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Asymptotic Stabilization of Motion about an Unstable Orbit: Application to Spacecraft Flight in Halo Orbit

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***Abstract*— Recently developed control techniques extend the traditional $\mathcal{H}_\infty$ framework to periodic linear time-varying systems. This paper applies these techniques to the problem of asymptotic stabilization of an unstable orbit. The problem of stabilization of a spacecraft placed in an unstable orbit about the interior Sun-Earth libration point is considered and it is shown that the control effort required by this method is reasonable and can be provided by a low-thrust propulsion system such as an ion engine.**

I. Introduction

The circular restricted three body problem describes the motion of a small particle under the gravitational influence of two massive bodies. This simplified version of the general three-body problem has been used to design missions for spacecraft in the Sun-Earth system. The restricted three-body problem has five libration points. In a synodic coordinate system, chosen such that the two massive bodies are fixed in the coordinate system, the libration points have the property that a test particle placed at a Libration point will theoretically remain there forever.

The vicinity of the interior libration point (L_1) of the Sun-Earth system is an ideal place for a solar observatory as the Sun's surface is always available for observation and the Earth is near enough for good communication. However the exact L_1 point is unsuitable as radio signals from the satellite would disappear in solar noise. Fortunately there are periodic orbits, known as Halo orbits, in the vicinity of this point which would be suitable. These periodic orbits are however unstable. One of the first detailed study of this problem was done by the Barcelona group [1] and [2] for the ISEE-3 (International Sun-Earth Explorers) mission. This paper addresses the issue of asymptotic stabilization of a spacecraft relative to these unstable periodic orbits.

The problem of station-keeping for satellites in unstable orbits is not a new one [3]–[5]. Though typically approached from a dynamical systems perspective [6], Scheeres, Hsiao and Vinh [7] recently phrased the station-keeping problem using trajectory control and proposed a strategy in which the control law, by creating additional center manifolds where the spacecraft could be flown achieves Lyapunov stability about the unstable orbit. In a similar vein, the problem is viewed in this paper from a controls perspective and formulated in terms of asymptotic stabilization of a periodic linear time-varying (LTV) system. The approach here is to use recently developed tools that extend the $\mathcal{H}_\infty$ control synthesis techniques to LTV systems [8].

The application of these tools results in a controller that stabilizes the nonlinear system in the presence of disturbances. The thrust required by the control law is reasonable and can be implemented using a low-thrust propulsion device such as an ion engine. The advantage of the proposed methodology is the robust performance of the controller and also the considerable flexibility of the current formulation. The tools used in this paper can be extended to include the effects of the invariant manifolds in order to design a controller with greater fuel efficiency. Another advantage is the considerable ease with which the problem of designing controllers for *Formation Flight* of multiple spacecraft can be addressed. This is important as formation flight has been identified by NASA as key technology for the future scientific missions [9].

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II. Model of Motion and Problem Statement

A. Lagrangian Formulation

The equations of motion for the circular restricted three-body problem were adopted from the work done by Richardson [10], [11]. Here the Lagrangian for the system (Fig. 1) is constructed by considering the influence of the Earth and the Sun on the spacecraft as third-body perturbations.

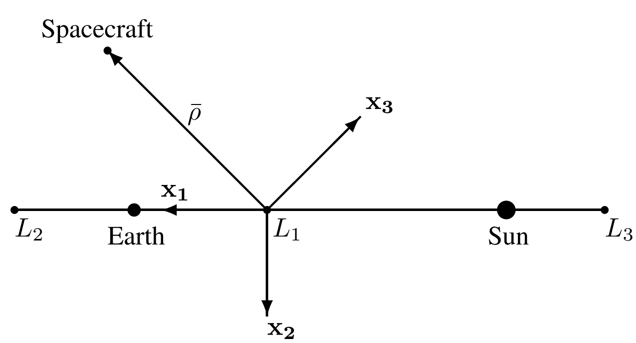

Fig. 1. Libration Point Geometry

$$L = \frac{1}{2}(\dot{\bar{\rho}}\cdot\dot{\bar{\rho}}) + GE\Big[\frac{1}{|\bar{r}_E - \bar{\rho}|} - \frac{\bar{r}_E\cdot\bar{\rho}}{|\bar{r}_E|^3}\Big] + GS\Big[\frac{1}{|\bar{r}_S - \bar{\rho}|} - \frac{\bar{r}_S\cdot\bar{\rho}}{|\bar{r}_S|^3}\Big], \tag{1}$$

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where $\bar{\rho}$ is the position vector of the satellite relative to the libration point, $\bar{\rho} = \mathbf{x_1} i + \mathbf{x_2} j + \mathbf{x_3} k$. The quantities GE and GS are gravitation constants of the Earth and the Sun respectively and the vectors $\bar{r}_E$ and $\bar{r}_S$ are position vectors of the Earth and Sun with respect to the libration point. This formulation allows the expression of the Lagrangian as a summation of Legendre polynomials, which provides computational facility when considering higher-order terms. The equations of motion through third order are given by

$$\begin{aligned}
\omega^2\ddot{\mathbf{x}}_\mathbf{1} - 2\omega\dot{\mathbf{x}}_\mathbf{2} - (1+2c_2)\mathbf{x_1} &= \frac{3}{2}c_3(2\mathbf{x_1}^2 - \mathbf{x_2}^2 - \mathbf{x_3}^2) \\
&\quad + 2c_4\mathbf{x_1}(2\mathbf{x_1}^2 - 3\mathbf{x_2}^2 - 3\mathbf{x_3}^2) + O(4), \\
\omega^2\ddot{\mathbf{x}}_\mathbf{2} + 2\omega\dot{\mathbf{x}}_\mathbf{1} + (c_2-1)\mathbf{x_2} &= -3c_3\mathbf{x_1}\mathbf{x_2} \\
&\quad - \frac{3}{2}c_4\mathbf{x_2}(4\mathbf{x_1}^2 - \mathbf{x_2}^2 - \mathbf{x_3}^2) + O(4), \\
\omega^2\ddot{\mathbf{x}}_\mathbf{3} + \lambda^2\mathbf{x_3} &= -3c_3\mathbf{x_1}\mathbf{x_3} - \frac{3}{2}c_4(4\mathbf{x_1}^2 - \mathbf{x_2}^2 - \mathbf{x_3}^2) \\
&\quad + \Delta\mathbf{x_3} + O(4).
\end{aligned} \tag{2}$$

The various parameters are tabulated in the Appendix.

Distance and time have been renormalized so that higher-order terms in the Legendre polynomial expansion fall off in a regular manner. This allows an estimate of the error, introduced by ignoring the higher-order terms, to be obtained. For further details refer to [10], [11].

B. Periodic Solution

The periodic nature of the solution can be seen by considering the linearized form of the equations of motion. The frequencies of oscillation in the in-plane and out-of-plane directions would not, in general, be equal. However, if the amplitudes of the in-plane and out-of-plane motions are sufficiently large then the nonlinear terms produce eigenfrequencies that are equal. This results in the so-called Halo orbit. The equation of the Halo orbit in physical space through the third-order is given by,

$$\begin{aligned}
\gamma_\mathbf{1}(t) &= a_{21}A_x^2 + a_{22}A_z^2 + (a_{23}A_x^2 - a_{24}A_z^2)\cos(2\lambda t) \\
&\quad - A_x\cos(\lambda t) + (a_{31}A_x^3 - a_{32}A_xA_z^2)\cos(3\lambda t), \\
\gamma_\mathbf{2}(t) &= KA_x\sin(\lambda t) + (b_{21}A_x^2 - b_{22}A_z^2)\sin(2\lambda t) \\
&\quad + (b_{31}A_x^3 - b_{32}A_xA_z^2)\sin(3\lambda t), \\
\gamma_\mathbf{3}(t) &= A_z\cos(\lambda t) + d_{21}A_xA_z(\cos(2\lambda t - 3) \\
&\quad + (d_{32}A_zA_x^2 - d_{31}A_z^3)\cos(3\lambda t),
\end{aligned} \tag{3}$$

where $\gamma_1(t)$, $\gamma_2(t)$ and $\gamma_3(t)$ are the coordinates along the $\mathbf{x_1}$, $\mathbf{x_2}$ and $\mathbf{x_3}$ axes respectively. Let $\Gamma(t)$ denote the Halo periodic orbit in phase-space, i.e. $\Gamma(t) = [\gamma_1(t), \dot{\gamma}_1(t), \gamma_2(t), \dot{\gamma}_2(t), \gamma_3(t), \dot{\gamma}_3(t)]$, also $\Gamma(T+t) = \Gamma(t)$, where T is the period of the orbit.

A family of periodic orbits exist and the periodic orbit proposed for the ISEE-3 mission was chosen for this work [10]. The values of the various constants, for this periodic orbit, are tabulated in the Appendix. For further details regarding how this third-order periodic solution was obtained refer [10], [11] and the references therein.

C. Problem Statement

The problem that is solved here is that of asymptotically stabilizing the motion of a space-craft of mass 1000kg, whose dynamics are governed by (2), about $\Gamma(t)$, by the application of thrust along the three directions: $\mathbf{x_1}$,$\mathbf{x_2}$ and $\mathbf{x_3}$.

III. 'H_∞ SYNTHESIS': PERIODIC LTV SYSTEMS

Techniques recently developed by Dullerud and Lall, introduced in [12] for the analysis and control of periodic LTV systems, were used to design a controller for the system. The usual discrete-time state-space description of an LTV system is given by

$$\begin{aligned}
x_{k+1} &= A_kx_k + B_ku_k \\
y_k &= C_kx_k + D_ku_k,
\end{aligned} \tag{4}$$

where x_k is the state of the system, u_k is the control input, y_k is the sensor measurement, and A_k, B_k, C_k and D_k are time-varying matrices that capture the dynamics of the system.

Consider an operator $\mathcal{A}$, defined in the following block-diagonal form,

$$\begin{bmatrix} A_0 & & & 0 \\ & A_1 & & \\ & & A_2 & \\ 0 & & & \ddots \end{bmatrix}.$$

Similar definitions can be made for $\mathcal{B}, \mathcal{C}$ and $\mathcal{D}$. Also define $\widetilde{x} = (x_0, x_1, x_2, \ldots)$ and with similar definitions for u and y. Now, the shift-operator is introduced as,

$$(\mathcal{Z}\widetilde{x}) = (0, x_0, x_1, x_2 \cdots). \tag{5}$$

It can be seen that an equivalent representation of the system (4) can be made in terms of block-diagonal operators.

$$\begin{aligned}
\widetilde{x} &= \mathcal{Z}\mathcal{A}\widetilde{x} + \mathcal{Z}\mathcal{B}\widetilde{u} \\
\widetilde{z} &= \mathcal{C}\widetilde{x} + \mathcal{D}\widetilde{u}
\end{aligned} \tag{6}$$

This formulations leads to an operator-based description of the system and a function, called the *system function*, which has many properties analogous to those of transfer functions for LTI systems, such as the induced norm being the maximum of a matrix norm over frequency. This framework thus allows techniques formerly restricted to LTI systems to be applied to LTV systems.

In particular, the traditional H_∞ analysis and synthesis problem for LTI systems can now be formulated for LTV systems. The basic set-up for control design is depicted in Fig. 2. **G** is the LTV system to be controlled, while **K** is the controller to be designed. The variables w are exogenous signals which consist of disturbances and tracking signals, z are the error signals that must be kept small, y are the sensor signals, while u are the control signals.

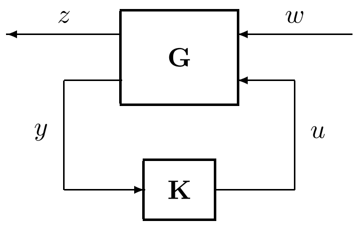

Fig. 2. Feedback interconnection

Let the system $\mathbf{G}$ be defined by the following discrete state space equations:

$$\begin{aligned} x_{k+1} &= A_k x_k + B_{1k} w_k + B_{2k} u_k, \\ z_k &= C_{1k} x_k + D_{11k} w_k + D_{12k} u_k, \\ y_k &= C_{2k} x_k + D_{21k} w_k. \end{aligned} \quad (7)$$

A controller $\mathbf{K}$ (a relation between the sensor signals y and the control variables u) is to be designed, of the form

$$\begin{aligned} x^K_{k+1} &= A^K_k x^K_k + B^K_k y_k, \\ u_k &= C^K_k x^K_k + D^K_k y_k, \end{aligned} \quad (8)$$

such that the closed loop system is stable and the map from w to z is minimized. A closed-loop realization of the system can be written as

$$\begin{aligned} x^L_{k+1} &= A^L_k x^L_k + B^L_k w_k, \\ z_k &= C^L_k x^L_k + D^L_k w_k, \end{aligned} \quad (9)$$

where $x^L_k = [x_k, x^K_k]^T$, has states of both G and K, and all the matrices are of appropriate dimensions. Equation (9) is in a form similar to (4), and once again using the shift-operator the system can be rewritten in an operator framework. Now following the approach detailed in [8], [12] the problem of determining a stabilizing controller K that minimizes the $w \to z$ norm can be cast in terms of a feasibility condition for linear operator inequalities.

The important difference to note here with respect to the standard formulation is that while the $\mathcal{H}_\infty$ control problem for LTI systems is stated using linear matrix inequalities, for LTV systems the results are in the form of linear operator inequalities. The problem therefore is typically infinite-dimensional. However, for the case of periodic LTV systems the repetitive structure of the block diagonal operators leads to a solution in the form of linear matrix inequalities, which can be solved by traditional means.

IV. Control Problem Formulation

To obtain the equations of motion in a form suitable for controller synthesis, the nonlinear equations of motion (2) are first linearized about the periodic orbit and then subsequently discretized. Consider the state of the system to be in the form $x = [\mathbf{x_1}, \dot{\mathbf{x}}_\mathbf{1}, \mathbf{x_2}, \dot{\mathbf{x}}_\mathbf{2}, \mathbf{x_3}, \dot{\mathbf{x}}_\mathbf{3}]^T$. Let $X(t) = \Gamma(t)$ and $\mathrm{x} = X + \delta\mathrm{x}$, where $\delta\mathrm{x}$ is assumed to be 'small' relative to the state X. Denote the equations of motion (2) compactly in the form $\dot{X} = F(X)$. Then,

$$\delta\dot{\mathrm{x}} = \dot{\mathrm{x}} - \dot{X} \quad (10)$$

$$\delta\dot{\mathrm{x}} = F(X + \delta\mathrm{x}) - F(X) \quad (11)$$

$$\delta\dot{\mathrm{x}} = F(X) + \frac{\partial F}{\partial X}\Big|_{\Gamma(t)} \delta\mathrm{x} + \ldots - F(X) \quad (12)$$

$$\delta\dot{\mathrm{x}} \sim A(t)\delta\mathrm{x} \quad (13)$$

$$A(t) = \frac{\partial F}{\partial X}\Big|_{\Gamma(t)} \quad (14)$$

$$A(t+T) = A(t). \quad (15)$$

Rewriting $\delta\mathrm{x}$ by x for notational comfort the equations of motion for the system, linearized about the nominal Halo orbit, are obtained.

$$\dot{x} = A(t)x. \quad (16)$$

The above equation governs the evolution of the system in the absence of control. In the presence of control (u) which is applied by means of three orthogonal thrusters and unmodeled gravitational forces (w) which act as disturbances, the system equation for deviations about the Halo orbit is given by,

$$\dot{x} = A(t)x + B_u u + B_w w, \quad (17)$$

where u and w are three-dimensional column vectors and

$$B_u = \begin{bmatrix} 0 & 0 & 0 \\ 1/M & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1/M & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/M \end{bmatrix}, \quad (18)$$

where M is the mass of the spacecraft in kg.

To determine the elements of B_w attention is once again turned to the celestial mechanics aspect of the problem and the assumptions and approximations made while determining the equations of motion. The most important perturbative effects are due to the eccentric nature of the Earth's orbit and the gravitational force of the moon, whose magnitudes are of third and fourth order respectively [10]. Perturbations due to forces from other planets are several orders of magnitude smaller. Solar radiation, another force not considered in the modeling, will only cause the locations of the libration points and the Halo orbits to move outwards, and as such can be ignored for present purposes.

Simple calculations show that the largest perturbative force per unit mass of the spacecraft due to the eccentric nature of the Earth's orbit approximately is given by

$$|w_E| = GM_{Earth}M|(\frac{1}{|\vec{r}_E|^2} - \frac{1}{|\vec{r}_e|^2})|, \quad (19)$$

where $\vec{r}_e$ is the position vector of the Earth at pericenter from the Libration point L_1). This force is less than about $0.6M$ normalized force units. To design a conservative controller the disturbing forces are taken to be of unit magnitude times the spacecraft mass in the normalized coordinates. Therefore, $B_w = B_u$. As the perturbations due to Earth's eccentric orbit are essentially of the order of once every year it would be ideal to incorporate a low-pass filter in our plant design. However this increases the order of the plant and therefore the order of the controller resulting in substantially longer computation times. Therefore a conservative controller is designed without the filter.

For deep space missions fuel efficiency is an important concern. In this regard ion engines have proven to be ten time more fuel efficient than other on-board chemical thrusters. A typical Xenon ion engine at full throttle provides a thrust of approximately 90mN [13] with a lifetime of 10,000 hours, making it an ideal choice for a Halo mission. It is therefore important to design a controller which will satisfactorily perform the mission while at the same time require thrust that is less than the maximum thrust that can be provided by the ion engine. Ion engines can be modeled quite realistically as first-order systems, but for the present purposes, as the time constant for the engines is several orders of magnitude smaller than the periods over which constant thrust will be maintained, it will be assumed that commanded thrust values can be reached instantaneously.

The parameter $t = [0, T]$ is now discretized for one period and indexed by $k = 0 \cdots N-1$ such that the time interval between two successive points on the periodic orbit is given by $\Delta T = T/N$. The continuous-time equation of motion about the point indexed by k is then given by

$$\dot{x} = A(k\Delta T)x + B_u u + B_w w, \qquad (20)$$

Each of these N continuous-time linear equations are converted into N discrete-time equations with a time-step δT using a zero-order hold. The discrete-time equation at the index k on the periodic orbit is then given by

$$x_{n+1} = \Phi_k x_n + \Gamma_{u,k} u_n + \Gamma_{w,k} w_n, \qquad (21)$$

Also,

$$\Phi_k = e^{\delta T A(k\Delta T)}, \qquad (22)$$

$$\Gamma_{u,k} = \int_0^{\delta T} e^{\tau A(k\Delta T)} d\tau B_u, \qquad (23)$$

$$\Gamma_{w,k} = \int_0^{\delta T} e^{\tau A(k\Delta T)} d\tau B_w. \qquad (24)$$

It is important to note that with a choice of $\delta T = \Delta T$, i.e if the time-step δT chosen for converting the N continuous-time equations into discrete-time equations is the same as the time-step ΔT chosen for the discretization of the periodic orbit, one can obtain the discrete-time state space representation of the periodic system in a form similar to (7). With this choice for δT the system matrices from (21) can be mapped to (7).

As the state of the plant is to be minimized so as to be 'close' to the nominal Halo orbit, while at the same time penalizing control effort, the following choices are made for the other matrices in (7),

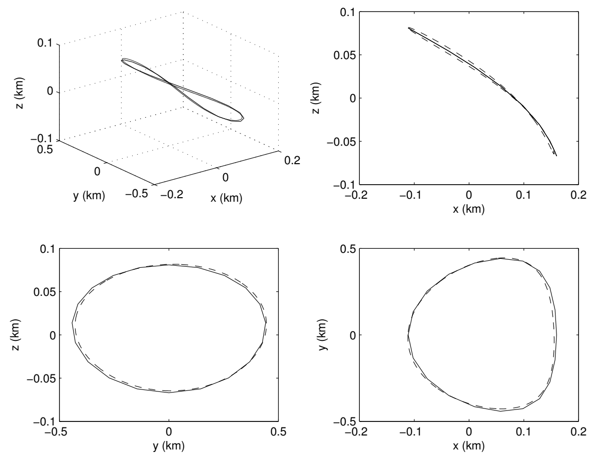

Fig. 3. Simulation of the controller implemented on the non-linear system. The dashed line indicates the controlled trajectory, the continuous line indicates the nominal Halo orbit.

$$C_{1k} = \begin{bmatrix} I_{6,6} \\ 0_{3,6} \end{bmatrix} \qquad D_{11k} = \begin{bmatrix} 0_{6,3} \\ I_{3,3} \end{bmatrix} \qquad D_{12k} = \begin{bmatrix} 0_{9,3} \end{bmatrix}. \qquad (25)$$

It is assumed for the present purposes that the sensor measurements are not corrupted by noise. Also the time taken to obtain positional information using ground-based antennas such as the Deep Space network is of the order of a few minutes which is small compared to the dynamics of the open-loop system and shall be ignored. Therefore designing a control system with a full state feedback C_{2k} is chosen to be the identity matrix, $I_{6,6}$ with D_{21k} set to zero. The periodic orbit was discretized at twenty points along the orbit, i.e. $N = 20$.

V. Controller Synthesis and Non-Linear Simulation Results

The LTV Toolbox developed by Dullerud and Lall [12] was used for control design and analysis. A controller which resulted in a stable closed loop system for the linearized system was designed and the resulting worst-case gain in the $\mathcal{L}_2$ sense from input to output (w to z) was found to be 1.4550.

Fig. 3 shows a plot of the simulated trajectory of the non-linear system (2) under the application of the $\mathcal{H}_\infty$ controller obtained from the Toolbox. The starting point for the simulation is on the Halo orbit. The system is also simulated in the absence of any disturbances.

Fig. 4 shows a plot of the control effort required to asymptotically stabilize a spacecraft with a mass of 1000kg. The effort is plotted for several periods to emphasize its nearly periodic nature, which is to be expected. The maximum force required is $\approx 15mN$, while the average force required is about $7mN$, $2mN$ and $0.2mN$ along the three axes: $\mathbf{x_1}$,$\mathbf{x_2}$ and $\mathbf{x_3}$. The maximum thrust required

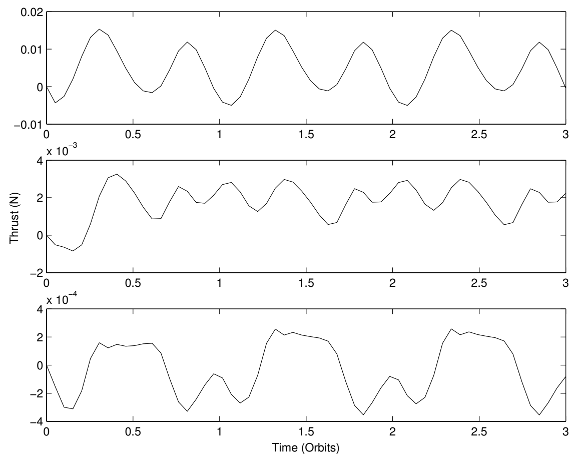

Fig. 4. Plot of the control effort required in N to control a spacecraft for 3 orbits.

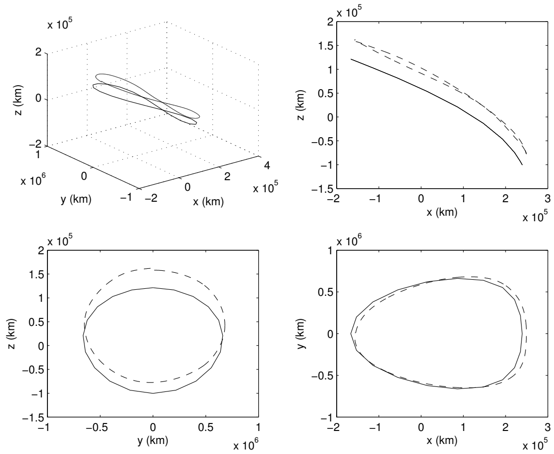

Fig. 5. Simulation of the controller implemented on the non-linear system with a disturbance of L_2 norm equal to one. The dashed line indicates the controlled trajectory, the continuous line indicates the nominal Halo orbit.

is less than $90mN$, which is the maximum that can be provided by current commercially available ionic engines. With the fuel efficiency associated with ionic engines the fuel costs are seen to be reasonable.

Fig. 5 shows a plot of the trajectory when the controller is implemented on the non-linear system while it is being subject to a disturbance of L_2 norm of about unit magnitude. As can be seen, the controller performs remarkably well for the non-linear system even in the presence of disturbance. The thrust required in this case (Fig. 6) is once again seen to be reasonable with the maximum required thrust $\approx 20mN$. It is interesting to note though that in the presence of disturbances the controller stabilizes the spacecraft about a different periodic orbit that is displaced from the initial Halo orbit. The significance of this observation is being investigated further by the authors.

The controller was also seen to stabilize the system when the starting point was substantially off the Halo orbit. The controller obtained is thus seen to provide robust performance in the face of deviation from ideal initial conditions and also in the presence of disturbances.

A post-fact Floquet analysis of the closed-loop system was done to determine stability. The stability of periodic systems can be determined by computing the eigenvalues of the Monodromy matrix, $M = \Phi(t_0 + T, t_0)$, where Φ is the state transition matrix of the closed-loop system. A periodic system is stable when the magnitude of all the complex eigenvalues of the Monodromy matrix is less than one. For the controlled system of the spacecraft about the Halo orbit, the eigenvalues of the Monodromy matrix are all less than one and the system is stable.

VI. Directions for Future Work

One promising extension of the current formulation of the $\mathcal{H}_\infty$ framework for LTV systems would be to take into account the local stable and unstable manifolds and penalize the elements of the state differently at different times.

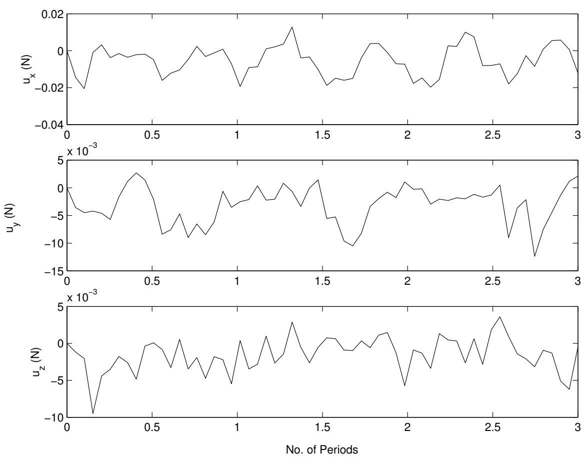

Fig. 6. Plot of the control effort required to control a spacecraft for 3 orbits while being subject to a disturbance of L_2 norm approximately equal to one.

Essentially the output of the system z would be weighted with a time varying matrix constructed by using the local invariant manifolds. It is hoped that with this framework the control effort would be utilized more effectively.

Another extension of the work is to consider the problem of *Formation Flight* in the Halo orbit. Plans for future space observatories include interferometric imagers with the spacecraft separated by thousands of kilometers. The Halo orbits with their relatively tame instability would serve as ideal locations for these observatories. The method employed in this paper lends itself rather well to incorporating multiple spacecrafts in the system and to designing a stabilizing controller for such a constellation of spacecraft.

Acknowledgements

We would like to thank Cédric Langbort, for interesting discussions regarding this and related problems.

APPENDIX

Values of the various constants in (2) and (3) calculated for the interior libration point of the Sun-Earth system.

a_{21}	$2.092695581 * 10^0$	a_{22}	$2.482976703.10^{-1}$
a_{23}	$-9.059647954 * 10^{-1}$	a_{24}	$-1.044641164 * 10^{-1}$
a_{31}	$7.938201951 * 10^{-1}$	a_{32}	$8.268538529 * 10^{-3}$
b_{21}	$-4.924458751 * 10^{-1}$	b_{22}	$6.074646717 * 10^{-2}$
b_{31}	$8.857007762 * 10^{-1}$	b_{32}	$2.301982738 * 10^{-2}$
d_{21}	$-3.468654605 * 10^{-1}$	d_{31}	$1.904387005 * 10^{-2}$
d_{32}	$3.980954252 * 10^{-1}$	c_2	$4.06107 * 10^0$
c_3	$3.0201 * 10^0$	c_4	$3.03054 * 10^0$
K	$3.22927 * 10^0$	ω	$9.85050176 * 10^{-1}$
λ	$2.086453455 * 10^0$	Δ	$2.9221445425 * 10^{-1}$
A_z	$110,000$ (km)	A_x	$206,000$ (km)

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