

# Adaptive throttle controller design based on a nonlinear vehicle model

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**Abstract**—Based on study of nonlinearities of vehicle longitudinal model and its simplification, a model reference adaptive controller for throttle control is designed using a simplified nonlinear vehicle model and its stability in the presence of unmodeled dynamics is proved using Lyapunov stability theory in this paper. Since the simplified nonlinear model is time invariant when gear is fixed and it meets requirement of time invariant for designing adaptive control system, the controller based on the simplified nonlinear model has better performance of convergence than that based on the simplified linear model. Simulation results on a full order nonlinear vehicle longitudinal model show that the adaptive controller based on simplified nonlinear model can reject disturbances that arise from parameter errors and is robust to unmodeled dynamics. Furthermore it has better performance of convergence than controller based on the simplified linear model.

## I. INTRODUCTION

Speed tracking is an important part of Adaptive Cruise Control (ACC). Many methods, such as PID and LQ, have been used and some achievements have been obtained [1]-[3]. But modeling errors such as unmodeled dynamics, parameter errors, etc., have not been fully taken into account when designing controllers. In fact variations of vehicle and environment parameters such as vehicle mass, slope, aerodynamic drag force and rolling resistance [4] significantly affect controller performances and even make system unstable. In order to achieve good performance in the presence of modeling errors,  $H_\infty$  theory and robust adaptive theory are used [5]-[8]. But there are some practical difficulties when using these kinds of methods. The linear model and bounds of model error for designing  $H_\infty$  controller are difficult to determine. In [8] a first order model is obtained by Taylor linearization method. Although the

Manuscript received September 20, 2003.

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model is very simple, its parameters change with operating points. Actually this simple linear model is time varying, which is in conflict with requirement of time invariant for designing adaptive control system.

In this paper, a simplified nonlinear model is used to describe vehicle longitudinal dynamics. It is time invariant when gear is fixed and it meets requirement of time invariant for designing adaptive control system. Using this simplified nonlinear model, a model reference adaptive controller for throttle control is designed. Theoretical analysis and simulation results show that the adaptive controller based on the simplified nonlinear model can reject disturbances that arise due to parameter errors, unmodeled dynamics, and has better performance of convergence than that based on the simplified linear model.

## II. LONGITUDINAL VEHICLE MODEL

Figure 1 shows the basic blocks and inputs, outputs of a longitudinal vehicle model [9]. The output of the engine subsystem is engine torque that is a nonlinear function of throttle angle and engine speed. A first order system is used to describe the dynamics of engine. The transmission subsystem is responsible for transferring engine torque to drivetrain. It is an automatic transmission with hydraulic torque coupling and four forward transmission gears. The gear state is a nonlinear function of throttle angle and vehicle speed. The input of the drivetrain subsystem is drive torque and its outputs are vehicle speed, acceleration or deceleration which are affected by road condition, aerodynamic drag and vehicle mass. In this paper only longitudinal control using throttle control is considered so braking torque is set to zero.

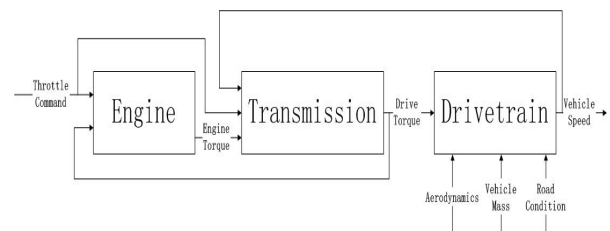


Fig. 1. Vehicle longitudinal model

Since the above nonlinear model is very complicated, it is difficult to use this nonlinear model to directly design a throttle controller. Taylor linearization method is used to simplify the above nonlinear vehicle longitudinal model [8] and high order error is neglected. The following linear model is obtained.

$$\frac{\bar{V}(s)}{\bar{\theta}(s)} = \frac{b_0}{(s+p_1)(s+p_2)(s+p_3)} \quad (1)$$

where

$$\bar{V} = V - V_0$$

$$\bar{\theta} = \theta - \theta_0$$

$V$  is vehicle speed

$\theta$  is throttle angle

$(V_0, \theta_0)$  is operating points

$b_0, p_i, i=1,2,3$  are functions of the operating points

For all operating points, the following is obtained

$$0 < p_3 < 0.4 \text{ and } \left| \frac{\text{Re}(p_i)}{p_3} \right|_{\min} > 17, i=1,2$$

So  $p_3$  is the dominant pole and the fast modes can be neglected, leading to a simple model of

$$\frac{\bar{V}(s)}{\bar{\theta}(s)} = \frac{b_0}{s+p_3} \quad (2)$$

This first order model is very simple, but its parameters change with the operating points. For ACC system, vehicle speed is always modified according to surroundings. This model is actually time varying. Furthermore the high order error is neglected during linearization procedure. Model errors of this simple linear model include not only unmodeled dynamics and parameter errors but also high order error. Normally, the bigger the model errors are, the more serious the negative effects to the control system are.

Considering the above problems, a simplified nonlinear model is used to describe the vehicle longitudinal dynamics in this paper:

$$M\dot{V} = \frac{R}{r}T_e - C_aV^2 - F_f \quad (3)$$

where

$M$  is total vehicle inertia

$R$  is torque ratio of transmission

$r$  is radius of wheel

$C_a$  is aerodynamic drag coefficient

$F_f$  is the resistance force of road

$T_e$  is engine torque.

Compared with the simplified linear model, it has the following advantages:

1) When gear is fixed, it is time invariant. Although vehicle speed is modified constantly according to surroundings for ACC system, gear shifting does not happen often. So parameters of this simplified nonlinear model are constant for a long time and this model acts as a time invariant system. Even if gear shifting occurs, the parameter errors due to gear shifting can be treated as initial parameter errors and will be rejected due to tuning ability of the adaptive controller.

2) The dynamics of powertrain is neglected when setting up the simplified nonlinear model. Main model errors of this nonlinear model are unmodeled dynamics and parameter errors.

It is well known that the design techniques for adaptive systems are based on theory for time invariant plants, so the simplified nonlinear model is more suitable for designing adaptive controller. In the next section, these two simplified models are used to design adaptive controllers for throttle control respectively.

### III. ADAPTIVE CONTROLLER DESIGN

In this section, the two simplified model are used to design the adaptive controllers respectively. In order to distinguish between these two controllers, adaptive controller based on the simplified linear model is called controller A and the other is called controller B.

Based on the linear model, controller A is described as follows [8]:

$$\begin{aligned} \theta &= f(V_d) - k_1(V - V_d) + k_2 \\ \dot{k}_i &= \begin{cases} 0, k_i \geq k_{ui}, \varepsilon X_i > 0 \\ 0, k_i \leq k_{li}, \varepsilon X_i < 0, i=1,2 \\ \gamma_i \varepsilon X_i, otherwise \end{cases} \end{aligned} \quad (4)$$

where

$V_d$  is the desired vehicle speed

$k_i$  is estimated parameter

$\theta_0 = f(V_0)$  describes the relationship of  $\theta_0$  and  $V_0$

$k_{ui}$  and  $k_{li}$  are respectively upper bound and lower bound of  $k_i$

$\varepsilon$  is the normalized error signal

$X_1 = V - V_d$  and  $X_2 = 1$  are measured signals

Using the first order linear model an adaptive controller can be obtained, but it is not proper, since this linear model is time varying which is in conflict with requirement of time invariant for designing adaptive control system. To overcome this shortcoming, the simplified nonlinear model is used to design adaptive controller for throttle control in the following.

The objective of throttle control is to make vehicle track the reference speed  $V_m$  described by

$$\dot{V}_m + a_m V_m = a_m V_d \quad (5)$$

where

$a_m > 0$  is chosen based on the requirements of system response speed, riding quality, etc.

The structure of the control system is shown in Fig. 2.

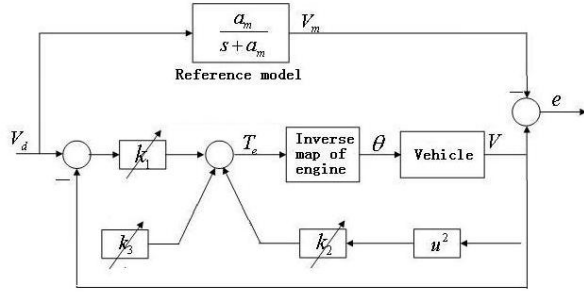


Fig. 2. Structure of the control system

The desired engine torque is

$$T_e = \mathbf{k}^T \mathbf{W} \quad (6)$$

where

$$\mathbf{k}^T = [k_1, k_2, k_3] \text{ is estimated parameter}$$

$$\mathbf{W}^T = [V_d - V, V^2, 1] \text{ is measured signal.}$$

Actual model parameter is

$$\mathbf{k}^* = \left[ \frac{a_m M r}{R}, \frac{r C_a}{R}, \frac{F_f r}{R} \right]^T$$

Using the inverse of engine torque map, the throttle angle is:

$$\theta = \text{MAP}^{-1}(T_e, \omega_e) \quad (7)$$

where

$$\omega_e \text{ is engine speed}$$

$\text{MAP}^{-1}$  is the inverse of engine torque map

A new error signal is defined as

$$\varepsilon = e - \frac{\varepsilon m^2}{s + a_m} \quad (8)$$

where

$$m^2 = m_s$$

$$\dot{m}_s = -\delta_0 m_s + T_e^2$$

$$m_s(0) = 0$$

$\delta_0$  is determined by the poles of the transfer function of powertrain.

$$e = V - V_m \text{ is the tracking error}$$

The adaptive law is

$$\dot{\mathbf{k}} = -\Gamma \varepsilon \mathbf{W} \quad (9)$$

where

$$\Gamma = \begin{bmatrix} \gamma_1 & 0 & 0 \\ 0 & \gamma_2 & 0 \\ 0 & 0 & \gamma_3 \end{bmatrix}$$

$$\gamma_i > 0, i = 1, 2, 3 \text{ is adaptive gain}$$

For robustness, the adaptive laws can be modified to

$$\dot{k}_i = \begin{cases} 0, k_i > k_{ui}, \dot{k}_i > 0 \\ 0, k_i < k_{li}, \dot{k}_i < 0 \\ -\gamma_i \varepsilon W_i, \text{otherwise} \end{cases} \quad (10)$$

where

$$k_{ui} \text{ and } k_{li} \text{ are upper bound and lower bound of } k_i$$

$$W_i \text{ is the element of } \mathbf{W}.$$

#### IV. ROBUSTNESS ANALYSIS OF CONTROLLER B

In the last section, model reference adaptive controllers for throttle control are designed based on simplified linear and nonlinear model respectively. The simplified nonlinear model is time invariant when gear is fixed, but the dynamics of powertrain are neglected. In this section, Lyapunov stability theory is used to prove stability of the control system in the presence of unmodeled dynamics.

Taking the dynamics of powertrain into account, when gear is fixed, the nonlinear model is

$$M \dot{V} = \frac{R}{r} T_e G(s) - C_a V^2 - F_f \quad (11)$$

where

$$G(s) = \frac{M(s)}{N(s)} \text{ is the transfer function of powertrain}$$

The model error due to unmodeled dynamics is defined as

$$\Delta(s) = \frac{M(s) - N(s)}{N(s)}$$

Substituting (12) into (11), so that

$$M\dot{V} = \frac{R}{r}T_e(1 + \Delta(s)) - C_aV^2 - F_f \quad (13)$$

Substituting (5) into (13), so that

$$\dot{e} + a_m e = \frac{R}{Mr}\Phi^T W + \eta \quad (14)$$

where

$$\Phi^T = k - k^* \text{ is the parameter error}$$

The disturbance  $\eta$  is defined as

$$\eta = \frac{R}{Mr}T_e\Delta(s) \quad (15)$$

The following Lyapunov function is chosen.

$$V = \varepsilon^2 + \frac{R}{2Mr}\Phi^T \Gamma^{-1} \Phi \quad (16)$$

The time derivative of  $V$  is

$$\dot{V} = \varepsilon \dot{\varepsilon} + \frac{R}{Mr}\Phi^T \Gamma^{-1} \dot{\Phi} \quad (17)$$

By the definition of the new error signal described by (8) therefore

$$\dot{\varepsilon} = \dot{e} + a_m e - \varepsilon m^2 - a_m \varepsilon \quad (18)$$

The parameters are time invariant when gear is fixed, so that

$$\dot{\Phi} = \dot{k} \quad (19)$$

Substituting (18), (14) and (19) into (17), so that

$$\begin{aligned} \dot{V} = & -\varepsilon^2 m^2 - a_m \varepsilon^2 + \varepsilon \left( \frac{R}{Mr}\Phi^T W + \eta \right) \\ & + \frac{R}{Mr}\Phi^T \Gamma^{-1} \dot{k} \end{aligned} \quad (20)$$

Substituting the adaptive law (9) into (20), so that

$$\dot{V} = -\varepsilon^2 m^2 - a_m \varepsilon^2 + \varepsilon \eta \quad (21)$$

Then the following is obtained

$$\dot{V} \leq -\varepsilon^2 m^2 - a_m \varepsilon^2 + |\varepsilon \eta| \quad (22)$$

Using

$$-\left(0.5|\varepsilon m| - \left|\frac{\eta}{m}\right|\right)^2 = -\frac{\varepsilon^2 m^2}{4} + |\varepsilon \eta| - \frac{\eta^2}{m^2}$$

the following is obtained

$$\dot{V} \leq -\frac{3\varepsilon^2 m^2}{4} - \left(\frac{|\varepsilon m|}{2} - \left|\frac{\eta}{m}\right|\right)^2 + \frac{\eta^2}{m^2} \quad (23)$$

From [10] the following theorem is known.

Let

$$y = H(s)[u]$$

where

$$H(s) \text{ is a strictly proper transfer function.} \quad (14)$$

If  $H(s)$  is analytic in  $\text{Re}[s] \geq -0.5\delta$  for some  $\delta > 0$

then

$$|y_t| \leq \frac{1}{\sqrt{p}} \|(s+p)H(s)\|_\infty^\delta \|u_t\|_2^\delta \quad (15)$$

where

$$p \geq \delta \text{ is an arbitrary constant.}$$

It is difficult to determine the model errors,  $\Delta(s)$ , due to the unmodeled dynamics of powertrain, but simply by experience the range of the poles of  $\Delta(s)$  can be

determined. Choosing  $\delta_0$  that makes  $\Delta(s)$  analytic in

$$\text{Re}[s] \geq -0.5\delta_0$$

then according to the above theorem, the following is obtained

$$|\eta_t| \leq \frac{1}{\sqrt{\delta_0}} \left\| \frac{R}{Mr}(s + \delta_0)\Delta(s) \right\|_\infty^{\delta_0} \|T_{et}\|_2^{\delta_0} \quad (24)$$

And then

$$\left| \frac{\eta}{m} \right| \leq \frac{1}{\sqrt{\delta_0}} \left\| \frac{R}{Mr}(s + \delta_0)\Delta(s) \right\|_\infty^{\delta_0} = \sqrt{C} \quad (25)$$

where

$C$  is a constant.

Substituting (25) into (23), so that

$$\dot{V} \leq -\frac{3\varepsilon^2 m^2}{4} - \left(\frac{|\varepsilon m|}{2} - \left|\frac{\eta}{m}\right|\right)^2 + C \quad (26)$$

From (26) we know that if  $V$  is greater than a certain

constant, then  $\dot{V} < 0$ . This property of  $V$  implies that the control system is stable and  $e, \varepsilon, \Phi \in L_\infty$ .

## V. SIMULATION RESULTS

In the last section, stability of controller B in the presence of unmodeled dynamics is proved using Lyapunov stability

theory. The analysis of section 2 shows that the simplified nonlinear model is more suitable for designing adaptive controller. In order to test the performance of controller A and controller B, they are both applied in the full order nonlinear vehicle longitudinal model [9] in this section. The initial control parameters can be chosen arbitrarily between the upper and lower bounds. The parameters of the controllers are chosen as follows in the simulation.

1) Adaptive controller A

$$a_m = 0.5$$

$$\mathbf{F} = \text{diag}[3 \quad 0.5]$$

$$k_{u1} = 4, k_{l1} = -100$$

$$k_{u2} = 60, k_{l2} = -60$$

2) Adaptive controller B

$$a_m = 0.5$$

$$\mathbf{F} = \text{diag}[2.5 \quad 0.0005 \quad 5]$$

$$k_{u1} = 193, k_{l1} = 35$$

$$k_{u2} = 0.043, k_{l2} = 0.0094$$

$$k_{u3} = 314.3, k_{l3} = -32$$

In order to test the performance of the controllers, two acceleration scenarios and a deceleration scenario are used in simulation. One acceleration scenario is at low speed and the other is at high speed. The desired and reference vehicle speed are shown in Fig. 3.

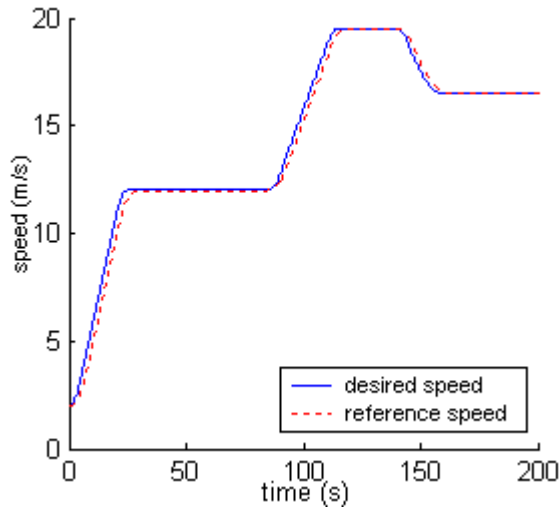
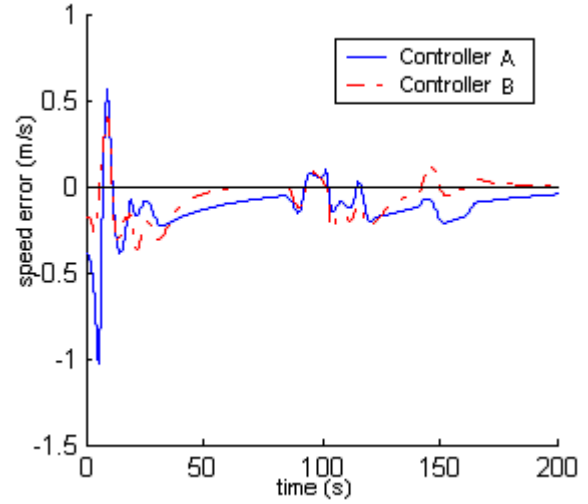
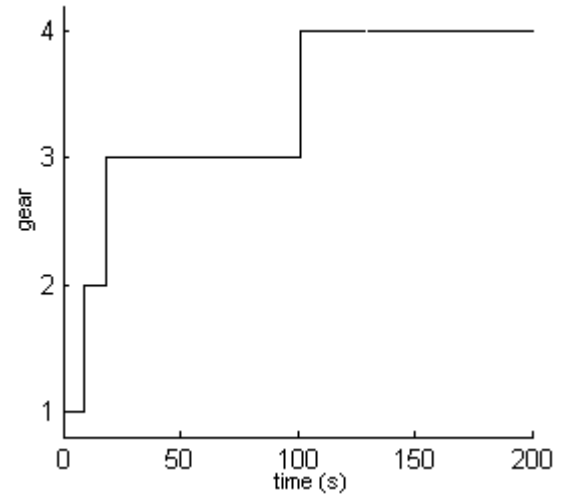


Fig. 3. Desired and reference speed

And the results are shown in Fig. 4.



(a) Speed tracking errors of both controllers



(b) Gear state

Fig. 4. Simulation results

From  $t=0$  s to  $t=20$  s, the desired speed increases to 12 m/s with an acceleration of  $0.5 \text{ m/s}^2$ . From the simulation results, tracking errors are large. One reason is that initial parameters of controllers are chosen arbitrarily, which results in large parameter errors. Another reason is that gear shifting is frequent during this period, which results in sharp change of real parameters. When the desired speed is stable, tracking error of controller B converges to 0 more rapidly than that of controller A.

From  $t=90$  s to  $t=100$  s, the desired speed increases to 19.5 m/s with an acceleration of  $0.3 \text{ m/s}^2$ . In this period, only one shift happens, and due to the accommodation ability, the parameter errors become smaller, so the tracking errors become smaller too. It is the same as the last accelerating scenario. Tracking error of controller B converges to 0 more rapidly than that of controller A when the desired speed is stable.

From  $t=135$  s to  $t=155$  s, the desired speed decreases to 16.5 m/s with a deceleration of  $-0.3$  m/s<sup>2</sup>. No shift occurs in this period. From the simulation results, performance of convergence and tracking ability of controller B is better than that of controller A.

The tracking error bounds of controller A of three scenarios are 1 m/s, 0.2 m/s and 0.2 m/s respectively and that of controller B are 0.45 m/s, 0.2 m/s and 0.1 m/s respectively. From the simulation results, It is concluded that throttle control system using controller B converges to desired speed more quickly and has smaller tracking errors, which coincides with the conclusion in section 2 that the simplified nonlinear model is more suitable for designing adaptive controller.

## VI. CONCLUSION

In this paper, model reference adaptive controllers are designed for throttle control. Before designing the controller, two simple vehicle models, one is a first order system and the other is a nonlinear one, are analyzed and the following characteristics are obtained.

- 1) Parameters of the simplified linear model are functions of operating points, so in fact it is time varying. The main model errors are unmodeled dynamics, parameter errors and high order error, which is neglected in Taylor linearization.
- 2) Simplified nonlinear model is time invariant when gear is fixed. Its main model errors are unmodeled dynamics and parameter errors.

Since adaptive control is mainly used to time invariant or slowly time-varying plants, and for ACC system, vehicle speed should be modified frequently to keep safe distance to front vehicle, it is concluded that the simplified nonlinear model is more suitable for designing adaptive controller.

Based on the simplified nonlinear model, a model reference adaptive controller for throttle control is designed. Theoretical analysis and simulation results show that the adaptive throttle controller has the following advantages:

- 1) The controller can reject disturbance that arise due to parameter errors because of the learning capacity.
- 2) The control system is stable in the presence of unmodeled dynamics.
- 3) Since the simplified nonlinear model is more suitable for designing adaptive controller, controller B has better performance of convergence and tracking ability than controller A.

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