# Application of congestion control algorithms for the control of a large number of actuators with a matrix network drive system 

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#### Abstract

Systems with a large number of actuators can be built more efficiently by reducing the number of amplifiers using a matrix network drive system. The system with network drive architecture has limited capacity in terms of power related resources. In this paper, congestion control algorithm, along with several other algorithms, is presented for scheduling of the network drive system. For the network drive system, the load to the network is the output of the Pulse Width Modulators of each actuator. As the demand for the actuators increases, the duty ratio of the PWM outputs will increase. The objective of the scheduling strategy is to maximize the average duty ratio of the PWM power input to the actuator without causing instability of the network modulator, and to minimize the effect of the instability on the system performance when it occurs. Four different protocols are presented and compared using simulation.


## I. Introduction

Demands for systems with large number of actuators are increasing. Multi-DOF mechatronic systems, including robots, vehicles, and machine tools can benefit from having more actuators. Although the number of actuators increases significantly, the number of drivers and cables may not increase linearly as long as these actuators do not have to work at the same time. Using a matrix network drive system, a smaller number of drive amplifiers are connected to a larger number of actuators instead of having a dedicated drive amplifier connected to each actuator. Reducing the number of drive amplifiers will reduce the weight and cost of the system.

As shown in Fig.1, the problem of controlling a system with limited drive capacity can be converted to a problem of transmitting signals through a channel with limited capacity. Due to the limited resource nature of the drive system, it has characteristics analogous to a communication network. For example, as the input demand to the network drive system increases, instability occurs and the system performance drops sharply. In a communication network, when too much traffic is offered, the congestion sets in and performance degrades sharply. The congestion control algorithms of the communication network can be applied to the network drive system to increase the performance of the system.


Fig. 1 Control system with limited drive capacity
Sharing resources while maintaining control of each actuator can be done by switching the drivers. Fig. 2 shows the matrix architecture for sharing the drive amplifiers. Switch matrix architecture is used to drive $\mathrm{M} * \mathrm{~N}$ actuators with just $\mathrm{M}+\mathrm{N}$ drive amplifiers. Actuators that need high precision control cannot be used in this type of architecture, but systems with lots
of actuators that do not necessarily require high precision control can benefit from this architecture.
By activating the appropriate pair of switches in each column and each row of the matrix, one can turn on any actuator in the network at any instant. By turning on the switch for the $\mathrm{m}^{\text {th }}$ row and the $\mathrm{n}^{\text {th }}$ column, the actuator on the corresponding row and column, denoted $\mathrm{A}_{\mathrm{mn}}$, will be turned on.


Fig. 2 Switch matrix architecture for sharing the drive amplifiers.
In the network, each actuator must have a diode connected in series to ensure that the current can pass through each actuator in only one direction. Otherwise, more than one actuator will be turned on when a specific combination of switches are connected to the power source.

## II. System Description

The control system for the networked actuator system consists of; Controllers for each actuator; Pulse Width Modulators; Network Modulator and Matrix Switch Drive Architecture. Fig. 3 shows the block diagram of the control system. The controllers for each actuator can be designed without taking into account the network drive system. The Pulse Width Modulator changes the output signal of the controller into a constant voltage, varying width PWM signal. The central network modulator modulates the PWM signal to be suitable for the matrix drive architecture. The design of protocol to be used for the network modulator is the topic that we are interested in.


Fig. 3 Block diagram of control system for vast DOF matrix array actuators

The actuator is equivalent to the decoder of the whole system, which would transform the output of the matrix switch drive architecture into a waveform of the desired output. Typically, an actuator will possess characteristics of a low pass filter, which would change the digital signals into analog signals with certain output levels.

## III. Scheduling Strategies

The subsystem consisting of a network modulator and the matrix form drive architecture is likely to show an unstable state, depending on the average duty ratio of the input commands that are given to the actuators. Since the resources are being shared, the system cannot support the full time activation of all the actuators. Figure 4 shows the concept of instability occurring at a certain duty ratio of input commands. As the incoming demand increases, the total penalty will accumulate and may not be able to recover. This is similar to congestion occurring in the communication network when there is too much load on the network. Depending on the protocol being used, the duty ratio of the input commands at which the divergence occurs can vary.


Fig. 4 Concept of instability as the duty ratio increases

Therefore, the objective of the scheduling strategy is to maximize the average duty ratio of the PWM power input to the actuator without causing instability of the network modulator. Average duty ratio can be considered as a throughput in communication networks. Second objective is to minimize the effect of the instability on the system performance when it occurs.
Four different protocols are presented and their results are compared using simulation. Throughout the paper, the error is defined as follows. Let $D_{i j}\left(T_{k}\right)$ be the time length of the PWM pulse generated for actuator at node $i, j$ at the beginning of the $k$ th PWM sampling period, $T_{k}$. Let $K_{i j}(t)$ be the length of actual activation time of the actuator at node $i, j$ from time $T_{k}$ to $t$, where $T_{k}<t \leq T_{k+1}$.


Fig. 5 Evaluating error of an actuator at time $t$
The error of the actuator at node $i, j$ at time $\mathrm{t}, E_{i j}(t)$ is,

$$
\begin{equation*}
E_{i j}(t)=a_{i j}\left(D_{i j}\left(T_{k}\right)-K_{i j}(t)\right)^{2} \tag{1}
\end{equation*}
$$

where $a_{i j}$ is a weighting factor that is to be determined in proportion to the effect of the actuator at node $i, j$ on the total system performance.

## A. Sequential Scheduling

In this protocol, the network modulator simply allocates power to each actuator sequentially in a round robin fashion by turning on two switches, one-by-one, one in a row and the other in a column. This protocol is similar to first-come first-serve protocol, which is the very basic method of queuing.


Fig. 6 Conceptual waveform of sequential scheduling protocol (shaded area represents the output of PWM modulator, solid line represents output of network modulator)

Figure 6 shows the waveform generated by the protocol in a single PWM sampling period. The shaded area represents the PWM modulated signal, which is the input to the network modulator. The solid lines represent the actual power supplied to each actuator. The switching sequence is shown at the top row. Only a single actuator is activated at a time.
B. Demand-based Scheduling

In this protocol, the activation of the actuator is scheduled based on their demand for power. The idea is to schedule the switching such that the actuator that has a larger duty ratio is activated before the one that has smaller duty ratio at the sampling period $\mathrm{T}_{k}$. The demand of power for each actuator, $D_{i j}\left(T_{k}\right)$, is evaluated, and the actuator with the highest demand, $K$, is chosen to be activated.

$$
\begin{equation*}
[K]=\underset{i, j}{\arg } \operatorname{Max}\left[i, j \leq N .\left[D_{i j}\left(T_{k}\right)\right]\right. \tag{2}
\end{equation*}
$$

## C. Fair Scheduling

This protocol adopts the fair queuing algorithm used as a crucial component of effective congestion control in communication networks. The fair queuing suggests that the packet with the smallest size has the priority to be transmitted first. This will block illbehaved sources from dominating the bandwidth, thereby degrading the system performance. This concept is adopted, and the actuator with the smaller duty ratio is serviced before the one with the larger duty ratio. The demand of power for each actuator, $D_{i j}\left(T_{k}\right)$, is evaluated, and the actuator with the lowest demand, $K$, is chosen to be activated.

$$
\begin{equation*}
[K]=\underset{i, j}{\arg } \operatorname{Min}_{1 \leq i, j \leq N}\left[D_{i j}\left(T_{k}\right)\right] \tag{3}
\end{equation*}
$$

This protocol is opposite of the demand-based scheduling. The comparisons of the two protocols are made with simulations later on.

## D. Max demand row/ column based Scheduling

Now, instead of turning on the actuators one at a time, multiple actuators are activated. For example, by turning on a switch for the $\mathrm{m}^{\text {th }}$ row and all the switches on the column, all the actuators on $\mathrm{m}^{\text {th }}$ row will be activated. The row or column that has the maximum error is selected, and activated. The error of an actuator, $e_{i j}(t)$, is evaluated with the same method as shown in Figure 5.

$$
\begin{equation*}
e_{i j}(t)=a_{i j}\left(D_{i j}\left(T_{k}\right)-S_{i j}(t)\right)^{2} \tag{4}
\end{equation*}
$$

Total error of the $\mathrm{i}^{\text {th }}$ row, $E_{R i}$, is the sum of the demands of the actuators at the same row as given in the following equation.

$$
\begin{equation*}
E_{R i}=\sum_{j=1}^{N} e_{i j} \quad E_{C i}=\sum_{i=1}^{N} e_{i j} \tag{5}
\end{equation*}
$$

The error of the actuators that are on the same row and same column are added up and compared to find the row or column that has the maximum demand. From the evaluated demands, the row or column with the maximum demand is chosen.

$$
\begin{equation*}
[K, R / C]=\underset{i, R / C}{\arg } \operatorname{Max}\left[E_{R i}, E_{C i}\right] \tag{6}
\end{equation*}
$$

The actuators on the chosen row or column will be turned on. The total error, $\mathrm{E}_{\text {total }}$, is calculated for performance evaluation by adding up all the errors of the actuators.

$$
\begin{equation*}
E_{\text {total }}=\sum_{i=1}^{N} \sum_{j=1}^{N}\left(e_{i j}\right) \tag{7}
\end{equation*}
$$

Due to the fact that multiple actuators can be activated at the same time, the average duty ratio of the actuator that the network can service before becoming unstable will be higher.

## IV. Results and Discussion

Simulations have been done for the four different protocols described in the previous chapter. The system used for the simulation consists of 16 actuators, in a four by four matrix structure. The simulations are focused on investigating the stability behavior of the network modulator and the matrix drive architecture.

The outputs of the PWM modulators of each actuator are generated as a Poisson arrival process, where the duty ratio of each sampling period is a number of arrivals in a single sampling period. The arrival rate is the average duty ratio of the output of the PWM modulator. The outputs of the PWM modulators are the inputs to the network modulator.

Changing the arrival rate of the process simulates the change of the average duty ratio of the input commands to the network modulator. In order to simplify the simulation, the average duty ratios of the inputs are set to be equal for all the actuators. The duty ratios of the input commands to the network modulator have been changed until the total error blows out without being recovered.


Fig. 7 Performance of Sequential Scheduling

Fig. 7 shows the total error of the system using the sequential scheduling. The total error is a average of errors of all the actuators at the time instant. As the average duty ratio of the input commands to the network modulator becomes larger than $6 \%$, the instability starts to occur. Since there are sixteen actuators and only one actuator can be activated at once, $6 \%$ stability limit is a reasonable result.


Fig. 8 Performance of Demand-based Scheduling
Fig. 8 shows the performance of the demand-based scheduling. The demand-based scheduling protocol shows a better performance compared to the sequential scheduling protocol. It is due to the fact that the errors of the actuators are more evenly distributed by the demand-based protocols, than the sequential protocol. The sequential protocol can accumulate a large error at a certain actuator, thereby increasing the total error.


Fig. 9 Performance of Fair Scheduling
Fig. 9 shows the performance of the fair scheduling protocol. The total error of the fair scheduling is greater than the total error of the demand-based scheduling. But when the errors of each actuator are compared, the protocols will show a different behavior.

For the fair scheduling, the performance of the actuator with largest demand deteriorates a lot, but the performance of other actuators will not deteriorate. But for the demand-based scheduling, performance of all the actuators will deteriorate, with all the actuators having similar error level.


Fig. 10 Error of each actuator plotted when fair scheduling is used


Fig. 11 Error of each actuator plotted when demand-based scheduling is used

Simulations have been done to compare these two protocols for the case when the average duty ratio of each actuator is no longer equal. Fig. 10 shows the case when fair scheduling is used. The average duty ratio of each actuator is distributed uniformly, from lowest value of $0 \%$ to highest value of $13 \%$, with an average duty ratio of $6.5 \%$. So now, each actuator has different demand for power. Only one actuator shows error increasing exponentially, but the errors of other
actuators are maintained near zero. Fig. 11 shows the errors of all 16 actuators for the system using demandbased scheduling, with the same input condition. All the actuators show an exponential increase of error. This result shows that when fair scheduling is used, even though the total average error is higher, actuators with low demand are activated with small error. But when demand-based scheduling is used, the errors are evenly distributed among the actuators. The total average error is lower but all the actuators have large error level.


Fig. 12 Performance of Max demand row/column Protocol
Finally, the max demand row/column protocol is shown in Fig. 12. The max demand row/column protocol shows an instability occurring at an average duty ratio of around $22 \%$, for a system with sixteen actuators. Since the maximum fraction of actuators that can be activated at once is $1 / 4$, it is reasonable to see a system blow out at a duty ratio of around $25 \%$, for a system that has four actuators in the same row or column.

## V. Conclusion

Scheduling protocols for controlling matrix drive system has been developed and compared using simulation. By introducing a network modulator, power signals are treated similar to information signals to be transmitted through a network. This enabled the design of protocols similar to ones used in communication networks.

## VI. References

[1] Forouzan, Behrouz A., "Data communication and networking," McGraw-Hill Companies, Inc. New York, NY. 2001
[2] Murilo G. Coutinho, Peter M. Will, "The Intelligent Motion Surface: A hardware/software tool for the assembly of meso-scale devices,"

Proc. International Conference on Robotics and Automation, pp.1755-1760, April 1997
[3] G. Katz, L.T. Baker, G. Tu, "A MOS LSI capacitive keyboard interface chip," IEEE Journal of SolidState Circuits, , Volume 13 Issue 5, Oct 1978, pp. 561-565
[4] R. Mukherjee, T. F. Christian and R. A. Thiel, "An actuation system for the control of multiple shape memory alloy actuators," Sensors and Actuators A: Physical,Volume 55, Issues 2-3, 1996

