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Adaptive Position Servo Control of Permanent Magnet Synchronous Motor

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***Abstract*—The Permanent Magnet Synchronous Motor (PMSM) has been widely used in many accurate servo control systems as it has excellent control qualities, large torque coefficient, small ripple torque, and so on. However, the model uncertainty of PMSM has an important effect on the accuracy of control systems. In this paper, the dynamic model for PMSM systems is established in an experimental method. The adaptive model following control law is presented for the position servo system of PMSM, and the motor velocity signal of the adaptive controller is estimated with the velocity observer. The experimental results verify the effectiveness of the adaptive control method and show that the position of the PMSM can follow the output of the reference model well.**

I. Introduction

With the development of the technology of power electronics control, Nd-Fe-B material and motor design, permanent magnet synchronous motors (PMSM) get wide applications in many control systems, especially in the accurate servo control systems because they have excellent control capabilities, large torque coefficient, small ripple torque, and many other good qualities [1], [2]. However, due to the couple non-linearity of direct axis and quadrature axis currents, load variety of the plant and mechanical friction, the model uncertainty of PMSM system has a great impact on the accuracy of control systems [3], [4].

Generally, the robust control and the adaptive control methods are used to eliminate the effects of uncertainty for systems with uncertainty and mechanical friction [5], [6]. The adaptive control method has more and more applications in industrial and robotic control since it can effectively control systems with model or parameter uncertainty [7], [8].

In this paper, firstly, the model uncertainty of the PMSM system is analyzed and the model is established experimentally. Secondly, the adaptive model following control (AMFC) law is proposed for the position servo system. In order to simplify the realization, the controller is designed on the basis of the order-reduced model of the PMSM system. The velocity signal of the motor used for the adaptive controller is obtained using the velocity observer. Finally, the experimental results of the adaptive control method are given and they show that this method can control the PMSM system with model uncertainty and friction more effectively.

II. Model of PMSM System

The block diagram of the PMSM system in this paper is shown as Fig. 1. The system consists of the PMSM and its driver.

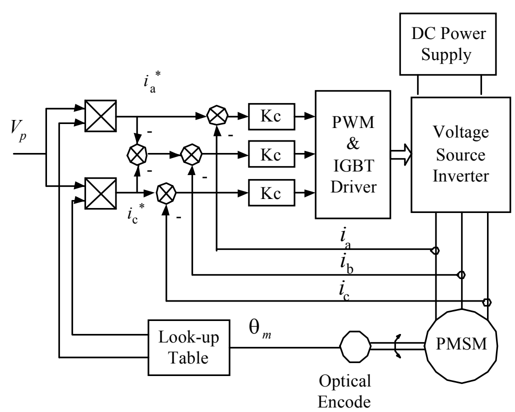

Fig. 1. Diagram of the PMSM system.

Ignoring the effect of magnetic saturation, magnetic hysteresis and eddy current, the model of the surface-mounted PMSM in synchronous coordinate can be written as [3], [9]:

$$\begin{cases} u_d = ri_d + L\ pi_d - \omega_e L\ i_q \\ u_q = ri_q + L\ pi_q + \omega_e L\ i_d + \omega_e \sqrt{\frac{3}{2}}\psi_f \end{cases} \tag{1}$$

$$T_{em} = \sqrt{\frac{3}{2}} P_m \psi_f i_q = K_T i_q \tag{2}$$

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$$T_{em} = T_l + J\frac{d^2\theta_m}{dt} + D\frac{d\theta_m}{dt} \tag{3}$$

where, u_d, u_q are direct and quadrature axis voltages respectively, and i_d, i_q the direct and quadrature axis currents respectively; r is the resistance of motor; p is the differential operator, i.e. $p = \frac{d}{dt}(\cdot)$; ω_e is the motor's electrical velocity; ψ_f is the fundamental amplitude of magnetic linkage in the winding of one phase due to the permanent magnet on rotor; L is the synchronous inductance; P_m is the number of pole pairs; K_T is the torque coefficient, and $K_T = \sqrt{3/2}P_m\psi_f$; J is the inertia constant of the rotor and load; D is the coefficient of friction; T_l is the load torque.

The state equation of the PMSM system considering the motor and its driver in Fig.1 is [10]:

$$\begin{bmatrix} \frac{di_d}{dt} \\ \frac{di_q}{dt} \\ \frac{d\omega_r}{dt} \end{bmatrix} = \begin{bmatrix} -r'/L & P_m\omega_r & 0 \\ -P_m\omega_r & -r'/L & -\sqrt{\frac{3}{2}}P_m\psi_f/L \\ 0 & \sqrt{\frac{3}{2}}P_m\psi_f/J & -D/J \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ \omega_r \end{bmatrix}$$

$$+\begin{bmatrix} 0 \\ K_cK_s/L \\ 0 \end{bmatrix} V_p + \begin{bmatrix} 0 \\ 0 \\ -1/J \end{bmatrix} T_l \tag{4}$$

where, ω_r is the mechanical velocity of the motor, and $\omega_r = \omega_e / P_m$; K_c is the gain of the current adjuster; K_s is the gain of inverter; K_f is the coefficient of current feedback; $r' = r + K_cK_sK_f$, is the equivalent resistance of motor considering the driver; V_p is the instruction to the motor driver.

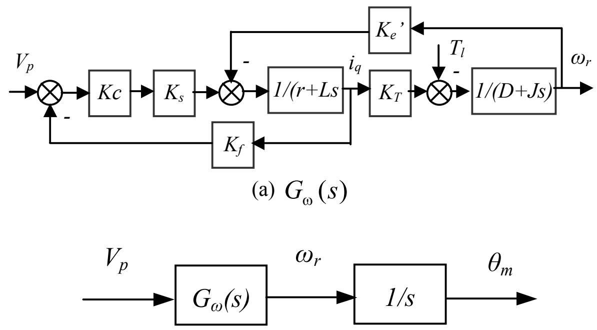
It can be seen from (4) that the PMSM system is a nonlinear coupling system. If the coupling effect of direct axis current on the quadrature axis voltage is regarded as the disturbance to the coefficient of EMF, the transfer function diagram of the PMSM system for control instruction to the velocity and position output can be shown as Fig. 2 (a) and (b).

Therefore, the transfer functions can be written as:

$$G_\omega(s) = \frac{K_cK_s}{K_e'} \frac{1}{\frac{JL}{K_e'K_T}s^2 + \frac{r'J}{K_e'K_T}s + 1} \tag{5}$$

$$G_\theta(s) = \frac{\theta_m(s)}{V_p(s)} = \frac{G_\omega(s)}{s} \tag{6}$$

where, $K_e' = \sqrt{\frac{3}{2}} P_m\psi_f + P_m L i_d$, is the equivalent coefficient of EMF considering the effect of direct axis current.



(a) $G_\omega(s)$

(b) $G_\theta(s)$

Fig. 2. Simplified diagram of PMSM system

The dynamic direct axis current is always existent even if the vector control method is used for PMSM. The parameter of the transfer function is time-variational. Furthermore, the variety of the load and the friction cause the model uncertainty of the PMSM system. Fig. 3 is the experimental frequency response curves of the PMSM system with combination of different rotor positions and different inputs of motor driver. It can be seen that the model uncertainty of the PMSM system really exists.

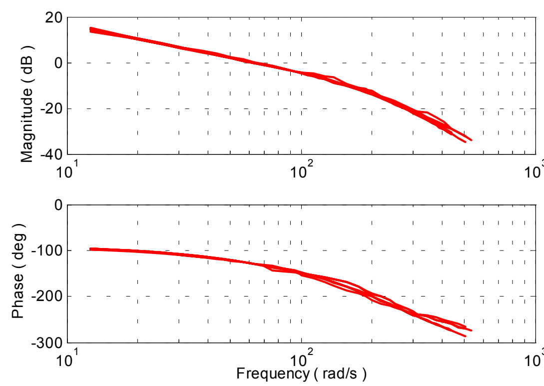


Fig. 3. Frequency characteristic of the PMSM system.

The center of the curve cluster is taken as the nominal response curve. The nominal model of PMSM system can be expressed as (7) with parameter optimization for a three--order transfer function from (5) and (6).

$$G_\theta(s) = \frac{72.210}{s(2.278\times10^{-5}s^2 + 7.721\times10^{-3}s + 1)} \tag{7}$$

After the model of PMSM system is established, the adaptive position servo controller will be designed next.

III. Adaptive Model Following Control of Systems with Uncertainty

Supposing the nominal transfer function of the plant is:

$$G(s) = \frac{b_0}{s^n + \sum_{i=0}^{n-1} a_i s^i} \tag{8}$$

The differential equation of (8) can be expressed as:

$$p^n y(t) + \left[\sum_{i=0}^{n-1} a_i p^i \right] y(t) = b_0 u(t) \qquad (9)$$

The transfer function and the differential equation of the reference model are chosen as:

$$G_m(s) = \frac{b_{m0}}{s^n + \sum_{i=0}^{n-1} a_{mi} s^i} \qquad (10)$$

$$p^n y_m(t) + \left[\sum_{i=0}^{n-1} a_{mi} p^i \right] y_m(t) = b_{m0} r(t) \qquad (11)$$

The output error between the reference model and the plant is defined as:

$$e(t) = y_m(t) - y(t) \qquad (12)$$

The control input u_p is chosen so that the output of the plant can track that of the reference model closely. Furthermore, the tracking performance should be insensitive to the uncertainty of the plant.

$$u_p = u_{p1} + u_{p2} \qquad (13)$$

Here, u_{p1} is the control input that can make the plant output exactly follow that of the reference model when the plant is an ideal system.

$$u_{p1} = K_r r(t) + [\sum_{i=0}^{n-1} K_{ai} p^i] y_m(t) + K_{er} e(t) \qquad (14)$$

Formula (15) can be obtained by substituting (14) into (9) and considering the definition of the output error.

$$p^n e(t) + \left[\sum_{i=1}^{n-1} a_i p^i \right] e(t) + (a_0 + b_0 K_{er}) e(t) = \left[\sum_{i=0}^{n-1} (a_i - a_{mi} - b_0 K_{ai}) p^i \right] y_m(t) + (b_{m0} - b_0 K_r) r(t) \qquad (15)$$

Let $K_{ai} = (a_i - a_{mi}) / b_0$, $i = 0, \cdots, n-1$, $K_r = b_{m0} / b_0$. Configure K_{er} so that the equation $s^n + \left[\sum_{i=1}^{n-1} a_i s^i \right] + (a_0 + b_0 K_{er}) = 0$, has stable latent roots. Now, (15) is asymptotically stable, i.e. $\lim_{t \to \infty} e(t) = 0$. Thus the output of the plant follows that of reference model well. It is obvious that the satisfying response cannot be achieved by using u_{p1} only when the plant presents uncertainty. Hereby, u_{p2} must be introduced to reduce the effect of uncertainty.

$$u_{p2} = \Delta K_r r(t) + [\sum_{i=0}^{n-1} \Delta K_{ai} p^i] y_m(t) + \Delta K_{er} e(t) \qquad (16)$$

where, ΔK_r, ΔK_{ai}, and ΔK_{er} are adaptive parameters of the controller. Let

$$\Delta K_{ai}(t) = \int_0^t L_{1i} v(\tau) p^i y_m(\tau) d\tau + L_{2i} v(t) p^i y_m(t) \qquad (17)$$

$$\Delta K_r(t) = \int_0^t L_3 v(\tau) r(\tau) d\tau + L_4 v(t) r(t) \qquad (18)$$

$$\Delta K_{er}(t) = \int_0^t L_5 v(\tau) e(\tau) d\tau + L_6 v(t) e(t) \qquad (19)$$

where, $L_{1i} > 0$, $L_{2i} > 0$ ($i = 0, \cdots n-1$); $L_i > 0$, ($i = 3, \cdots 6$); and

$$v(t) = D(p) e(t) = [\sum_{i=0}^{n-1} d_i p^i] e(t) \qquad (20)$$

$v(t)$ is the output error of the system plus the linear compensator $D(p)$. By choosing the coefficients of the compensator appropriately, (21) can satisfy the strictly positive real condition.

$$G_v(s) = \frac{[\sum_{i=0}^{n-1} d_i s^i]}{s^n + \left[\sum_{i=1}^{n-1} a_i s^i \right] + (a_0 + b_0 K_{er})} \qquad (21)$$

Also, the Popov integral inequality can be satisfied if the PI adaptations are synthesized according to (17)-(19). The system will be asymptotically hyperstable by the Popov's theorem [11]. Therefore, the output of the plant follows that of the reference model.

IV. Adaptive Control of PMSM System

A. Adaptive Position Control of PMSM System

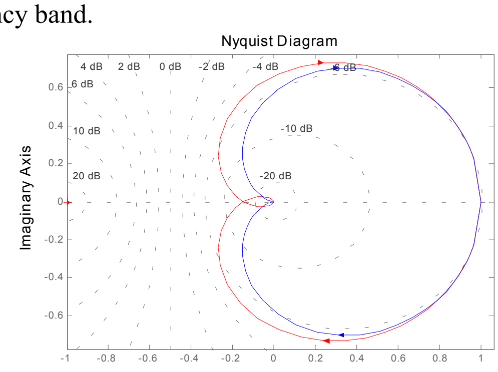
It is known from above that with increase of the model orders of the plant, the calculation quantity of the adaptive controller will increase, and sometimes the controller will be unreliable. Hereby, the adaptive control is implemented after the plant is controlled with simple proportional position loop. The proportional gain of the position controller is chosen as $K_p = 1.0$ so that the system has enough amplitude and phase margin. The closed-loop transfer function of the PMSM after simple control is:

$$G_p(s) = \frac{72.210}{2.278 \times 10^{-5} s^3 + 7.721 \times 10^{-3} s^2 + s + 72.210}$$

With Padé reduction technique [12], the order-reduced transfer function can be written as:

$$G_{re}(s) = \frac{9352.35}{s^2 + 129.54 s + 9352.35}$$

The Nyquist diagrams of $G_p(s)$ and $G_{re}(s)$ are shown in Fig. 4. It can be seen that the order-reduced system is approximate to the original one within the middle and low frequency band.



. Fig. 4. Nyquist diagram of original and reduced model.

According to the performance requirement of the PMSM system, the reference model is chosen as:

$$G_m(s) = \frac{15791.37}{s^2 + 100.53s + 15791.37}$$

The adaptive control law can be chosen from (13). The parameters K_r, K_{ai}, and K_{er} are chosen following the procedure described in section III:

$K_r = 1.68849$, $K_{a0} = -0.68849$, $K_{a1} = 0.00310$ $K_{er} = 2.0$.

ΔK_r, ΔK_{ai}, and ΔK_{er} come from (17)-(20).

According to the requirement of the Popov's theorem for the strictly real condition, the coefficients in $v(t) = d_1 pe(t) + d_0 e(t)$, are $d_0 = 2$ and $d_1 = 0.10$ respectively. For a 2-order system, it is strictly real if and only if $d_0 > 0$ and $d_1 > d_0 / a_1$.

B. Velocity Observer of PMSM

From (20) and (21), it is known that in order to satisfy the strictly real condition, the differential signal of the adaptive system's output error is needed. From (2), the differential signal of the plant output is also needed. However, in this paper only the position transducer is used for the PMSM system. Although the velocity signal can be estimated from the discrete differential of the position, this solution based on the position data only has significant limitations of accuracy and noise [13], since the sample time in this paper is 0.0002s. Fig. 5 is the discrete differential of the motor position. It displays serious high frequency noise. This velocity signal cannot be used for the adaptive controller.

The observer shown in Fig. 6 is used to estimate the velocity of PMSM [13]. The nominal motor model in Fig. 6 is chosen as (7) with experimental method. Fig. 7 is the output of the observer. Fig. 7 (a) shows that the output of the observer is very close to the PMSM position. Consequently, the estimated velocity of the observer approximates well to the real velocity of the motor.

C. Experimental Results of the Adaptive Control

Fig. 8 is the AMFC diagram of the PMSM system. The controller is implemented on an industry computer.

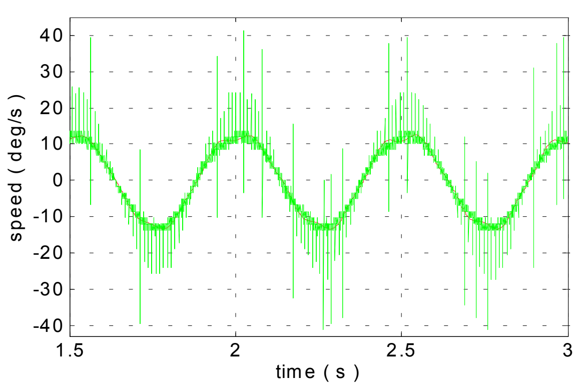


Fig. 5. Discrete differential of the PMSM position

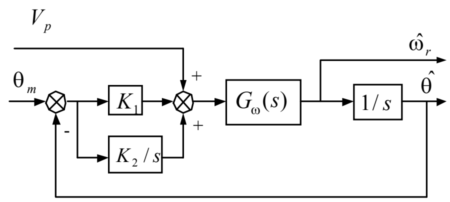
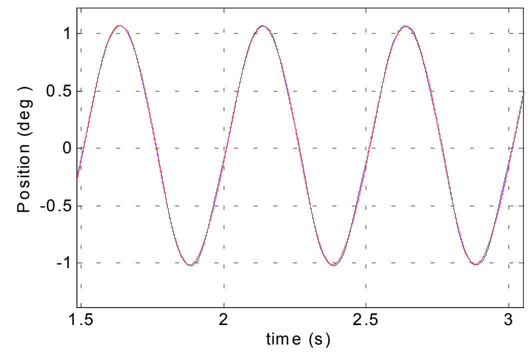
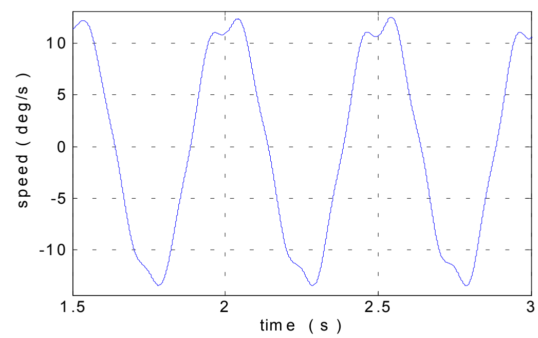


Fig. 6. Diagram of the velocity observer.



(a) Positions



(b) Speed

Fig. 7. Output of the velocity observer.

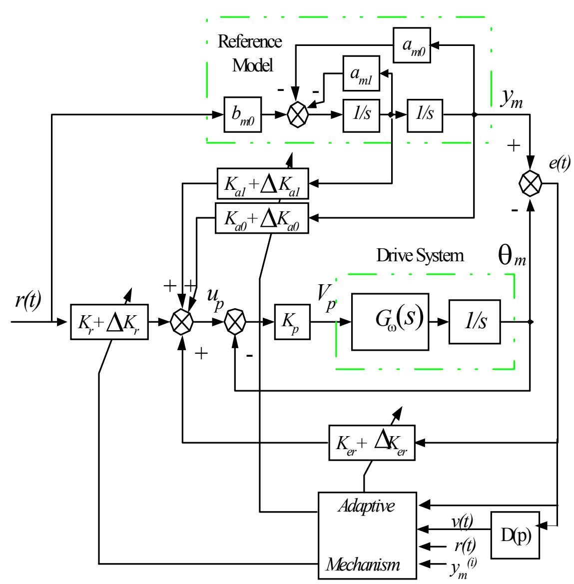


Fig. 8. AMFC diagram of the PMSM system.

Fig. 9 shows the output of reference model and the position of the PMSM with the unit step input. It indicates that good performance of model following is achieved, and the final static error reaches 0.005°, which satisfies the system's requirement.

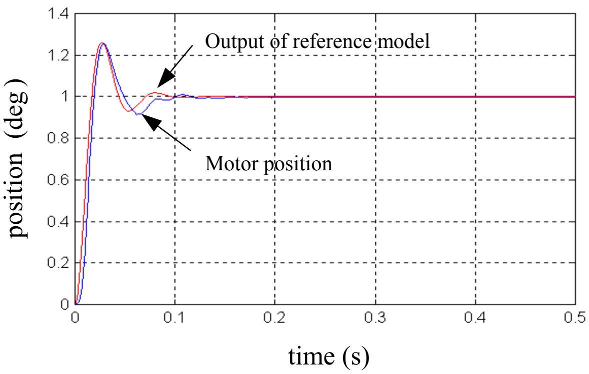


Fig. 9. Step response of PMSM.

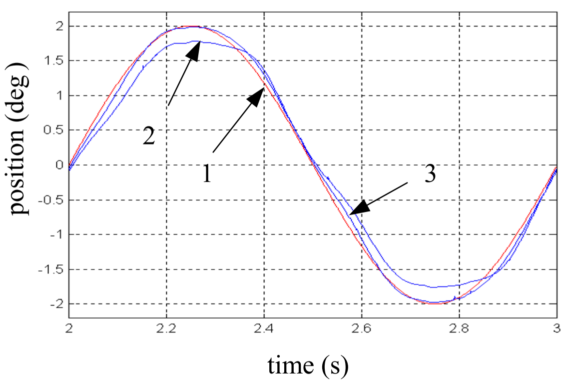
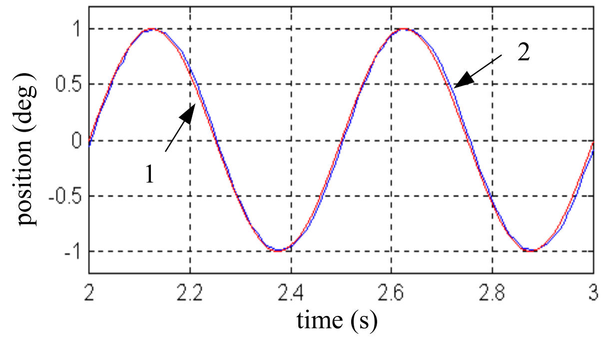


Fig. 10. PMSM response of sine input.

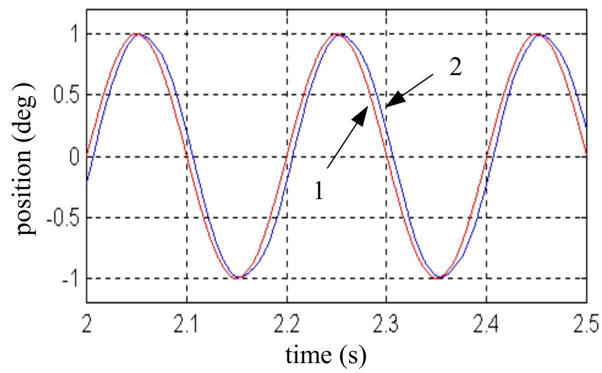
Fig. 10 is the response of the PMSM to 1Hz sine input. Curve 1 is the control instruction to the system, curve 2 is the motor's position with the controller of u_{p1} in the nominal operating condition and curve 3 is the motor position with adaptive controller.

It is known from Fig.10, that the motor position cannot follow the output of reference model well with the controller designed for the nominal operating condition. The error between the motor position and instruction is big within the time ranges from 2.1s to 2.4s and 2.6s to 2.9s. The reason is that the motor position is close to its moving peak within these two time slices, the absolute velocity within the periods of time is small, and the static friction becomes the main factor of the system's uncertainty. The adaptive controller is insensitive to the uncertainty and makes the motor position follow the reference model output well.

Fig. 11 is the motor responses to the sine instruction of 2Hz @ 1° and 5Hz @ 1°. Curve 1 is the instruction and curve 2 is the motor position. It shows that the good servo performance of PMSM is reached with the adaptive control law.



(a) 2Hz @ 1°



(a) 5Hz @ 1°

Fig. 11. Response of sine input.

V. Conclusion

In this paper, the uncertainty of the PMSM system is analyzed, and the model is established using experimental methods. With respect to the model uncertainty, the adaptive model following controller is designed for the position servo control of the PMSM. The velocity signal used for the adaptive controller is estimated with the velocity observer. The experimental results show that good position servo performance of the PMSM is achieved. The adaptive controller is insensitive to the model uncertainty.

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