Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

Hierarchical Intelligent Sliding Mode Control: Application to Stepper Motors

Bernardo Rincon Marquez, Alexander G. Loukianov, and Edgar N. Sanchez, *Member, IEEE*

Abstract: **In this paper, one method of robust control based on sliding mode and fuzzy logic techniques is presented. It combines hierarchical high gain control with fuzzy logic to design a discontinuous control for nonlinear multivariable systems in order to eliminate chattering in presence of disturbances. The proposed approach is then used to design a robust controller for a stepper motor. Simulation results are presented to illustrate the applicability of the approach.**

I INTRODUCTION

A simple and good technique for robust control is the sliding mode control [9, 10], since it is composed of two clear steps: first, selection of a sliding surface such that a sliding mode equation on this surface is robust in presence of disturbances, and second, design of a discontinuous control which stabilizes the projection motion of the closed loop system on the sliding surface subspace.

It is known that the sliding mode motion is invariant with respect to disturbance, which satisfies the matching condition [2]. There are two cases for controlling systems with unmatching condition: measured disturbances, and, unmeasured ones. For the second case, the problem can be solved using high gain control, but it can produce chattering [10] due to defects on the control devices.

In this paper we propose a new control scheme using combination of the sliding mode control, block control [10] and fuzzy logic control [3] techniques to eliminate chattering on the closed-loop system for both matched and unmatched unknown disturbances. Note that the sliding mode fuzzy logic controller was investigated for systems with matched disturbances in [1, 4, 5, 8, 11]. This paper is organized as follows: In section 2, the proposed method, which combines the Sliding Mode and Fuzzy Logic techniques, is shown. The stability conditions for the Sliding Mode are established, and the gains are defined based on the disturbance bounds. In section 3, an application example is presented using the proposed method to permanent magnet stepper motor control, and, in section 4, the results of simulation of this example are shown. Finally some relevant conclusions are stated.

Work supported by Conacyt, Mexico, under Grant 36960A
B. R. M. Author is with the F.I.M.E. of Colima University, Campus Coquimatlán, Col. 28400 México, e-mail: brm@ucol.mx.
A. G. L. Author is with Guadalajara University, Campus Lagos de Moreno, Jal. 47460 México, e-mail: louk@gdl.cinvestav.mx.
E. N. S. Author is with the CINVESTAV IPN Guadalajara Unit, P. O. Box 31-438, Guadalajara, Jal. 44550, México, e - mail: sanchez@gdl.cinvestav.mx

II CONTROL METHOD

Let consider a multiple input multiple output (MIMO) nonlinear system subject to disturbances

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{B}(\mathbf{x})\mathbf{u} + \mathbf{G}(\mathbf{x})\mathbf{w} \qquad (1)$$
$$\mathbf{y} = \mathbf{h}(\mathbf{x})$$

where $\forall t$, $\mathbf{x}(t) \in R^n$ is the state vector, $\mathbf{u} \in R^m$ is the control input bounded as

$$\|\mathbf{u}\| \le k_{\max} \qquad k_{\max} > 0$$

$\mathbf{w} \in R^q$ represents an external disturbance, which is unknown but bounded, $\mathbf{y} \in R^p$ is the output, $\mathbf{f}(\mathbf{x}) \in R^n$ and $\mathbf{B}(\mathbf{x}) \in R^{n\times m}$ are sufficiently smooth and bounded functions, $\mathbf{G}(\mathbf{x}) \in R^{n\times q}$ is an unknown but bounded function, and $\mathbf{f}(0) = 0$.

We assume that there exists a nonlinear transformation that reduces the system (1) to the so-called Block Controllable Form with disturbances [7]:

$$\begin{aligned} \dot{\boldsymbol{x}}_1 &= \boldsymbol{f}_1(\boldsymbol{x}_1) + \boldsymbol{B}_1(\boldsymbol{x}_1)\boldsymbol{x}_2 + \mathbf{G}_1(\boldsymbol{x}_1)\boldsymbol{w} \\ \dot{\boldsymbol{x}}_i &= \boldsymbol{f}_i(\bar{\mathbf{x}}_i) + \boldsymbol{B}_i(\bar{\mathbf{x}}_i)\boldsymbol{x}_{i+1} + \boldsymbol{G}_i(\bar{\mathbf{x}}_i)\boldsymbol{w}, \quad i = 2,...,r-1 \\ \dot{\boldsymbol{x}}_r &= \boldsymbol{f}_r(\mathbf{x}) + \boldsymbol{B}_r(\mathbf{x})\boldsymbol{u} + \boldsymbol{G}_r(\mathbf{x})\boldsymbol{w} \end{aligned} \qquad (2)$$
$$\boldsymbol{y} = \boldsymbol{x}_1$$

with $\mathbf{x} = (\boldsymbol{x}_1,...,\boldsymbol{x}_r)^T$, $\bar{\mathbf{x}}_i = (\boldsymbol{x}_1,...,\boldsymbol{x}_i)^T$, $\boldsymbol{B}_i \ne 0$, and $\boldsymbol{f}_i(\bar{\mathbf{x}}_i)$ and $\boldsymbol{B}_i(\bar{\mathbf{x}}_i)$ are sufficiently smooth and bounded functions.

For electromechanical systems the following structure is frequently considered

$$n_1 \le n_2 \le \ldots \le n_r \le m$$

where n_i is the vector size, corresponding to state $\boldsymbol{x}_i$, and, m is the input dimension.

If the disturbance $\boldsymbol{w}$ satisfies the so called *matching condition*, that is, there is a scalar function $\lambda(\mathbf{x})$ such that

$$\mathbf{G(x) = B(x)\lambda(x)}$$

then the sliding mode motion is invariant with respect to external disturbances [2]. The aim of this paper is to design a discontinuous control which provides robustness for the closed-loop system, and chattering free motion in presence of both matched and unmatched disturbances. The control procedure consists of the following steps:

Suppose that the output $\boldsymbol{y}$ is required to track the reference signal $\boldsymbol{x}_{ref}$. Using the block control technique [7], we introduce the following recursive transformation:

$$z_1 = x_1 - x_{ref} \equiv \Phi_1(x_1, x_{ref}) \quad (3a)$$

$$z_2 = f_1(x_1) + b_1(x_1)x_2 - \dot{x}_{ref} + k_1\Phi_1(x_1, x_{ref}) \equiv \Phi_2(\bar{x}_2, x_{ref,2}) \quad (3b)$$

$$z_{i+1} = \bar{B}_i(\bar{x}_i)x_{i+1} + f_i(\bar{x}_i) + k_i\Phi_i(\bar{x}_i, x_{ref,i}) \equiv \Phi_{i+1}(\bar{x}_{i+1}, x_{ref,i+1}) \qquad i+1 = 3,...,r \quad (3c)$$

where $z = (z_1,...,z_r)^T$ is a new variables vector, $k_i > 0$, $x_{ref,2} = (x_{ref}, x_{ref}^{(1)})^T$, $x_{ref,i} = (x_{ref}, x_{ref}^{(1)},..., x_{ref}^{(i-1)})^T$, $\bar{B}_i = \bar{B}_{i-1}B_i$.

The transformation (3a)-(3c) reduces the system (2) to the following desired form:

$$\dot{z}_1 = -k_1 z_1 + z_2 + \bar{G}_1(z_1, x_{ref}, w) \quad (4a)$$

$$\dot{z}_i = -k_i z_i + z_{i+1} + \bar{G}_i(\bar{z}_i, x_{ref,i}, w), \quad i = 2,...,r-1 \quad (4b)$$

$$\dot{z}_r = \bar{f}_r(z, x_{ref,r}) + \bar{B}_r(z, x_{ref,r})u + \bar{G}_r(z, x_{ref,r}, w) \quad (4c)$$

where $\bar{z}_i = (z_1,...,z_i)^T$, $\bar{f}_r(z, x_{ref,r})$ is a continuous and bounded function, $\bar{B}_r = \bar{B}_{r-1}B_r$, and $\bar{B}_r \neq 0$.

In order to generate sliding mode in (4a)-(4c) a natural choice for the switching function is $s = z_r$ (3c). Then the desired dynamics of the closed-loop system for the case of unknown $\boldsymbol{w}$, can be selected as

$$\dot{s} = \bar{f}_r(z, x_{ref,r}) - k_r\bar{B}_r(z, x_{ref,r})\frac{\bar{B}_r(z, x_{ref,r})s}{\left\|\bar{B}_r(z, x_{ref,r})s\right\|} + \bar{G}_r(z, x_{ref,r}, w) \qquad k_r > 0 \quad (5)$$

From (4c) and (5) a discontinuous control strategy can be obtained as:

$$u = -k_r\frac{\bar{B}_r(z, x_{ref,r})s}{\left\|\bar{B}_r(z, x_{ref,r})s\right\|} \quad (6)$$

In order to derive the stability condition, we use a positive definite function $\mathbf{V_r}(s) = \frac{1}{2}s^2$. Then from

$$\dot{\mathbf{V}}_r \leq -\|s\|\left[k_r\left\|\bar{B}_r(z, x_{ref,r})\right\| - \left\|\bar{f}_r(z, x_{ref,r}) + \bar{G}_r(z, x_{ref,r}, w)\right\|\right]$$

we can obtain

$$k_r \geq \left\|u_{eq}(z, x_{ref,r}, w)\right\| \quad (7)$$

where the equivalent control u_{eq} defined as a solution of $\dot{s} = 0$ (4c) such as

$$u_{eq} = \bar{B}_r^{-1}(z, x_{ref,r})\left[\bar{f}_r(z, x_{ref,r}) + \bar{G}_r(z, x_{ref,r}, w)\right] \quad (8)$$

Under condition (7), the state converges to the surface $s = 0$ and the sliding mode motion occurs on this surface in a finite time. This motion is described by the following $(r-1)^{th}$ order system:

$$\dot{z}_1 = -k_1 z_1 + z_2 + \bar{G}_1(z_1, x_{ref}, w) \quad (9a)$$

$$\dot{z}_i = -k_i z_i + z_{i+1} + \bar{G}_i(\bar{z}_i, x_{ref,i}, w), \quad i = 2,...,r-2 \quad (9b)$$

$$\dot{z}_{r-1} = -k_{r-1}z_{r-1} + \bar{G}_{r-1}(\bar{z}_{r-1}, x_{ref,r-1}, w) \quad (9c)$$

If $G_i(\bar{x}_i)$ on (2) is bounded, then, transformation (3a)-(3c) is bounded too, therefore $\bar{G}_i(\bar{z}_i, x_{ref,i}, w)$ in (9a)-(9c) is bounded.

The following assumption on the bounds of the unknown terms in (4a)-(4c), is stated:

A1) There exist positive constants q_{ij} and d_i such that

$$\left\|\bar{G}_1(\bar{z}_1, x_{ref}, w)\right\| \leq q_{11}\|z_1\| + d_1 \quad (10a)$$

$$\left\|\bar{G}_2(\bar{z}_2, x_{ref,2}, w)\right\| \leq q_{22}\|z_2\| + k_1 q_{21}\|z_1\| + d_2 \quad (10b)$$

$$\left\|\bar{G}_3(\bar{z}_3, x_{ref,3}, w)\right\| \leq q_{33}\|z_3\| + k_2 q_{32}\|z_2\| + k_1^2 q_{21}\|z_1\| + d_3 \quad (10c)$$

$$\left\|\bar{G}_i(\bar{z}_i, x_{ref,i}, w)\right\| \leq q_{i,i}\|z_i\| + \sum_{j=1}^{i-1} k_j^{(i-j)} q_{i,j}\left\|z_j\right\| + d_i, \quad i = 4,...,r-1. \quad (10d)$$

To achieve the robustness property with respect to unknown but bounded uncertainty, the controller gains $k_1,\dots,k_{n-1}$ have to be chosen in a hierarchically way, from the smallest to the biggest. Thus, since $\bar{G}_1(\bar{z}_1, x_{ref}, w)$ in (10a) does not depend on k_1, the value of this coefficient can be chosen so high that the term $k_1 z_1$ in (9a) will be dominant. By the block linearization procedure, the term $\bar{G}_2(\bar{z}_2, x_{ref,2}, w)$ in (10b) depends on k_1 but not on $k_2,\dots,k_{r-1}$. Then for fixed k_1, the appropriate choice of k_2 value provides the domination of term $k_2 z_2$ for the second block of (9b), and so on.

In order to establish the required hierarchy for the control gains which ensures stability of the sliding mode motion (9a)-(9c), we choose a Lyapunov function candidate V for the system (9a)-(9c) as a sum of Lyapunov function candidates for the each block of (9a)-(9c), namely

$$V = \sum_{i=1}^{r-1} V_i, \qquad V_i = \tfrac{1}{2}z_i^2, \quad i = 1,...,n-1$$

and calculate the derivatives $\dot{V}_i$, $i = 1,...,r-1$ step by step from the first block to the last block of (9a)-(9c).

At the first step, differentiating the Lyapunov function candidate $V_1 = \frac{1}{2}z_1^2$ along the trajectories of (9a) and using assumption A1, namely (10a), we get

$$\dot{V}_1 = -k_1 z_1^2 + z_1 z_2 + z_1\bar{G}_1(\bar{z}_1, x_{ref}, w) = -|z_1|\left[(k_1 - q_{11})|z_1| - |z_2| - d_1\right]$$

which is negative in the region

$$|z_1| > \frac{1}{k_1 - q_{11}} |z_2| + \frac{d_1}{k_1 - q_{11}} .$$

Therefore, the state ultimately enter the domain of subspace (z_1, z_2) defined by

$$|z_1| \le \alpha_{12} |z_2| + \beta_{12}$$

where the parameters α_{12} and β_{12} defined as

$$\alpha_{12} = (k_1 - q_{11})^{-1} \text{ and } \beta_{12} = \alpha_{12} d_1$$

are positive if the following condition holds:

$$k_1 > q_{11} \tag{11}$$

At the second step, following similar lines to those of the first block, the derivative $\dot{V}_2$ of the Lyapunov function candidate $V_2 = \frac{1}{2} z_2^2$ calculated along the trajectories of the second block of (9b), under conditions (10a), (10b) and (11), is given by

$$\begin{aligned} \dot{V}_2 &= -k_2 z_2^2 + z_2 [z_3 + \overline{\boldsymbol{G}}_2 (\overline{\boldsymbol{z}}_2, \boldsymbol{x}_{ref,2}, \boldsymbol{w})] \\ &\le -|z_2| \; [\; (k_2 - q_{22})|z_2| - |z_3| - k_1 q_{21} |z_1| - d_2 \,] \\ &\le -|z_2| [\; (k_2 - q_{22} - k_1 q_{21} \alpha_{12}) |z_2| - |z_3| - k_1 q_{21} \beta_{12} - d_2 \,] \end{aligned}$$

which is negative if

$$(k_2 - q_{22} - k_1 q_{21} \alpha_{12}) |z_2| - |z_3| - k_1 q_{21} \beta_{12} - d_2 > 0 .$$

Hence, the state ultimately enter the domain of the subspace (z_1, z_2, z_3) defined by

$$|z_2| \le \alpha_{23} |z_3| + \beta_{23}$$

and consequently

$$|z_1| \le \alpha_{13} |z_3| + \beta_{13}$$

where the scalar parameters α_{23}, β_{23} α_{13} and β_{13} defined as

$\alpha_{23} = (k_2 - q_{22} - k_1 q_{21} \alpha_{12})^{-1}$,

$\beta_{23} = \alpha_{23} (k_1 q_{21} \beta_{12} + d_2)$, $\alpha_{13} = \alpha_{12} \alpha_{23}$ and

$\beta_{13} = \alpha_{12} \beta_{23} + \beta_{12}$

are positive if the values of k_1 and k_2 satisfy the following inequalities

$$k_1 > q_{11} \quad \text{and} \quad k_2 > q_{22} + k_1 q_{21} \alpha_{12} \tag{12}$$

Proceeding along the same procedure for the $(i-1)^{th}$ block of the system (9a)-(9c), then the convergence domain of the subspace $(z_1, z_2, ..., z_{i-2}, z_{i-1}, z_i)$, is

$$\begin{aligned} |z_1| &\le \alpha_{1,i} |z_i| + \beta_{1,i} \\ |z_2| &\le \alpha_{2,i} |z_i| + \beta_{2,i} \\ &\vdots \\ |z_{i-1}| &\le \alpha_{i-1,i} |z_i| + \beta_{i-1,i} \end{aligned} \tag{13}$$

where $\alpha_{j,i} = \alpha_{j,i-1} \alpha_{i-1,i}$,

$$\alpha_{i-1,i} = \left(k_{i-1} - q_{i-1,i-1} - \sum_{j=1}^{i-2} k_j^{(i-j)} q_{i-1,j} \alpha_{j,i-1} \right)^{-1},$$

$$\beta_{j,i} = \alpha_{j,i-1} \beta_{i-1,i} + \beta_{j,i-1}, \quad j = 1,...,i-1 .$$

At the next step, taking again the derivative of the Lyapunov function $V_i = \frac{1}{2} z_i^2$ along the trajectories of the i^{th} block of (9a)-(9c), and using (10a)-(10d), we obtain

$$\begin{aligned} \dot{V}_i &= -k_i z_i^2 + z_i [z_{i+1} + \overline{\boldsymbol{G}}_i (\overline{\boldsymbol{z}}_i, \boldsymbol{x}_{ref,i}, \boldsymbol{w})] \\ &\le -k_i z_i^2 + |z_i| \left(|z_{i+1}| + q_{i,i} |z_i| + \sum_{j=1}^{i-1} k_j^{i-j} q_{i,j} |z_j| + d_i \right). \end{aligned}$$

Using now (13), we can majorize $\dot{V}_i$ as

$$\begin{aligned} \dot{V}_i \le -|z_i| \Big[& \Big(k_i - q_{i,i} - \sum_{j=1}^{i-1} k_j^{(i-j)} q_{i,j} \alpha_{j,i} \Big) |z_i| \\ & - |z_{i+1}| - \sum_{j=1}^{i-1} k_j^{(i-j)} q_{i,j} \beta_{j,i} - d_i \Big] . \end{aligned}$$

From this equation, it follows that

$$|z_i| \le \alpha_{i,i+1} |z_{i+1}| + \beta_{i,i+1} \tag{14}$$

where the parameters

$$\alpha_{i,i+1} = \left(k_i - q_{i,i} - \sum_{j=1}^{i-1} k_j^{(i-j)} q_{i,j} \alpha_{j,i} \right)^{-1}$$

and $\beta_{i,i+1} = \alpha_{i,i+1} \left(\sum_{j=1}^{i-1} k_j^{(i-j)} q_{i,j} \beta_{j,i} - d_i \right)$,

$i = 4,...,n-1$ are positive if the condition

$$k_i > q_{i,i} + \sum_{j=1}^{i-1} k_j^{(i-j)} q_{i,j} \alpha_{j,i} \tag{15}$$

holds. Substitution of (14) in (13) gives the following set of inequalities for the subspace $(z_1, z_2, ..., z_{i-2}, z_{i-1}, z_i, z_{i+1})$:

$$\begin{aligned} |z_1| &\le \alpha_{1,i+1} |z_{i+1}| + \beta_{1,i+1} \\ |z_2| &\le \alpha_{2,i+1} |z_{i+1}| + \beta_{2,i+1} \\ &\vdots \\ |z_{i-1}| &\le \alpha_{i-1,i+1} |z_{i+1}| + \beta_{i-1,i+1} \\ |z_i| &\le \alpha_{i,i+1} |z_{i+1}| + \beta_{i,i+1} \end{aligned} \tag{16}$$

where $\alpha_{j,i+1} = \alpha_{j,i} \alpha_{i,i+1}$ and $\beta_{j,i+1} = \alpha_{j,i} \beta_{i,i+1} + \beta_{j,i}$, $j = 1,...,i$, $i = 4,...,n-1$.

At the last step we have the domain of convergence of the subspace $(z_1, z_2, ..., z_{n-1})$ defined by the following inequalities:

$$|z_i| \le \alpha_{i,n-1} |z_{n-1}| + \beta_{i,n-1}, \; i = 1,...,n-2 .$$

These expressions are used to evaluate the derivative for the Lyapunov function candidate $V_{n-1} = \frac{1}{2} z_{n-1}^2$ along the trajectories of (9c), that is

$$\begin{aligned} \dot{V}_{n-1} &= -k_{n-1} z_{n-1}^2 + z_{n-1} \overline{\boldsymbol{G}}_{n-1} (\overline{\boldsymbol{z}}_{n-1}, \boldsymbol{x}_{ref,n-1}, \boldsymbol{w}) \\ &\le -k_{n-1} z_{n-1}^2 + |z_{n-1}| \;\; (q_{n-1,n-1} + \sum_{j=1}^{r-2} k_j^{n-1-j} q_{r-1,j} |z_j| \\ &\qquad + d_{n-1}) \\ &\le -\left(k_{n-1} - q_{r-1,n-1} - \sum_{j=1}^{r-2} k_j^{(n-1-j)} q_{r-1,j} \alpha_{j,n-1} \right) \; |z_{n-1}|^2 \\ &\quad + \left(\sum_{j=1}^{r-2} k_j^{(n-1-j)} q_{r-1,j} \beta_{j,n-1} + d_{r-1} \right) \; |z_{n-1}| \end{aligned}$$

If k_{n-1} is chosen such that the condition

$$k_{n-1} > q_{n-1,n-1} + \sum_{j=1}^{r-2} k_j^{(n-1-j)} q_{n-1,j} \alpha_{j,n-1} \tag{17}$$

holds, then we obtain

$$\dot{V}_{n-1} = -2\alpha_{n-1}V_{r-1} + \beta_{n-1}\sqrt{2V_{n-1}}$$

with positive

$$\alpha_{n-1} = k_{n-1} - q_{n-1,n-1} - \sum_{j=1}^{n-2} k_j^{(n-1-j)} q_{n-1,j}\alpha_{j,n-1}$$

and $\beta_{n-1} = \sum_{j=1}^{n-2} k_j^{(n-1-j)} qn_{r-1,j}\beta_{j,n-1} + d_{n-1}$.

By the Comparison Lemma [6], we have

$$|\mathbf{z}_{n-1}(t)| \le \gamma_{n-1,n-1} \exp\left[\tfrac{1}{2}\alpha_{n-1}(t-t_0)\right] + h_{n-1} \qquad (18)$$

where $\gamma_{n-1,n-1} = |z_{n-1}(t_0)| - h_{n-1}$, and $h_{n-1} = \dfrac{\beta_{n-1}}{\alpha_{n-1}}$.

Thus

$$\lim_{t\to\infty} \sup |z_{n-1}(t)| \le h_{n-1} \qquad (19)$$

Therefore, using the obtained upper (18) and ultimate (19) bounds on the solution $z_{n-1}(t)$, and the inequalities (16), and going back, from the $(r-1)^{th}$ block to the first block of (9a)-(9c), we can find step-by-step upper estimations and ultimate bounds on the solutions $z_{n-2}(t),\ z_{n-3}(t), \ldots,\ z_1(t)$.

In order to reduce the effect of the unknown disturbances action in (5), that is, ensure inequalities (12), (15) and (17) for given bounds (10a)-(10d), and, respectively, increase the region of the sliding mode stability (7), it is needed to increase the value of the controller gain k_n in (6).

This high gain, however, can produce chattering due to some defects of the control devices. To solve this problem we propose to adjust the value of the gain k_n depending on the value of s and the equivalent control u_{eq} (8)

$$\boldsymbol{u}_{eq} = \overline{\boldsymbol{B}}_r^{-1}(\boldsymbol{z}, \boldsymbol{x}_{ref,r})\left[\overline{\boldsymbol{f}}_r(\boldsymbol{z}, \boldsymbol{x}_{ref,r}) + \overline{\boldsymbol{G}}_r(\boldsymbol{z}, \boldsymbol{x}_{ref,r}, \boldsymbol{w})\right]$$

For the case of unknown disturbances the value of u_{eq} can be obtained by filtering the control signal as [9]

$$\tau\dot{u}_{eq} + u_{eq} = u$$

The k_n gain modification can be done by using a fuzzy logic scheme. For the case of a bounded control, the value of k_n begins with $k_n = k_{n,\max}$ and then, as s tends to zero, the value of k_n decreases smoothly up to $k_n = k_{n,\min}$, avoiding chattering.

The block diagram of the closed-loop system with block transformation and sliding mode fuzzy logic controller ($SMFLC_{BC}$) is presented in the Figure 1.

The diagram consists of the following parts:

<u>Block Control</u>. This block transforms state **x** to the new coordinate **z**

$$T_{BC}: \mathbf{x} \to \mathbf{z}, \text{ such that } \mathbf{z} = T_{BC}(\mathbf{x})$$

where the map T_{BC} is defined by (3a)-(3c), and computes the value of the following variables:

$$s = z_n \text{ and}$$

$$\boldsymbol{u}_{eq} = \overline{\boldsymbol{B}}_r^{-1}(\boldsymbol{z}, \boldsymbol{x}_{ref,r})\left[\overline{\boldsymbol{f}}_r(\boldsymbol{z}, \boldsymbol{x}_{ref,r}) + \overline{\boldsymbol{G}}_r(\boldsymbol{z}, \boldsymbol{x}_{ref,r}, \boldsymbol{w})\right].$$

<u>*Fuzzy Controller:*</u> This block uses only two inputs: $In_1 = \|s\|$ and $In_2 = \|u_{eq}\|$, and determine the gain k_n such that to satisfy the stability condition (7). This controller consists of the following components:

Input Normalization, which scales inputs.

Fuzzification, which transforms the crisp input values in fuzzified values

$$F: \quad In \to Lin \quad \text{such that } F(In_i) = Lin(i,j)$$

where $In_i \in In$ is a crisp input value defined on the universe of discourse *In*, and *Lin*(*i, j*) is the corresponding fuzzified input value.

Inference Mechanism, which uses the following type rule:

Rule m: If (Lin(1, j) and Lin(2, k)) Then CR(j, k)

where *CR*(*j, k*) is the corresponding k_d gain value for the rule consequent.

Defuzzification which is based on the weighted mean defuzzification method [3] to produce a scalar value k_d calculated as

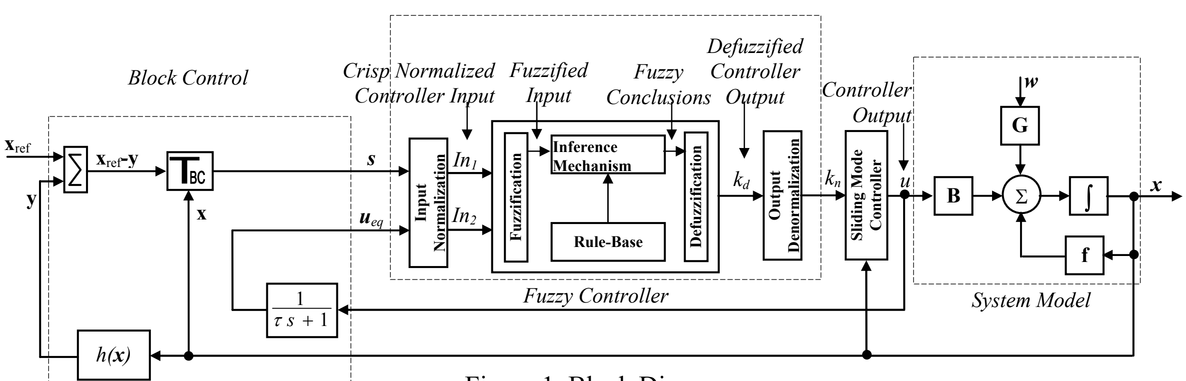

Figure 1. Block Diagram

$$k_d = \frac{\sum_{j=1}^{ne_1}\sum_{k=1}^{ne_2} LAnt(j,k)\, CR(j,k)}{\sum_{j=1}^{ne_1}\sum_{k=1}^{ne_2} LAnt(j,k)} \tag{20}$$

where $LAnt(j,k) = \min(LIn(1,j), LIn(2,k))$ is the premise quantification of the active rule, and ne_i for $i=1,2,$ is the fuzzy set size.

Denormalization which multiplies normalized fuzzy controller output (20) with denormalization factor (*scale*), $k_n = k_d \cdot scale$, such that the system (5) stays stable.

Sliding Mode Controller: which implements (6) to obtain $\boldsymbol{u}$ with $k_r = [k_{r1},...,k_{rm}]$ and $k_{rm} = k_n$.

III STEPPER MOTOR CONTROL

In this section, we apply the proposed scheme to control a permanent magnet stepper motor. Its mathematical model is given by

$$\begin{aligned}
\frac{d\theta}{dt} &= \omega \\
\frac{d\omega}{dt} &= \frac{1}{J}[-K_m i_a \sin(N_r\theta) + K_m i_b \cos(N_r\theta) - B_v\omega - \tau_l] \\
\frac{di_b}{dt} &= \frac{1}{L}[-Ri_b - K_m\omega\cos(N_r\theta) + u_b)] \\
\frac{di_a}{dt} &= \frac{1}{L}[-Ri_a + K_m\omega\sin(N_r\theta) + u_a]
\end{aligned} \tag{21}$$

where, θ is the angular position; ω is the shaft speed; i_a and i_b are the currents in phases *A* and *B* respectively; u_a and u_b are the voltages in phases *A* and *B,* respectively; J is the moment of inertia; *R* and *L* are the resistance and inductance in each of the phase windings, N_r is the number of rotor teeth, K_m is the motor torque constant, B_v is the viscous friction and τ_l presents the load torque perturbation.

Selecting the following state variables, $x_1 = \theta$, $x_2 = \omega$, $x_3 = i_b$, and, $x_4 = i_a$, the inputs as $u_1 = u_b$, and, $u_2 = u_a$, the system (21) is represented as a block controllable system consisting of tree blocks, and subject to the unknown disturbance , $\tau_l = w_1$.

$$\begin{aligned}
[\dot{x}_1]_1 &= [x_2] \\
[\dot{x}_2]_2 &= [-a_2x_2 + b_1(x_1)x_3 - b_2(x_1)x_4] - g_1w_1 \\
\begin{bmatrix}\dot{x}_3\\ \dot{x}_4\end{bmatrix}_3 &= \begin{bmatrix}-c_1b_1(x_1)x_2 - a_3x_3\\ c_1b_2(x_1)x_2 - a_4x_4\end{bmatrix} + b_0\begin{bmatrix}u_1\\ u_2\end{bmatrix}
\end{aligned}$$

where $b_1(x_1) = \frac{K_m}{J}\cos(N_rx_1)$, $b_2(x_1) = \frac{K_m}{J}\sin(N_rx_1)$, $g_1 = \frac{1}{J}$, $a_2 = \frac{B}{J}$, $a_3 = a_4 = \frac{R}{L}$, $c_1 = \frac{J}{L}$, and, $b_0 = \frac{1}{L}$.

Suppose that the output $y = x_1$ is required to track the reference signal x_{1ref}. Following the block transformation procedure, first we define the tracking error as $z_1 = x_1 - x_{1ref}$, and

$$\dot{z}_1 = x_2 - \dot{x}_{1ref}. \tag{22}$$

Then a desired dynamics for z_1 is introduced as

$$\dot{z}_1 = -k_1z_1 + z_2. \tag{23}$$

Solving (22) and (23) for z_2, we obtain

$$z_2 = k_1x_1 + x_2 - k_1x_{1ref} - \dot{x}_{1ref}$$

Then

$$\begin{aligned}
\dot{z}_2 = (k_1 - a_2)x_2 &+ b_1(x_1)x_3 - b_2(x_1)x_4 \\
&- k_1\dot{x}_{1ref} - \ddot{x}_{1ref} - g_1w_1
\end{aligned}$$

From this equation and the desired dynamics

$$\dot{z}_2 = -k_2z_2 + z_3 - g_1w_1$$

we have

$$z_3 = f_3(x_1, x_2) + b_1(x_1)x_3 - b_2(x_1)x_4 + \varphi(t)$$

where z_3 is a new variable,

$f_3 = k_1k_2x_1 + (k_1 + k_2 - a_2)x_2$,

$\varphi(t) = -k_1k_2x_{1ref} - (k_1 + k_2)\dot{x}_{1ref} - \ddot{x}_{1ref}$. In order to have a nonsingular transformation, we introduce a new variable z_4

$$z_4 = -b_2(x_1)x_3 - b_1(x_1)x_4$$

such that the matrix $\mathbf{B}_2 = \begin{bmatrix} b_1(x_1) & -b_2(x_1) \\ -b_2(x_1) & -b_1(x_1)\end{bmatrix}$ has full rank.

In order to obtain the control action, first we define the switching functions as

$$\begin{bmatrix}s_1\\ s_2\end{bmatrix} = \begin{bmatrix}f_3(x_1, x_2)\\ 0\end{bmatrix} + \begin{bmatrix} b_1(x_1) & -b_2(x_1) \\ -b_2(x_1) & -b_1(x_1)\end{bmatrix}\begin{bmatrix}x_3\\ x_4\end{bmatrix} + \begin{bmatrix}\varphi(t)\\ 0\end{bmatrix}$$

Then the projection motion on the subspace s_1, s_2 is governed by

$$\begin{bmatrix}\dot{s}_1\\ \dot{s}_2\end{bmatrix} = \begin{bmatrix}\bar{f}_3(\mathbf{x})\\ \bar{f}_4(\mathbf{x})\end{bmatrix} + b_0\mathbf{B}_2\begin{bmatrix}u_1\\ u_2\end{bmatrix} + \begin{bmatrix}\dot{\varphi}(t)\\ 0\end{bmatrix}$$

where $\mathbf{x} = (x_1, x_2, x_3, x_4)^T$; $\bar{f}_3(\mathbf{x})$ and $\bar{f}_4(\mathbf{x})$ are continuous functions. The control strategy is selected as

$$\begin{bmatrix}u_1\\ u_2\end{bmatrix} = -\mathbf{B}_2^{-1}\begin{bmatrix}k_3 sign(s_1)\\ k_4 sign(s_2)\end{bmatrix}$$

and the sliding mode stability conditions are

$$b_0k_3 > \left|\bar{f}_3(\mathbf{x}) + \dot{\varphi}(t)\right| \text{ and } b_0k_4 > \left|\bar{f}_4(\mathbf{x})\right|.$$

Under these conditions the state converges to the sliding manifold $s_1 = 0, s_2 = 0$, and when this manifold is reached the sliding mode motion is described by the second order system with unknown nonvanishing perturbation

$$\dot{z}_1 = -k_1 z_1 + z_2$$
$$\dot{z}_2 = -k_2 z_2 + w_1 .$$

In order to reduce the disturbances influence, we apply the proposed sliding mode fuzzy logic control scheme for adjusting the gain k_4 such that $k_4 \le u_0$ with $u_0 > 0$.

IV SIMULATION RESULTS

In this section, simulation results are presented for the Permanent Magnet Stepper Motor with parameters: $L = 10\,mH$, $R = 8.4\,\Omega$, $J = 3.6 \times 10^{-6}\ Kg\,m^2$, $k_m = 0.05\,Vs/rad$, $N_r = 50$, $B = 1 \times 10^{-4}\ Nms/rad$. The maximal supplied voltage is $u_0 = 2\ V$. Figures 2 and 3 shows the behavior of the state (x_1, x_2, x_3, x_4) as well as of the new state (z_1, z_2, z_3, z_4) .

Figure 2 displays Block Control Tracking (BCT) results in presence of disturbance (0.5 seg, 10% of nominal torque) without Fuzzy Logic Control (FLC), there we observe chattering and disturbances effects. On the other hand, Figure 3 displays BCT with Fuzzy Logic Control (FLC) results using the proposed approach, where the chattering is reduced.

V CONCLUSIONS

As can be seen, the proposed scheme performance is encouraging. For the simulations, we assume that the disturbance can not be measured, which is an extreme situation. However, the proposed hierarchical sliding mode control approach with the fuzzy logic control, improves the system behavior, reducing chattering and guaranteeing stability.

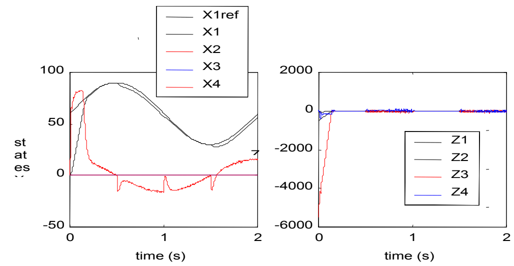

Figure 2: BCT with disturbances without FLC.

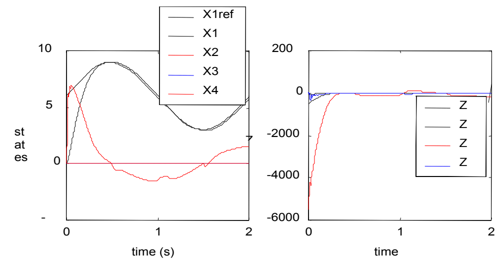

Figure 3: BCT with FLC and disturbances

VI REFERENCES

1. Alexík M, Vittek J. (1994) Adaptive sliding mode control of position servo system. IFAC, *Workshop on Sliding Modes*. Smolenice, Slovakia., September 7-10, pp 278-283.
2. Drajenovic B. (1969) The invariance conditions in variable structure systems. Automatica; 5: 287-295.
3. Driankov D, Hellendoorn H, Reinfrank M. (1996) An Introduction to Fuzzy Control. Springer-Verlag, USA.
4. Ha Q.P, Nguyen Q.H, Rye D.C, Durrant-Whyte H.F. (2001) Fuzzy sliding-mode controllers with applications. IEEE Trans Ind Electron; 48.1: 38-46.
5. Kaynak O, Erbatur K, Ertugrul M. (2001) The fusion of computationally intelligent methodologies and sliding-mode control-a survey. IEEE Trans Ind Electron; 48.1: 4-17.
6. Khalil H.K. (1996) Nonlinear Systems. 2nd ed. Prentice-Hall, New-Jersey.
7. Loukianov A.G. (1998) Nonlinear block control with sliding mode. Automation and Remote Control; 59.7: 916-933.
8. Palm R, Driankov D, Hellendoorn H. (1997) Model Based Fuzzy Control - Fuzzy Gain Schedulers and Sliding Mode Fuzzy Controllers. Springer-Verlag, Germany.
9. Utkin V.I. (1992) Sliding Modes in Control Optimization. Springer-Verlag, USA.
10. Utkin V.I, Guldner J, Shi J. (1999) Sliding Mode Control in Electromechanical Systems. Taylor and Francis.
11. Wong L.K, Leung F.H.F, Tam P.K.S. (2001) A fuzzy sliding controller for nonlinear systems. IEEE Trans Ind Electron; 48.1: 32-37.