Robust PI/PID parameter surfaces for a class of fractional-order processes

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Abstract: Recently, PID tuning rules based on integer-order model set approach has been developed. This paper shows how they can be enhanced through extending the set of *a priori* admissible systems to fractional-order form. Firstly, such set covers wider range of real process plants. Secondly, a new parameter affecting the model set span was introduced. It can help to reach the proper robustness/performance ratio especially in the case when the system has a lower order which is known. The authors believe that the procedure of fully automatic computing of robust PI/PID parameter surfaces will in the future lead to huge dataset that will serve as a base for deriving mature 2DOF PI/PID tuning rules based on various requirements.

Keywords: Robust PID tuning rules, fractional-order systems, process control, Nyquist plot shaping, robust control performance, sensitivity function, approximation function

1. INTRODUCTION

The producers of control systems and compact controllers aim at improving their products by intelligent features that shorten the time of installing into production lines. Clearly, such features should be serviceable without high-level knowledge of control theory, i.e. based on relatively simple tuning rules and procedures that can be automated.

Over the years, the evolution of PID tuning rules started from well known Ziegler-Nichols method (Ziegler and Nichols, 1942) which uses two-parameter [1] process model obtained from step response. The similar approach was followed and revised e.g. in Cohen and Coon (1953) or Åström and Hägglund (2006) where already three characteristic numbers were used. The original Ziegler-Nichols rules were refined also in Hang et al. (1991) by adding set-point weighting. These approaches can be denoted as indirect tuning rules. Their reliability is limited due to the fact that they were derived for simple process models, usually first/second order plus dead time (Luyben, 2001; Chen and Seborg, 2002). The more pragmatic approach was used in Åström and Hägglund (2006) where a large test batch of 134 processes was proposed, but only in integer-order form.

When a system model structure is known (gray box), there are other analytic approaches worth to mention, e.g. based on direct pole placement (Levine, 1996) and dominant pole design (Åström and Hägglund, 1995), or optimization based methods (Panagopoulos et al., 2002). These existing methods are often too complex, time consuming and can not be fully automated. Therefore, the control system producers and industrial practitioners still prefer simple empirical methods instead of exact math theory (Åström and Hägglund, 2006; Leva, 2001; Padula and Visioli, 2011). Thus, this work follows the direction initiated by Åström and Hägglund (2006).

The novel approach overcomes the well known drawbacks of empirical methods (see e.g. Ho et al. (1997) for nice overview) while the simplicity of resulting tuning procedure is preserved. The key paradigm is the *model set*. It is an exactly defined set of all processes satisfying two types of information: *a priori* assumption about the process candidate models and information obtained from identification experiment. Similarly to modern control theory, the model set represents after mapping into frequency domain the system uncertainty. By the theory referred in this paper and automatic numerical procedure developed, it can be guaranteed that the computed surfaces provide a controller which meets closed loop design specifications in frequency domain for arbitrary process from the model set.

The *a priori* information used in this paper separates from all possible linear systems relatively small set of all-pole fractional processes [2]. However, such set is wide enough to cover majority of real industrial plants (even with dead-time or stable zeros (Skogestad, 2003)). Compared to other works, e.g. Rotač (1984); Luo and Chen (2009), neither the number of poles nor the total process order is *a priori* limited. Moreover, it is used as a new additional tuning parameter. Point out, that in majority of works (see e.g. Podlubny (1999); Luo and Chen (2009); Yeroglu and Tan (2011)), a standard fractional order differential equation is used at process model side which is difficult to analyse. These approaches are not very suitable neither for describing essentially monotone processes nor for deriving analytical PID tuning rules from simple set of experimental data.

[1] Point out that two parameters can deal only with process gain and time-scale and are not able to assess the normalized dead-time which has been proven as critical for computing controller parameters

[2] i.e. having monotone step response with different level normalized dead-time defining how difficult the process is to control

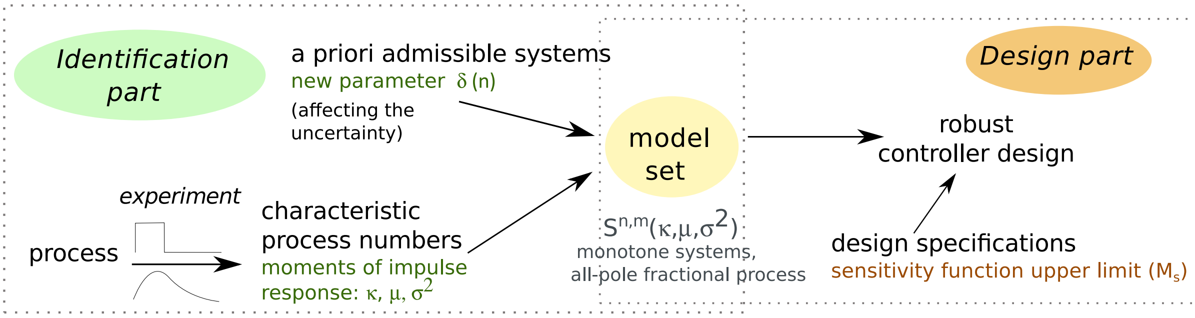

Fig. 1. Principal idea of the controller design based on the model set approach.

Further, it is assumed that, by processing the plant output, first three moments of impulse response are estimated and considered as the only known experimental data, i.e. the three parameter process description is preserved. Remind that the utilization of signal time moments in control is not new and appeared e.g in Maamri and Trigeassou (1993) and Bentayeb et al. (2006).
This paper provides a comprehensive insight into new approach rising from several initial research lines dealing with fractional-order model set and published recently (see e.g. Schlegel and Čech (2014)). The work was motivated by particular industrial requirements: extend the set of *a priori* admissible processes in order to ensure applicability on wider range of real plants namely with distributed parameters, provide parameter for fine adjusting of robustness / performance ratio based on additional process knowledge (upper order limit). Point out that, in accordance to majority of research works, sensitivity function upper limit is still considered as primary (coarse) tuning 'knob'. Hence we developed an automated procedure which outputs, based on defined sensitivity function limit, are surfaces of robust controller parameters which can further approximated via analytic functions for direct computation of robust PI and PID controller parameters [3].

The authors have used similar approach to develop a relay based PID tuner recently (Schlegel and Čech, 2005; Schlegel, 2002).

The rest of the paper is organized as follows: Section 2 reminds the problem formulation to make the work more self-contained. It also provides the technique for parameterization of all so-called *extremal processes*. Section 3 drafts the numerical optimization procedure and provides in full detail its directly applicable results – analytical PID tuning rules. Section 4 brings illustrative example validating the whole method. Conclusions and ideas for future work are given in Section 5.

2. PROBLEM FORMULATION

Let us formulate the problem comprehensively depicted in Fig. 1. Consider the standard SISO feedback control loop with 1-DOF PI/PID controller

$$C(s) = K\left(1 + \frac{1}{T_i s} + \frac{T_d s}{\frac{T_d}{N}s + 1}\right), \quad K, T_i, T_d, N \in \mathbb{R}^+ \tag{1}$$

where K is the proportional gain, and T_i, T_d are integral and derivative time constant, respectively, and a fixed parameter N determines the time constant of a derivative term filter [4]. Our final aim is to design free parameters K, T_i, T_d in such a way that the closed loop meets specific requirements on robust control performance (Section 3) with respect to arbitrary process from specific set described further.

2.1 Identification chain and admissible processes

In Charef et al. (1992), there was shown that the all-pole fractional-order transfer function in the form

$$P(s) = \frac{K}{\prod_{i=1}^{p}(\tau_i s + 1)^{n_i}}, \quad p \in \mathbb{N}, \ K, \tau_i, n_i \in \mathbb{R}^+, i = 1, \ldots, p \tag{2}$$

describes very well the majority of *essentially monotone* processes (see Åström and Hägglund (2006) for detailed definition) even with arbitrary dead-time.

Instead of numbers obtained from the step response using its tangent line in the inflexion point, the impulse response $h(t)$ time moments expressed as

$$m_i = \int_0^\infty t^i h(t)\mathrm{d}t, \qquad i = 0, 1, 2 \tag{3}$$

are used in this approach. This leads to well accepted 3-parameters process description (see Section 1). Further, it was proven in Schlegel et al. (2003a), that these moments (3) can be uniquely mapped to another numbers

$$\kappa = m_0, \ \mu = \frac{m_1}{m_0}, \ \sigma^2 = \frac{m_2}{m_0} - \frac{m_1^2}{m_0^2},$$

that give the *characteristic numbers* [5] and for process (2) it results to the following relations [6]

$$\kappa = K, \qquad \mu = \sum_{i=1}^{p} \tau_i n_i, \qquad \sigma^2 = \sum_{i=1}^{p} \tau_i^2 n_i. \tag{4}$$

Further, the key paradigm is the set of *a priori* admissible processes with predefined maximum order and consistent with experimentally obtained characteristic numbers,

[3] Point out, that the parameters of approximation functions are at this stage of research confidential.

[4] Remind, that in practice, the value of N is usually chosen according to noises in the measured signals and known before the tuning procedure starts. The proper choice of N can be also well automated.

[5] The impulse response moments (3) or equivalently the numbers (4) can be obtained from the process step response or may be estimated from process input/output data.

[6] Hence, for (2), κ is equivalent to the static gain, μ is related to residual time constant, and finally σ^2 corresponds to the steepness of step response in the inflection point.

more formally stated by following definition which has been already given in full detail in Schlegel and Čech (2014). Here just their summary is given in order to make the document more self-contained [7].

Definition 1. (Model set). The transfer function $P(s)$ is *admissible* if and only if

(i) $P(s)$ is in the form (2), $n_i \geq 1, \forall i, \sum_{i=1}^{p} n_i \leq n$, where $n \in \mathbb{R}^+$ is the total order of the process.
(ii) $P(s)$ is consistent with experimental data, thus fulfils (4). The set of all admissible transfer functions will be called *model set* and denoted as $\mathcal{S}^n(\kappa, \mu, \sigma^2)$.

Proposition 1. Let $n \geq 1$, then the model set $\mathcal{S}^n(\kappa, \mu, \sigma^2)$ is not empty if and only if

$$\frac{1}{n} \leq \frac{\sigma^2}{\mu^2} \leq 1. \tag{5}$$

If the strict inequality (5) is satisfied then the model set contains for given characteristic numbers (κ, μ, σ^2) infinite number of processes. After mapping into frequency domain, the model set creates a connected area called *value set* for each frequency $\omega > 0$.

Definition 2. (Value set). The set $\mathcal{V}^n_\omega(\kappa, \mu, \sigma^2) = \left\{ P(s)|_{s=\mathrm{j}\omega} : P(s) \in \mathcal{S}^n(\kappa, \mu, \sigma^2) \right\}$ will be called the *value set* of $\mathcal{S}^n(\kappa, \mu, \sigma^2)$ at the frequency $\omega > 0$.

The value set boundary is generated by so called *extremal transfer functions.*

Definition 3. (Extremal transfer functions). The admissible transfer function $P(s) \in \mathcal{S}^n(\kappa, \mu, \sigma^2)$ will be called *extremal*, if there exists $\omega > 0$ such, that $P(\mathrm{j}\omega) \in \partial\mathcal{V}^n_\omega(\kappa, \mu, \sigma^2)$, where $\partial\mathcal{V}^n_\omega(\kappa, \mu, \sigma^2)$ denotes the value set boundary in complex plane. Let us denote the set of all extremal transfer functions as $\mathcal{S}^n_E(\kappa, \mu, \sigma^2)$.

For the *a priori* assumption (2) and condition (4) the set $\mathcal{S}^n_E(\kappa, \mu, \sigma^2)$ is independent on frequency ω and can be generated by transfer functions in very specific simple forms. The exact expressions for extremal transfer functions were given in Schlegel and Čech (2014). Here, this set is used as an input for consequential numerical procedure of computing PI/PID parameter surfaces.

Proposition 2. Without loss of generality, the fractional process (2) can be normalized in gain, i. e. $\bar{\kappa} = 1$, and in time, thus $\bar{\mu} = 1$. The remaining parameter $\bar{\sigma}^2 = \sigma^2/\mu^2$ then has a meaning similar to normalized dead time.

The parameter $\bar{\sigma}^2$ is hence used as one of the independent variables for the purposes of following computational procedure.

3. NUMERIC PROCEDURE FOR ROBUST PI (PID) CONTROLLER DESIGN

The set of all admissible processes resulting from previous section can be considered as a process uncertainty model entering the numeric procedure for calculation of robust controller parameters, which is based on generalized robustness regions method (Schlegel et al., 2003b; Shafiei and Shenton, 1997; Ruszewski, 2008). The regions are computed with respect to the design requirement for *maximum peak of sensitivity function* for the extremal processes.

Definition 4. (Design task formulation). Let for fixed real number $n \geq 3$ and characteristic numbers κ, μ, σ^2 be fulfilled the condition (5). The aim is to compute the parameters $(K,\ T_i)$ of the controller $C(s)$ (1) especially with $T_d = 0$ for PI controller (or with fixed ratio T_i/T_d and fixed filter order N in case of PID controller formula) in such a way to minimize the ratio

$$T_i/K, \tag{6}$$

while the following two conditions (*on robust control performance*) are satisfied for any $P(s) \in \mathcal{S}^n(\kappa, \mu, \sigma^2)$:

(i) *(Robust stability)* Nyquist plot $L(\mathrm{j}\omega) \triangleq C(\mathrm{j}\omega)P(\mathrm{j}\omega)$ fulfils closed loop stability condition, i.e. it does not encircle critical point $(-1, \mathrm{j}0)$.
(ii) *(Stability margin)* $L(\mathrm{j}\omega)$ lies outside [8] a circle U with a center $c = -1$ and radius $r = 1/M_s$, i.e.

$$\forall\omega \geq 0 : L(\mathrm{j}\omega) \notin U(c, r), \tag{7}$$

where M_s is an upper limit of sensitivity function $S(j\omega) \triangleq \frac{1}{1+L(\mathrm{j}\omega)}$, i.e. $\sup_\omega |S(j\omega)| \leq M_s$.

By following the conditions in Definition 4 for particular $P(s) \in \mathcal{S}^n_E(\kappa, \mu, \sigma^2)$, equations for circle and its tangent line are derived

$$(u - c)^2 + v^2 = r^2,$$
$$(u - c)u_1 + vv_1 = 0,$$

where $[u, v]$ denotes the complex point of $L(j\omega)$, $[a, b]$ the complex point of $P(j\omega)$, $[x, y]$ the complex point of $C(j\omega)$, and $*_1$ corresponding derivatives, so $u = ax - by$, $v = ay + bx$, $u_1 = a_1x + ax_1 - b_1y - by_1$, $v_1 = a_1y + ay_1 + b_1x + bx_1$.

Especially, for PI controller $C(s) = K + K_I/s$, when $K_I = K/T_i$, we get $x = K$, $y = -K_I/\omega$, $x_1 = 0$ and $y_1 = K_I/\omega^2$. Using Grőbner basis technique with lexicographic order $K \succ_{lex} K_I$, the following basis $\mathcal{B}$ is obtained

$$\mathcal{B} = [p_4K_I^4 + p_3K_I^3 + p_2K_I^2 + p_1K_I + p_0, q_kK + q_2K_I^2 + q_1K_I + q_0],$$

where $p_4 = \left(a^2 + b^2\right)^4$, $p_3 = 2\,(a^2+b^2)^2(a^2b_1-2\,aba_1-b_1b^2)c\omega^2 - 2\,bc(a^2+b^2)^3\omega$, $p_2 = (c({a_1}^2+{b_1}^2)b^4+2\,a^2c({a_1}^2+{b_1}^2)b^2+a^4c({a_1}^2+{b_1}^2))c\omega^4 - 2\,(a^2+b^2)((-2\,c^2b_1+r^2b_1)b^3+(-4\,ac^2a_1+ar^2a_1)b^2+2\,a^2b_1(c^2+1/2\,r^2)b+a^3r^2a_1)\omega^3+(a^2+b^2)^2b^2c^2\omega^2$, $p_1 = -2\,((({a_1}^2+{b_1}^2)c^2 - r^2{b_1}^2)b^3 - 3\,ab^2r^2a_1b_1 + a^2(({a_1}^2+{b_1}^2)c^2 - 2\,r^2{a_1}^2 + r^2{b_1}^2)b+a^3r^2a_1b_1)c\omega^5+((-2\,c^2b_1+2\,r^2b_1)b^4-4\,a(c^2-1/2\,r^2)a_1b^3+2\,a^2b_1(c^2+r^2)b^2+2\,a^3r^2a_1b)c\omega^4$, $p_0 = -(c-r)(c+r)(((-{a_1}^2-{b_1}^2)c^2+r^2{b_1}^2)b^2+2\,aa_1bb_1r^2+a^2r^2{a_1}^2)\omega^6$, $q_k = (a_1a^2c+2\,acb_1b-ca_1b^2)\omega^3$, $q_2 = -(a^2+b^2)^2$, $q_1 = -c(a^2b_1-2\,aba_1-b_1b^2)\omega^2+bc(a^2+b^2)\omega$, $q_0 = -aa_1c^2\omega^3+aa_1r^2\omega^3-bb_1c^2\omega^3+bb_1r^2\omega^3$.

The solution of this system of equations (four pairs of K_I, K) creates four branches in $K_I - K$ plane parametrized by ω which creates the boundary of robustness regions (together with parameters axes K_I, K from assumption $K_I > 0$, $K > 0$). Moreover, only one of them contains optimal solution (K_I^*, K^*) minimizing the criteria (6). For

[7] Unlike in Schlegel and Čech (2014), here it is directly considered, that the minimal order of arbitrary pole can be at least one.

[8] Such type of task is, from robustness perspective, more relevant then just the requirement for fixing $L(j\omega)$ to some point and also more complex for solution.

better imagination, specific regions are depicted in the Figure 2[9].

This procedure is repeated for all $P(s) \in \mathcal{S}_E^n(\kappa, \mu, \sigma^2)$ (respective selected samples from boundary of value sets). Finally from all suboptimal solutions (K_I^*, K^*), the optimal one which minimizes (6) while the conditions in definition 4 are fulfilled for the set $\mathcal{S}^n(\kappa, \mu, \sigma^2)$.

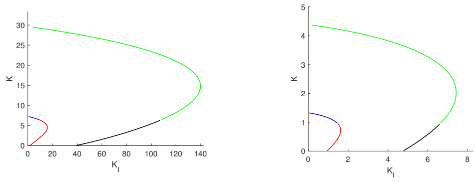

Fig. 2. Examples of robust regions curves in $K_I - K$ plane, left for $\delta = 0.25, \sigma^2 = 0.8$, right for $\delta = 0.1, \sigma^2 = 0.3$. The red lines contain the optimal solution minimizing (6).

According to the Proposition 2, we can consider the model set which is normalized in time and gain, so we deal now with the processes from the set $\mathcal{S}^n(1, 1, \sigma^2)$.

The key novelty of this paper is inclusion of the parameter n as *additional tuning parameter*, because this parameter affects the uncertainty of the model set – in the sense that the area of value sets can be shaped by it (for smaller n the value sets are reduced, for growing n are increased, see Fig. 8). Obviously, we can map this parameter also to the interval $(0, 1)$ and define *design task parameter* $\delta \triangleq \frac{2}{n}$. For such normalized set, let's us denote the PI parameters as $\bar{K}$, $\bar{T}_i$ (for PID it was considered fixed $N = 10$ and $\bar{T}_d = \frac{\bar{T}_i}{4}$). The shape of surfaces for optimal parameters $\bar{K}$ and $\bar{T}_i$ for specific M_s is depicted in Fig. 3. The idea of the whole procedure how to obtain controller parameters is summarized in Fig. 5.

4. EXAMPLE

Consider that a system is a black box which belongs to the class (2). As a case study, let us assume that inside a box there is a fractional order dynamics described by

$$P(s) = \frac{2}{(2.47s + 1)^{2.21}(4.52s + 1)} \tag{8}$$

which leads according to (4) to characteristic numbers $\kappa = 2$, $\mu = 10$, $\sigma^2 = 34$ and after normalization $\bar{\sigma}^2 = \sigma^2/\mu^2 = 0.34$. It is shown in Table 1 how the controller parameters depend on new δ parameter. The values $(\bar{K}, \bar{T}_i)$ are obtained from computed surfaces for $M_s = 1.6$ and PID controller formula. The denormalized parameters (K, T_i) are computed according to the rules in Fig. 5.

The example brings two key claims:

Claim 1. In case of additional *a priori* knowledge of maximum process order n the controller parameters can be adjusted by proper choice of δ which reduces the model uncertainty, see Fig. 8. However, the robust control performance is still guaranteed for the whole model set as confirmed also by Fig. 6 and 7. It flows out directly from the numerical tuning procedure.

[9] Anyway, for PID controller (1), the numerical procedure for computing the roots of the higher order polynomials was used.

Table 1. Normalized and denormalized PID controller parameters for process (8), $M_s = 1.6$, $N = 10, T_d/T_i = 0.25$

δ	n	$\bar{K}$	$\bar{T}_i$	K	T_i
0.5	4	1.85	0.62	0.93	6.2
0.4	5	1.48	0.60	0.74	6.0
0.33	6	1.32	0.59	0.66	5.9
0.0667	30	1.13	0.57	0.57	5.7

Claim 2. One can adjust the controller parameters even without knowledge of maximum process order n. In this case, it could happen that design specifications defined by M_s are not met. However, the robust stability is still guaranteed. This was not fully justified in this paper and will be elaborated in the future.

Anyway, the new tuning parameter δ is a relevant instrument for safe adjusting of controller parameters which could have big impact e.g on maximum value of controller output during step response as shown on Fig. 9.

5. CONCLUSIONS

The paper presented outputs of fully automatic procedure of computing robust PI/PID parameter surfaces. The key novelty is the new tuning parameter δ which helps, together with sensitivity function upper limit, to handle the robustness/performance trade-off especially when the default tuning seems to be too conservative and there is some *a priori* assumption about the system total order. The robust surfaces can be approximated via analytic relations for PI/PID controller tuning. We assume that the procedure will be used for computing rich data sets of controller parameters that will be further used as a base deriving final analytic tuning rules for full 2DOF PID controller.

ACKNOWLEDGEMENTS

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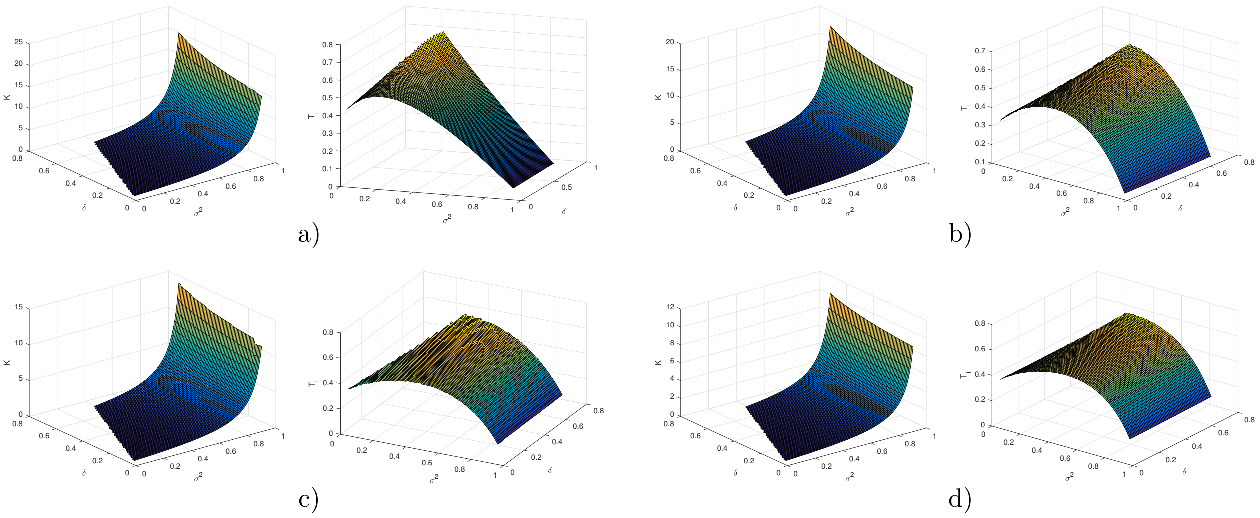

Fig. 3. PI controller's parameters for requirement on sensitivity function restriction by circle (7), compared for different M_s– a) for $M_s = 2$, b) for $M_s = 1.8$, c) for $M_s = 1,6$, and d) for $M_s = 1.4$, in each subfigure – controller gain $\bar{K}$ (left) and integral time constant $\bar{T}_i$ (right) (dzepending on $\bar{\sigma}^2$ and δ).

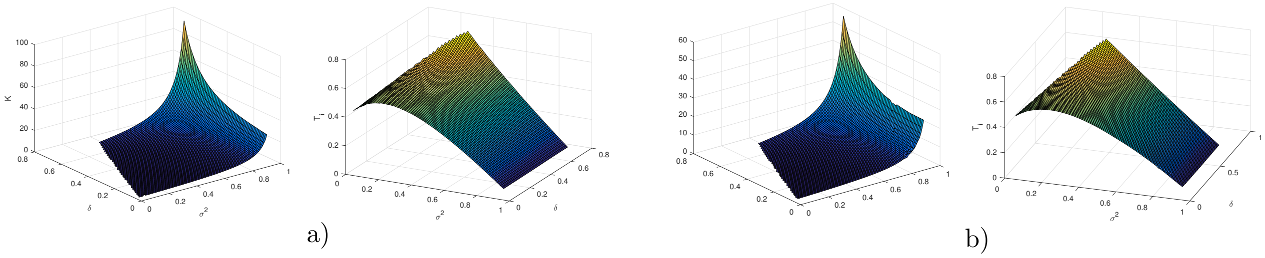

Fig. 4. PID controller's parameters for requirement on sensitivity function restriction by circle (7) for different M_s, in a) for $M_s = 2$, b) for $M_s = 1.6$, with fixed another parameters $N = 10$, $T_d/T_i = 1/4$, each subfigure contains – controller gain $\bar{K}$ (left), integral time constant $\bar{T}_i$ (right) (depending on $\bar{\sigma}^2$ and δ).

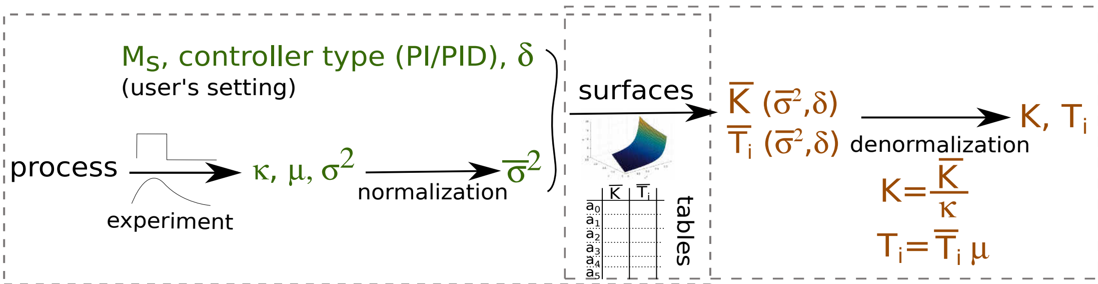

Fig. 5. Controller parameters acquisition chain.

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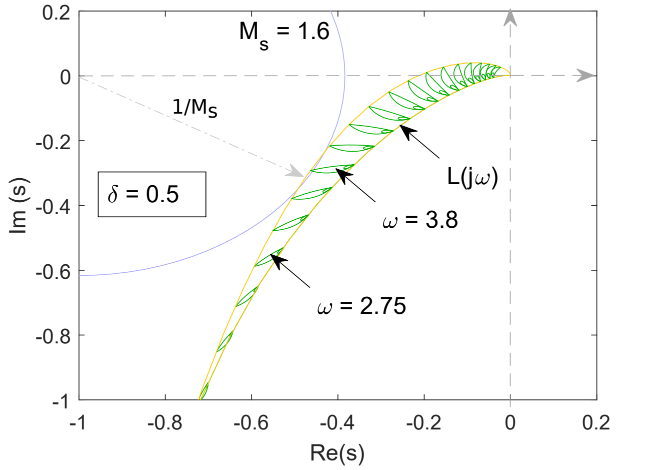

Fig. 6. Example: Nyquist plot plane for $\delta = 0.5$ ($n = 4$) for normalized controller parameters

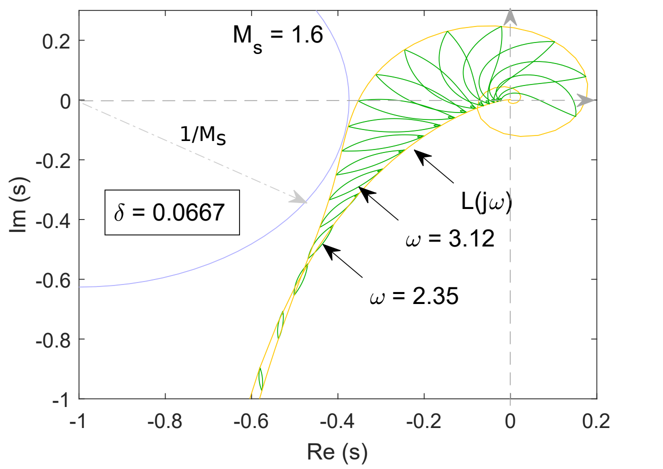

Fig. 7. Example: Nyquist plot plane for $\delta = 0.0667$ ($n = 30$) for normalized controller parameters

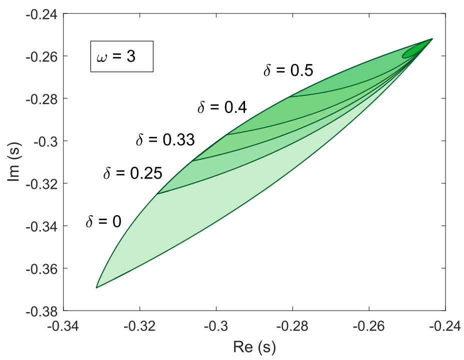

Fig. 8. Example: value set size shaped by new tuning parameter δ, for frequency $\omega = 3$ and $\bar{\sigma}^2 = 0.34$

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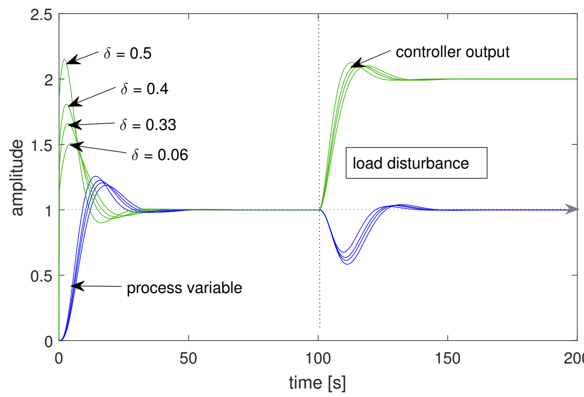

Fig. 9. Example: closed loop step responses and load disturbance responses dependent on δ

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