25), the performance specifications in (22) become

(a) 
$$\frac{\left| \frac{\omega_{gc}^{-\mu}}{K\lambda} \sqrt{1 + \tau^2 \omega_{gc}^2} \right| = \frac{1}{\left| P(j\omega_{gc}) \right|}$$
(26)  
(b) 
$$\frac{\tau \omega_{gc} \cos \frac{\pi\mu}{2} - \sin \frac{\pi\mu}{2}}{\cos \frac{\pi\mu}{2} + \tau \omega_{gc} \sin \frac{\pi\mu}{2}} = tg \left( -\pi + \phi_m - \varphi_p(j\omega_{gc}) \right)$$

To tune the FO-IMC controller, the system of nonlinear equations (26) need to be solved using either optimization techniques or graphical methods. However, the modulus  $|P(j\omega_{gc})|$  and phase  $\varphi_P(j\omega_{gc})$  have to be known. For which again the sine-test will be used.

#### 3. NUMERICAL EXAMPLES

In this section, fractional order PI based on KC autotuning method is applied to two heterogeneous dynamic systems. In each case, two additional fractional order controllers are also employed for comparison purposes: FO-PI gain-crossover autotuning and FO-IMC both using frequency specifications too.

## 3.1 Example 1

The first heterogeneous dynamic system considered is described by the following transfer function  $G_l(s)$ :

$$G_{1}(s) = \begin{bmatrix} \frac{1.64}{50s+1} & \frac{2.49}{250s+1} \\ \frac{2.56}{75s+1} & \frac{1.28}{275s+1} \end{bmatrix}$$
(27)

The transmission zeros are:

$$z_1 = -0.0035 z_2 = -0.0253$$
(28)

Then checking input-output pairings, with a RGA (Relative Gain Array) analysis of the multivariable process.

$$\Lambda_1 = \begin{bmatrix} -0.4910 & 1.4910\\ 1.4910 & -0.4910 \end{bmatrix}$$
(29)

RGA matrix  $\Lambda_1$  suggests that the pairing 1-2/2-1 is suitable, since the main diagonal has negative values. On the other hand, GM=3,  $\phi_m=PM=55^\circ$  and frequencies  $\varpi_1 = \omega_{gc1} = 0.05rad / seg$  and  $\varpi_2 = \omega_{gc2} = 0.01rad / seg$  are imposed as design constraints for both outputs of the system.

Table 1. Controller parameters for example 1

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Output	Controller	$k_p$	$k_i$	μ	λ
1	FO-PI (gain-crossover method)	0.733	0.118	0.923	
	FO-PI (KC method)	1.241	0.059	1.05	
	FO-IMC			1.389	64.15
2	FO-PI (gain-crossover method)	1.113	0.0233	0.930	
	FO-PI (KC method)	2.133	0.0077	1.10	
	FO-IMC			1.387	594.76

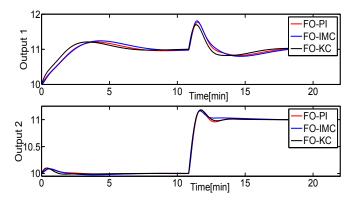


Fig. 3. Outputs of the first system with different fractional order controllers in reference tracking test

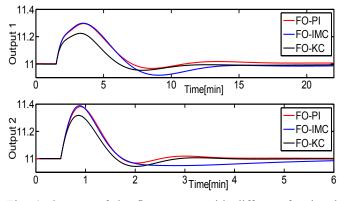


Fig. 4. Outputs of the first system with different fractional order controllers in disturbance rejection test

# 3.2 Example 2

The second heterogeneous dynamic system considered is described by the following transfer function  $G_2(s)$ :

$$G_2(s) = \begin{bmatrix} \frac{2.32}{100s+1} & \frac{4.24}{500s+1} \\ \frac{3.22}{150s+1} & \frac{2.14}{550s+1} \end{bmatrix}$$
(30)

The transmission zeros are:

$$z_1 = -0.0017 z_2 = -0.0132$$
(31)

Then checking input-output pairings, with a RGA (Relative Gain Array) analysis of the multivariable process.

$$\Lambda_2 = \begin{bmatrix} -0.5715 & 1.5715\\ 1.5715 & -0.5715 \end{bmatrix}$$
(32)

RGA matrix  $\Lambda_2$  suggests that the pairing 1-2/2-1 is suitable, since the main diagonal has negative values. On the other hand, GM=3,  $\phi_m=PM=55^\circ$  and frequencies  $\varpi_1 = \omega_{gc1} = 0.02rad / seg$  and  $\varpi_2 = \omega_{gc2} = 0.008rad / seg$  are imposed as design constraints for both outputs of the system.

Table 2. Controller parameters for example 2

Output	Controller	$k_p$	$k_i$	μ	λ	
1	FO-PI		0.040	0.988		
	(gain-crossover	0.443				
	method)					
	FO-PI	0.731	0.019	1.10		
	(KC method)	0.751				
	FO-IMC			1.389	229.08	
2	FO-PI		0.054	0.758		
	(gain-crossover	0.794				
	method)					
	FO-PI	2.818	0.004	1.15		
	(KC method)	2.010				
	FO-IMC			1.388	814.87	

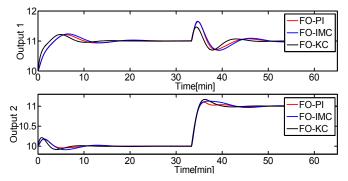


Fig. 5. Outputs of the second system with different fractional order controllers in reference tracking test

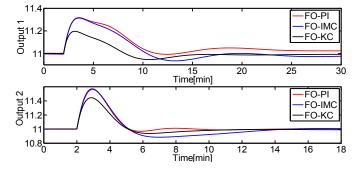


Fig. 6. Outputs of the second system with different fractional order controllers in disturbance rejection test

#### 4. ANALYSIS OF RESULTS

In this section, the results obtained for two heterogeneous dynamic systems are analized. For the first system, the controller parameters are given in the Table 1. These parameters are calculated according to the procedures described in section 2. Fig. 3 depicts the results for reference tracking performance for the fractional order controllers tuned with different methods, while, Fig. 4 illustrates the results for disturbance rejection performance for the results presented in Fig. 3 the controllers FO-PI gain-crossover autotuning method and FO-PI based on KC autotuning method obtain similar results in reference tracking, but better than FO-IMC controller. However, Fig.4 indicates that FO-PI based on KC autotuning method achieves a better disturbance

rejection than other controllers. Similarly, the controller parameters for the second system are given in Table 2. Fig. 5 and Fig.6 illustrate the performance results for reference tracking and disturbance rejection. In accordance with the results shown in Fig.5 and Fig.6 the FO-PI based on KC autotuning method achieves excellent load disturbance rejection, while maintaining a good reference tracking performance. Similar analyses are realized using performance indexes of the absolute integral error (*IAE*) and integral squared error (*ISE*) to evaluate the reference tracking and disturbance rejection respectively.

$$IAE = \sum_{t=0}^{\infty} |r_i(t) - y_i(t)|; ISE = \sum_{t=0}^{\infty} (r_i(t) - y_i(t))^2, \text{ with } i=1,2 (33)$$

The performance indexes calculated for two numerical examples are shown in the Table 3.

Table 3	. Performance	indexes	for the	different	controllers
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Example	Controller	Output 1		Output 2	
Example		IAE	ISE	IAE	ISE
1	FO-PI (gain-crossover method)	559.5	21.77	217.1	22.06
1	FO-PI (KC method)	545.5	12.33	171.9	18.14
	FO-IMC	742.6	25.46	295.6	24.04
2	FO-PI (gain-crossover method)	628.2	28.85	273.4	60.22
2	FO-PI (KC method)	536.7	9.22	315.0	52.58
	FO-IMC	774.7	26.65	412.4	79.41

According to the values of IAE for the first system. FO-PI gain-crossover autotuning and FO-PI based on KC autotuning obtain similar results. For the second system we have that FO-PI based on KC autotuning method achieves a better reference tracking performance for both outputs. Finally, ISE index indicated that the FO-PI based on KC autotuning method achieves excellent load disturbance rejection for both heterogeneous dynamic systems used. This is because the KC autotuner method is based on defining a 'forbidden region' in the Nyquist plane based on user-defined specs, which will guarantee the system margin requirements. In (De Keyser et al, 2017), it is reported the evaluation of this method to different type of systems obtaining good results.

# 5. CONCLUSIONS

In this paper, a fractional order PI based on KC autotuning method is presented. This method is based on defining a 'forbidden region' in the Nyquist plane based on user-defined specifications, which will guarantee the system margin requirements. The proposed method is applied to two heterogeneous dynamic systems. The performance of the FO-PI based on KC autotuning method is compared against two fractional order controllers based on PI gain-crossover autotuning method and Internal Model Control (IMC). The simulation results and numerical analysis show that the proposed method has better performance in disturbance rejection, while maintaining a good reference tracking performance. Further extension of this work could be the validation on a real heterogeneous dynamic systems where the system modeling is a heavy task.

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