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Closed-loop stochastic model predictive control in a receding horizon implementation on a continuous polymerization reactor example

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***Abstract*— In this paper we evaluate closed-loop stochastic model predictive control techniques on a nonlinear high density polyethylene fluidized bed example. Closed-loop MPC is a control strategy in which one optimizes feedforward signals while maintaining back-off to inequality constraints on the process variables. This back-off is kept minimal by using so-called closed-loop model prediction in which control plays a central role. The objective in this paper is to illustrate these novel techniques on a realistic simulator of an industrial size HDPE plant by enabling a grade change under persistent disturbances in the feeds.**

Keywords. Closed-loop MPC, stochastic disturbances, inequality constraints, polymerization, grade changes, nonlinear process control.

I. Introduction

The importance of plant operation within constraints has in the past twenty years led to the success of model predictive control (MPC), [16]. However, MPC does *not* provide a systematic way of dealing with (stochastic) disturbances. In [11] it is proposed to decompose the problem into an optimal Gaussian estimation and a deterministic prediction problem, which are solved as separate optimization problems in a receding horizon implementation. This view has become a main line of MPC research which considers stochastics in the past but not in the future. Three limitations of such open-loop MPC are that 1) no back-off is kept to the constraints, 2) there are no possibilities of shaping the process sensitivity, a basic characteristics of feedback design methods and 3) one falsely assumes the validity of the certainty equivalence property in the case of inequality constraints. In previous papers, [5, 6], we have formulated and solved a novel closed-loop prediction problem that tackles the first two inconsistencies related to the back-of and sensitivity optimization. The key issue is the decomposition of the predictive control problem in a feedforward trajectory and a feedback controller optimization. The idea is the following. Due to the back-off to the constraints, the controller retains its linear behavior such that a meaningful sensitivity function can be defined. In turn, this sensitivity function is used to choose a controller that minimizes that same back-off as in a bootstrap technique and consequently using feedforward we can optimize plant transitions while the inequality constraints are guaranteed not to be violated in the closed-loop. In a recent contribution, [7], we have showed that for any optimal controller of the closed-loop MPC problem (CLMPC), there exists an equivalent finite horizon LQG controller (FHLQG) which has the same performance. As it turns out, the optimal CLMPC strategy does share the separation property (in contrast to the certainty equivalence property), hence it can be decomposed in an optimal estimator and an optimal inequality constrained stochastic prediction problem. This has allowed us to deduce the receding horizon implementation for (CLMPC), opening the way for application to continuous processes. The purpose of this paper is to illustrate our control strategy via an implementation on a nonlinear continuous High Density Poly-Ethylene (HDPE) fluidized bed reactor having approximately 50 states and 2000 algebraic equations. The model is based on the dynamics presented in [2] and [13][1]. In several steps towards the implementation, detail is skipped due to space limitations. These details will be made available in [8].

II. Closed-loop model predictive control

Consider the discrete time-varying stochastic system

$$\begin{pmatrix} x_{k+1}(\xi) \\ z_k(\xi) \\ y_k(\xi) \end{pmatrix} = \begin{pmatrix} A_k & G_k & B_k \\ C_k^z & O & D_k^z \\ C_k & F_k & O \end{pmatrix} \begin{pmatrix} x_k(\xi) \\ w_k(\xi) \\ u_k(\xi) \end{pmatrix} \quad (1)$$

where ξ is a generic element of some sample space Ω connected to a Gaussian probability measure, $w_k(\xi)$ is a resulting white noise sequence with variance matrix W_k and with the property $G_k W_k F_k^T = 0$ (process and measurement noise are independent). Let us 'lift' this system, see also [4], over a time horizon of n samples. In order to represent this process slightly more compact, define the following stochastic processes

$$\mathbf{y}_k(\xi) = \begin{pmatrix} y_k(\xi) \\ \vdots \\ y_{k+n}(\xi) \end{pmatrix}, \mathbf{z}_k(\xi) = \begin{pmatrix} z_k(\xi) \\ \vdots \\ z_{k+n}(\xi) \end{pmatrix} \quad (2)$$

and so on, representing the part from each signal $\mathbf{u}, \mathbf{w}, \mathbf{y}, \mathbf{z}$ from sample k to sample $k+n$. The two stochastic processes $\mathbf{y}$ and $\mathbf{z}$ are the measured and performance output processes

[1]The model was implemented and extended by R.L. Tousain, (Delft University of Technology, The Netherlands), [18] and W. van Brempt, (IPCOS, Belgium) within the IMPACT project. We greatly appreciate that they have made their modelling efforts available to us.

respectively ($\mathbf{z}$ contains any variable which either appears in the objective function or in the constraints)

$$\begin{aligned}\mathbf{y}_k(\xi) &= G_{yx}x_k(\xi) + G_{yu}\mathbf{u}_k(\xi) + G_{yw}\mathbf{w}_k(\xi)\\ \mathbf{z}_k(\xi) &= G_{zx}x_k(\xi) + G_{zu}\mathbf{u}_k(\xi) + G_{zw}\mathbf{w}_k(\xi).\end{aligned}$$

The resulting initial condition $x_k(\xi)$ and the disturbance $\mathbf{w}_k(\xi)$ are uncorrelated (Gaussian) stochastic variables. Note that we assume that there is no feedthrough from the inputs to the measured outputs. Next to these equations, we define the deterministic reference signals

$$\begin{aligned}\mathbf{y}^r_k &= G_{yx}x^r_k + G_{yu}\mathbf{u}^r_k + G_{yw}\mathbf{w}^r_k\\ \mathbf{z}^r_k &= G_{zx}x^r_k + G_{zu}\mathbf{u}^r_k + G_{zw}\mathbf{w}^r_k.\end{aligned}$$

Consider the finite horizon observer, in its one step ahead predictor form, which consists of time varying dynamics [17], (take index relative to k)

$$\begin{aligned}\begin{pmatrix}\hat{x}_0(\xi)\\ \hat{x}_1(\xi)\\ \vdots\\ \hat{x}_n(\xi)\end{pmatrix} &= \begin{pmatrix} I\\ \Phi^e_{1,0}\\ \vdots\\ \Phi^e_{n,0}\end{pmatrix}\hat{x}_0(\xi) \qquad (3)\\ &+ \begin{pmatrix} O & O & \cdots & O\\ B_0 & O & \cdots & O\\ \vdots & \vdots & \ddots & \vdots\\ \Phi^e_{n,1}B_0 & \Phi^e_{n,2}B_1 & \cdots & O\end{pmatrix}\begin{pmatrix}u_0(\xi)\\ u_1(\xi)\\ \vdots\\ u_n(\xi)\end{pmatrix}\\ &+ \begin{pmatrix} O & O & \cdots & O\\ N_0 & O & \cdots & O\\ \vdots & \vdots & \ddots & \vdots\\ \Phi^e_{n,1}N_0 & \Phi^e_{n,2}N_1 & \cdots & O\end{pmatrix}\begin{pmatrix}y_0(\xi)\\ y_1(\xi)\\ \vdots\\ y_n(\xi)\end{pmatrix}\end{aligned}$$

where the transition matrix $\Phi^e_{k,j}$ for the observer system mapping $\hat{x}_j$ to $\hat{x}_k$ is given for $k > j$ by

$$\Phi^e_{k,j} = A^e_{k-1}A^e_{k-2}\cdots A^e_j, \quad \Phi^e_{j,j} = I, \quad A^e_k = A_k - N_kC_k$$

where N_k is the Kalman predictor gain. Suppose that the observer, (3) is put in its error dynamics form, where the state of the observer is the error between the state and its estimate $e_k(\xi) := x_k(\xi) - \hat{x}_k(\xi)$. Note that in some literature the notation $\hat{x}_{k|k-1}$ is customary to denote the estimate of the Kalman predictor $\hat{x}_k$. This dynamical system is given in recursive form by

$$e_{k+1}(\xi) = \underbrace{(A_k - N_kC_k)}_{A^e_k}e_k(\xi) + \underbrace{(G_k - N_kF_k)}_{G^e_k}w_k(\xi)$$

and put in its lifted form $\mathbf{e}_0(\xi) = G_{ee}e_0(\xi) + G_{ew}\mathbf{w}_0(\xi)$ in terms of (Φ^e_k, G^e_k)

$$\begin{aligned}\begin{pmatrix}e_0(\xi)\\ e_1(\xi)\\ \vdots\\ e_n(\xi)\end{pmatrix} &= \begin{pmatrix} I\\ \Phi^e_{1,0}\\ \vdots\\ \Phi^e_{n,0}\end{pmatrix}e_0(\xi)\\ &+ \begin{pmatrix} O & O & \cdots & O\\ G^e_0 & O & \cdots & O\\ \vdots & \vdots & \ddots & \vdots\\ \Phi^e_{n,1}G^e_0 & \Phi^e_{n,2}G^e_1 & \cdots & O\end{pmatrix}\begin{pmatrix}w_0(\xi)\\ w_1(\xi)\\ \vdots\\ w_n(\xi)\end{pmatrix}.\end{aligned}$$

The corresponding innovation sequence, [9], is given by

$$v_k(\xi) := y_k(\xi) - \hat{y}_k(\xi) = C_ke_k(\xi) + F_kw_k(\xi)$$

if we also put the innovations sequence in its lifted form we obtain

$$\mathbf{v}_0(\xi) = G_{ve}e_0(\xi) + G_{vw}\mathbf{w}_0(\xi) \qquad (4)$$

and in terms of the matrices $(\Phi^e_k, G^e_k, C_k, F_k)$

$$\begin{aligned}\begin{pmatrix}v_0(\xi)\\ v_1(\xi)\\ \vdots\\ v_n(\xi)\end{pmatrix} &= \begin{pmatrix} C_0\\ C_1\Phi^e_{1,0}\\ \vdots\\ C_n\Phi^e_{n,0}\end{pmatrix}e_0(\xi)\\ &+ \begin{pmatrix} F_0 & O & \cdots & O\\ C_1G^e_0 & O & \cdots & O\\ \vdots & \vdots & \ddots & \vdots\\ C_n\Phi^e_{n,1}G^e_0 & C_n\Phi^e_{n,2}G^e_1 & \cdots & F_n\end{pmatrix}\begin{pmatrix}w_0(\xi)\\ w_1(\xi)\\ \vdots\\ w_n(\xi)\end{pmatrix}.\end{aligned} \qquad (5)$$

Then, introduce a feedback of the form

$$\mathbf{u}_k(\xi) - \mathbf{u}^r = L_k(\hat{x}_k(\xi) - \hat{x}^r_k) + K(\mathbf{v}_k(\xi) - \mathbf{v}^r_k) \qquad (6)$$

for some non-anticipating controller $K \in \mathbf{K}$ where

$$\mathbf{K} = \{\sum_{i=1}^{n}\sum_{j=1}^{i} E_iK^{ij}E_j^T : \ K^{ij} \in \mathbb{R}^{n_u \times n_y}\}$$

where $E_i = (O, \ldots, O, I, O, \ldots, O)^T$. This leads to controllers of the form

$$K_0 = \begin{pmatrix} K^{11} & O & \cdots & O\\ K^{21} & K^{22} & \cdots & O\\ \vdots & \vdots & \ddots & \vdots\\ K^{n1} & K^{n2} & \cdots & K^{nn}\end{pmatrix}.$$

It is immediate that

$$\begin{aligned}\mathbf{z}_k(\xi) &= G^k_{zx}x_k(\xi) + G^k_{zu}\mathbf{u}_k(\xi) + G^k_{zw}\mathbf{w}_k(\xi) \qquad (7)\\ &= G^k_{zx}\hat{x}_k(\xi) + G^k_{zx}e_k(\xi) + G^k_{zu}\mathbf{u}_k(\xi) + G^k_{zw}\mathbf{w}_k(\xi).\end{aligned}$$

Thus many ingredients for a recursive implementation are there, however, to guarantee optimality, the variance matrices and the feedback law must also be computed recursively. From equations (7) and (6) we know that $\mathbf{z}_k(\xi)$ in closed-loop is some function of $\hat{x}_k(\xi), e_k(\xi), \mathbf{w}_k(\xi)$. Since $\hat{x}_k(\xi), e_k(\xi)$ are independent of $\mathbf{w}_k(\xi)$ it follows that we only need to keep track of the joint variance matrix V_k of $\hat{x}_k(\xi)$ and $e_k(\xi)$. Observe that the actual controls applied at each instant are given by L^1_k and K^{11}_k which are the first n_u rows of the control law L_k and K_k respectively (assume w.l.o.g. that the references trajectories are zero)

$$u_k(\xi) = L^1_k\hat{x}_k(\xi) + K^{11}_kv_k(\xi)$$

Using [10] as reference we can directly write

$$\begin{aligned}\begin{pmatrix}\hat{x}_{k+1}(\xi)\\ e_{k+1}(\xi)\end{pmatrix} &= \begin{pmatrix} A_k + B_kL^1_k & (N_k + B_kK^{11}_k)C_k\\ O & A_k - N_kC_k\end{pmatrix}\begin{pmatrix}\hat{x}_k(\xi)\\ e_k(\xi)\end{pmatrix}\\ &+ \begin{pmatrix}(N_k + B_kK^{11}_k)F_k\\ G_k - N_kF_k\end{pmatrix}w_k(\xi)\end{aligned}$$

such that the joint variance matrix is recursively given by

$$V_{k+1} = \begin{pmatrix} A_k + B_k L_k^1 & (N_k + B_k K_k^{11})C_k \\ O & A_k - N_k C_k \end{pmatrix} V_k \begin{pmatrix} \star & \star \\ \star & \star \end{pmatrix}^T + \begin{pmatrix} (N_k + B_k K_k^{11})F_k \\ G_k - N_k F_k \end{pmatrix} W_k \begin{pmatrix} \star \\ \star \end{pmatrix}^T, \; V_0 = \begin{pmatrix} O & O \\ O & P_0 \end{pmatrix}.$$

Since $e_k(\xi) \perp \hat{x}_k(\xi)$ for any k, the joint variance matrix V_k is block-diagonal for each k by construction, therefore the variance matrices can be constructed efficiently by the following Riccati recursions for the estimation error

$$P_{k+1}^e = A_k P_k^e A_k^T - N_k(C_k P_k^e C_k^T + F_k W_k F_k^T)N_k^T + G_k W_k G_k^T$$

with boundary condition $P_e^0 = P_0$ and next, given the recursion for the estimation error, one can obtain the recursion for the state-estimate

$$P_{k+1}^{\hat{x}} = (A_k + B_k L_k^1)P_k^{\hat{x}}(A_k + B_k L_k^1)^T + (N_k + B_k K_k^{11})(C_k P_k^e C_k^T + F_k W_k F_k^T)(N_k + B_k K_k^{11})^T$$

with boundary condition $P_0^{\hat{x}} = O$. L_k^1 and K_k^{11} are given externally in every cycle by the solution of the control problem, furthermore, the Kalman gain is given by

$$N_k = A_k P_k^e C_k^T (C_k P_k^e C_k^T + F_k W_k F_k^T)^{-1}$$

The factored variance matrix of the initial condition and disturbances are

$$P_k^x = E x_k(\xi) x_k(\xi)^T = P_k^{\hat{x}} + P_k^e = F_x F_x^T, \quad W_k = F_w F_w^T.$$

Then with everything as above, the closed-loop MPC problem is defined as

$$\textbf{(CLMPC)} \quad \begin{array}{cc} \min\limits_{\mathbf{z}_k^r \in \mathbb{R}^{n_z}, K \in \mathbf{K}_0} & f(\mathbf{z}_k^r) \\ r\sqrt{h_j^T Z_k(K) h_j} + h_j^T \mathbf{z}^r \leq g_j & \end{array} \tag{8}$$

where $Z_k = E(\mathbf{z}_k(\xi) - \mathbf{z}_k^r)(\mathbf{z}_k(\xi) - \mathbf{z}_k^r)^T$. The constraints above follow from a set of inequality constraints

$$h_j^T \mathbf{z} \leq g_j, \; j = 1, \ldots, m.$$

which the process must obey. The term $r\sqrt{h_j^T Z_k(K) h_j}$ appearing in (8) is added as back-off to these constraints to avoid frequent violation. This problem is a convex optimization problem that can be solved for the global optimum, see for background [5, 6, 7], using a solver for second-order cone problems, see [12].

III. Application to nonlinear systems

The HDPE plant as described in this paper is a smooth nonlinear regular set of differential algebraic equations (DAE) of the form

$$\dot{\bar{x}} = f(\bar{x}, \bar{m}, \bar{u}, \bar{w}), \quad g(\bar{x}, \bar{m}, \bar{u}, \bar{w}) = 0, \quad \bar{x}(t_0) = \bar{x}_0$$
$$\bar{y} = C_y \bar{m}, \quad \bar{z} = C_z \bar{m} \tag{9}$$

where m is a vector of algebraic variables. Let

$$(\bar{x}(t), \bar{m}(t), \bar{u}_k, \bar{w}_k), \quad \forall t \in [t_k, t_{k+1})$$

be the solution to (9) over the interval $[t_k, t_{k+1})$, where we use a bar to denote the solutions to the nonlinear system. The inputs are taken constant over the sample intervals. Let

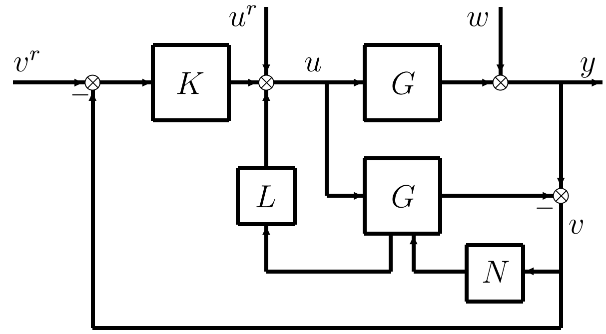


Fig. 1. Closed-loop predictive control using innovations and state feedback.

us sample these trajectories on the control sample time t_s, and define the discrete time processes $\bar{x}_k = \bar{x}(t_k), \bar{y}_k = \bar{y}(t_k), \bar{z}_k = \bar{z}(t_k)$. As before we stack these signals (boldfaced) and suppose we are in the l^{th} iteration of solving some dynamic optimization for which we seek an update on the inputs and performance outputs (approximate)

$$\bar{\mathbf{u}}^{l+1} = \bar{\mathbf{u}}^l + \mathbf{u}^r \;\rightarrow\; \bar{\mathbf{z}}^{l+1} \approx \bar{\mathbf{z}}^l + \mathbf{z}^r \tag{10}$$

then we measure the sensitivity of the performance with respect to these inputs perturbation using our linear time-varying models, see also [15], where the time varying dynamics are given by

$$A(t) = \frac{\partial f}{\partial x} - \frac{\partial f}{\partial m}\left(\frac{\partial g}{\partial m}\right)^{-1}\frac{\partial g}{\partial x},$$
$$B(t) = \frac{\partial f}{\partial u} - \frac{\partial f}{\partial m}\left(\frac{\partial g}{\partial m}\right)^{-1}\frac{\partial g}{\partial u},$$
$$G(t) = \frac{\partial f}{\partial w} - \frac{\partial f}{\partial m}\left(\frac{\partial g}{\partial m}\right)^{-1}\frac{\partial g}{\partial w}$$

and the output matrices are given by

$$C^y(t) = -C_y\left(\frac{\partial g}{\partial m}\right)^{-1}\frac{\partial g}{\partial x}, \quad F(t) = -C_y\left(\frac{\partial g}{\partial m}\right)^{-1}\frac{\partial g}{\partial w}$$
$$C^z(t) = -C_z\left(\frac{\partial g}{\partial m}\right)^{-1}\frac{\partial g}{\partial x}, \quad D^z(t) = -C_z\left(\frac{\partial g}{\partial m}\right)^{-1}\frac{\partial g}{\partial u}.$$

The inverse of $\partial g/\partial m$ exists by assumed regularity and smoothness of the DAE system. The discrete time system dynamics in (1) are then obtained by sampling these system matrices and converting them to discrete time. The approximation in sampling the matrices is usually quite good due to the small changes in system dynamics over the time increment t_s, while no approximation is made in converting the sampled system to discrete time. There is a strong connection to sampling of the sensitivity functions in nonlinear dynamic optimization once the discrete time systems are lifted, where the receding horizon LTV approach has the advantage that it is numerically very cheap while maintaining the iterative approvement as in SQP type of algorithms. In what follows we shall be concerned with transition control purposes and as in [13], we will consider objectives of the form

$$E\sum_{k=1}^{n}\Delta\bar{u}_k(\xi)^T R_k \Delta\bar{u}_k(\xi) + \int_0^T (\bar{z}(t,\xi)-\bar{z}^r)^T Q(t)(\bar{z}(t,\xi)-\bar{z}^r)dt$$

which we will interpret in the sense of its sampled version

$$E\sum_{k=1}^{n}(\bar{z}_k(\xi)-\bar{z}_k^r)^T Q_k(\bar{z}_k(\xi)-\bar{z}_k^r) + \Delta\bar{u}_k(\xi)^T R_k \Delta\bar{u}_k(\xi) \tag{11}$$

by employing some standard numerical integration technique such as the trapezoidal rule.

IV. Constrained finite horizon LQG control

The (**CLMPC**) is a computationally expensive optimization problem for large scale problems with many constraints and long prediction horizons due to the current status of many second order cone programming solvers. These solvers require the vectorization of the problem and due to the large number of parameters in the controller K, the size tends to blow-up, in particular the total number of Lagrange multipliers in primal-dual interior point methods. A way around this is to devise new algorithms exploiting the ellipsoidal structure of the (**CLMPC**), preferably treating the constraints recursively, which is for instance done for the maximal ellipsoid problem as discussed in a recent paper [3].

In light of the quadratic objective function (11) and the fact that the optimal solution to (**CLMPC**) is a finite horizon LQG controller, we will consider an approximate but very efficient control problem formulation. The idea is the following, in absence of any inequality constraints, (11) can be split (for the variational system dynamics) into a stochastic problem and a deterministic problem by the certainty equivalence property of LQG control. This motivates to split-up the (**CLMPC**) in a feedback and a feedforward problem.

1) Subproblem **CFHLQG**^{A}*:* The first step is to solve the minimal variance problem. Assume for ease of presentation that the inputs u are included in z, then this problem is of the structural form

$$\min_{M} \quad \text{tr } F_P^T(AMB+C)^T Q(AMB+C)F_P \tag{12}$$

We will be very explicit on A, M, B, C in the next section. Suppose one has solved this problem for the optimal $M^\star$, then the optimal variance matrix $Z^\star$ is also known, and the back-off terms using the ellipsoidal relaxation are readily computed as

$$\nu_j^\star = r\sqrt{h_j^T Z^\star h_j} \tag{13}$$

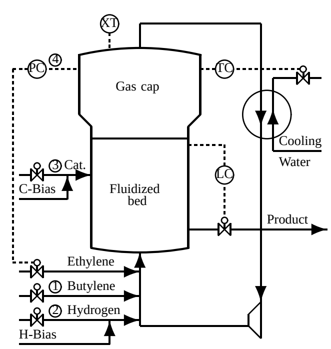
2) Subproblem **CFHLQG**^{B}*:* In the second step one solves the optimal transition, in which the back-off (13), to the constraints is used.

$$\begin{aligned} &\min_{\mathbf{u}^r} \quad (S\mathbf{z}^r - s)^T Q(S\mathbf{z}^r - s) \\ &\nu_j^\star + h_j^T(\bar{\mathbf{z}}^l + \mathbf{z}^r) \le g_j, \\ &\mathbf{z}^r = G_{zx}\mathbf{x}^r + G_{zu}\mathbf{u}^r + G_{zw}\mathbf{w}^r \end{aligned}$$

for some properly chosen matrix S and vector s related to the signals $\boldsymbol{\Delta}\bar{\mathbf{u}}^l, \bar{\mathbf{z}}^l, \bar{\mathbf{z}}^r$, the details are skipped due to space limitations. Notice that contrary to open-loop MPC, we can use two different tunings, one for reference tracking and one for disturbance rejection.

V. The continuous HDPE process

The HDPE process is a continuous fluidized bed reactor for gas phase polymerization using solid Ziegler-Natta catalyst particles. The schematic process layout is given in figure 2. Both the monomer Ethylene and the co-monomer Buthylene feeds enter at the bottom of the reactor where the gasses are blown with high velocity through the fluidized bed. The reactor is highly exothermic and therefore heat is removed by sending a relatively high recycle flow through the external heat exchanger. The process has three main basic control loops. The reactor temperature is controller by the cool water flow to the heat exchanger, the pressure in the reactor is controlled by the ethylene feed and the bed level is controlled by the product flow. The objective will



Performance outputs	
z_1	density (ρ)
z_2	melt-index (mi)
z_3	cool water flow
Measured outputs	
y_1	Ethylene gasfraction
y_2	Butylene gasfraction
y_2	Hydrogen gasfraction
Inputs	
$u_1(z_4)$	Buthylene feed
$u_2(z_5)$	Hydrogen feed
$u_3(z_6)$	Catalyst feed
Disturbances	
d_1	Hydrogen bias flow
d_2	Catalyst bias flow

Fig. 2. Process schematic gas-phase HDPE reactor

TABLE I
GRADE DEFINITIONS AND CONSTRAINTS

Grade		A	$\rightarrow$	B	Δ
z_1	[kg/m^3]	942.9		937.9	-5
z_2	[-]	1.1		3.1	$+2$
Constraints		lo		up	
z_3	[10^4kg/h]	2.5		5.2	

be to make a grade change in the density $z_1 = \rho$ and the melt-index $z_2 = \ln(\text{mi})$ as defined in table I. The specific weighting function is not that important, but is has the structure as in (11). The grade change must be done while satisfying the constraints on the process variables, see table I under two main persistent disturbances. The catalyst flow has a bias C-BIAS=$+0.025$kg/h and a bias in the hydrogen flow H-BIAS=-0.050kg/h. These disturbances are chosen for their counter effect on the grade change (density up, meltindex down) see also figure 3. To model these disturbances we have added continuous time disturbance models to our plant model of the form

$$\dot{d}_1 = 0, \quad d_1(t_0) = \xi_1, \quad \dot{d}_2 = 0, \quad d_2(t_0) = \xi_2$$

where ξ_1, ξ_2 are normally distributed with zero mean and standard deviations 0.1 and 0.025 respectively. The only other disturbances are the measurement errors on the gas composition in the gas cap which we have taken to be 5% of their nominal stead-state value. This turns out to be quite severe in the sense that it limits the speed of convergence of the Kalman filter and reduces the controller gain. In figure 3, we have plotted the optimal nominal reference trajectories for the transition obtained by a receding horizon implementation of the model predictive controller over an horizon of 8 hours with the objective function (11) without any disturbances. Also plotted are the actual trajectories if the biases on the flows are added, which force the system to deviate from the target grades considerably. In the lower right plot we have also plotted the back-off a standard MPC should take in open-loop prediction.

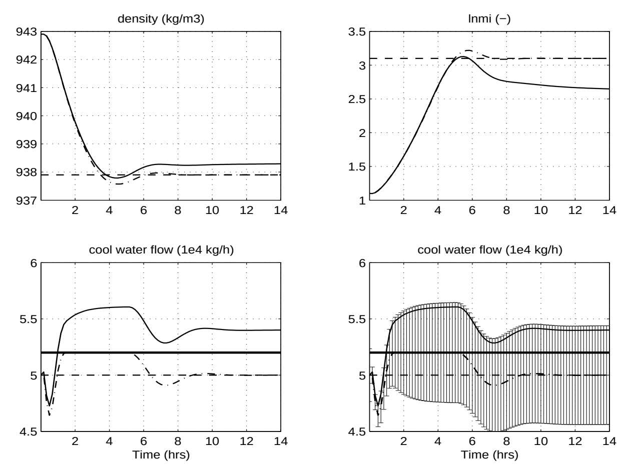


Fig. 3. Open-loop performance trajectories. Real performance (solid), reference performance (dash-dotted), target performance (dashed).

VI. Practical implementation issues

In [7], the receding horizon control law is deduced for the predictive control problems at hand and they rely on the state feedback properties of the solution of standard LQG control. Since we have a penalty in the rate of change of the inputs in (11), we must augment the discrete time system with the previous input sample. The optimal feedback gain is defined on this augmented system. Suppose one has a linear discrete time dynamical system

$$x_{k+1}(\xi) = A_k x_k(\xi) + B_k u_k(\xi) + G_k w_k(\xi)$$

define the extended state space system in which we keep track of the process state $x_k(\xi)$ as well as the input one step earlier u_{k-1} with the relative inputs Δu_k as the new inputs

$$x^+_{k+1}(\xi) = \begin{pmatrix} x_{k+1}(\xi) \\ u_k(\xi) \end{pmatrix} = \left(\begin{array}{cc|cc} A_k & B_k & B_k & G_k \\ O & I & I & O \end{array}\right) \begin{pmatrix} x_k(\xi) \\ u_{k-1}(\xi) \\ \hline \Delta u_k(\xi) \\ w_k(\xi) \end{pmatrix}$$

The optimal LQG solution will then be a state feedback of this extended state $x^+_k(\xi)$, or more precisely, the estimate of the extended state space $\hat{x}^+_k(\xi)$ including both the estimate of the process state as well as the estimate of the input. An observer for this system is obtained as

$$\hat{x}^+_{k+1}(\xi) = \left(\begin{array}{cc|cc} A_k & B_k & B_k & N_k \\ O & I & I & O \end{array}\right) \begin{pmatrix} \hat{x}^+_k(\xi) \\ \hline \Delta u_k(\xi) \\ v_k(\xi) \end{pmatrix}$$

The input is given by the state feedback on the extended system and the innovation sequence

$$\Delta u_k(\xi) = L^x_k \hat{x}_k(\xi) + L^x_k u_k(\xi) + K^{11}_k v_k(\xi) \qquad (14)$$

Applying this feedback to the stochastic system leads the estimation Riccati recursion, while for the estimate one finds

$$P^{\hat{x}}_{k+1} = \begin{pmatrix} A_k + B_k L^x_k & B_k(I + L^u_k) \\ L^x_k & I + L^u_k \end{pmatrix} P^{\hat{x}}_k \begin{pmatrix} \star & \star \\ \star & \star \end{pmatrix}^T + \begin{pmatrix} N_k + B_k K^{11}_k \\ K^{11}_k \end{pmatrix} (C_k P^e_k C^T_k + F_k W_k F^T_k) \begin{pmatrix} \star \\ \star \end{pmatrix}^T$$

The above analysis shows that the feedback must be defined on the extended state-space. The performance vector $\mathbf{z}$ is then found by lifting this extended system using the already available lifted system representation. To go from absolute inputs to relative inputs we have the following relations

$$\mathbf{u}_k(\xi) = J u_{k-1}(\xi) + T \boldsymbol{\Delta} \mathbf{u}_k(\xi) \qquad (15)$$

where

$$T = \begin{pmatrix} I & O & O & \cdots & O \\ I & I & O & \cdots & O \\ I & I & I & \cdots & O \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ I & \cdots & \cdots & I & I \end{pmatrix}, \quad J = \begin{pmatrix} I \\ I \\ I \\ \vdots \\ I \end{pmatrix}$$

hence the performance output is represented in the change in inputs

$$\mathbf{z}^+_k(\xi) = \begin{pmatrix} \mathbf{z}_k(\xi) \\ \boldsymbol{\Delta}\mathbf{u_k}(\xi) \end{pmatrix} = \begin{pmatrix} G^k_{zx} & G^k_{zu} J \\ O & O \end{pmatrix} \begin{pmatrix} \hat{x}_k(\xi) \\ u_{k-1}(\xi) \end{pmatrix} + \begin{pmatrix} G^k_{zx} \\ O \end{pmatrix} \mathbf{e}_k(\xi) + \begin{pmatrix} G^k_{zu} T \\ I \end{pmatrix} \boldsymbol{\Delta}\mathbf{u}_k(\xi) + \begin{pmatrix} G^k_{zw} \\ O \end{pmatrix} \mathbf{w}_k(\xi)$$

The feedback in lifted form is given by the feedback law

$$\boldsymbol{\Delta}\mathbf{u}_k(\xi) = L_k \hat{x}^+_{k-1}(\xi) + K_k \mathbf{v}_k(\xi) \qquad (16)$$

which is represented more conveniently in the notion of the previous sections as

$$\mathbf{z}^+_k(\xi) = (A^k M_k B^k + C^k) \begin{pmatrix} \hat{x}^+_k(\xi) \\ e_k(\xi) \\ \mathbf{w}_k(\xi) \end{pmatrix}$$

where the matrices A^k, B^k, M_k, C^k are given in terms of the system dynamics by

$$A^k = \begin{pmatrix} G^k_{zu} T \\ I \end{pmatrix}, B^k = \begin{pmatrix} I & O & O & O \\ O & I & O & O \\ O & O & G^k_{ve} & G^k_{vw} \end{pmatrix}$$

$$C^k = \begin{pmatrix} G^k_{zx} & G^k_{zu} J & G^k_{zx} & G^k_{zw} \\ O & O & O & O \end{pmatrix}, \qquad (17)$$

$$M_k = \begin{pmatrix} L^x_k & L^u_k & K_k \end{pmatrix}$$

and using the result of the previous paragraph it follows that in order to minimize the variance, we need the factor of the variance matrix $F_P F^T_P = \mathrm{diag}(P^{\hat{x}}_k, P^e_k, W_k)$ in (12).

VII. SIMULATION RESULTS

In figure (4) we have plotted the closed-loop results for the performance outputs z_1, z_2, z_3. In each subplot, three trajectories are plotted, namely the actual trajectories $\bar{\mathbf{z}}(\xi)$, the reference trajectories $\bar{\mathbf{z}}^r$ and the target grades. For the density and the melt-index ln(mi) we see no difference between the actual and the reference trajectories due to the tight control; the system closely follows the desired reference as it should. In the lower two subplots the cool water flow has been plotted. Due to the bias on the catalyst flow, the reactor temperature increases, however, due to the control action it nicely converges to the reference trajectory without violating the cool water constraint. In the lower right plot, the back off has been visualized via the errorbars as a measure for the variance, which reveals the envelope of trajectories the controller was expecting.

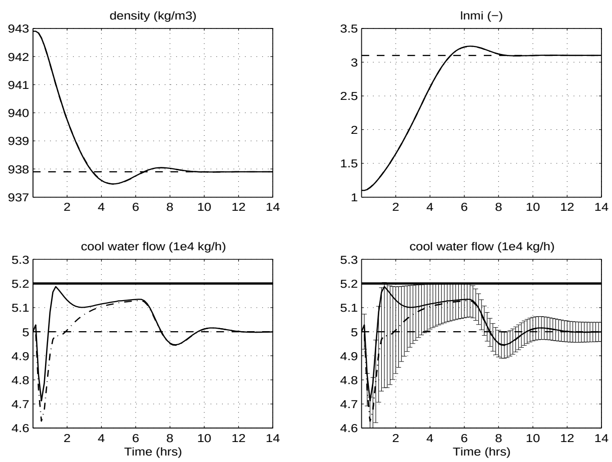


Fig. 4. Closed-loop performance trajectories. Real performance (solid), reference performance (dash-dotted), target performance (dashed).

VIII. CONCLUSIONS

The recent theoretical advances in closed-loop model predictive control were successfully applied to a nonlinear polymerization example of approximately 50 states. This shows that these techniques are very promising as tools in realistic process operation in which optimization and control are integrated to a high level. The direct feedback of closed-loop model predictive control, its ability to handle inequality constraints under stochastic disturbances and the ability to tune tracking performance and disturbance rejection separately are valuable aspects in chemical engineering applications.

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