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Stability and Convergence in Adaptive Systems*

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Abstract - **Sufficient conditions for adaptive control to ensure stability and convergence to a controller that is robustly stabilizing and performing are developed, provided that such a controller exists in the candidate controller pool. The results can be used to interpret any cost-minimizing adaptive scheme. An example of how a recently developed adaptive switching method can fail to select a stabilizing controller is presented, and a correction is proposed.**

Key words: adaptive control, stability, convergence, robustness, unfalsified control, model-free, learning

I. Introduction

Adaptive control algorithms aim to achieve stability and performance goals by using real-time experimental data to change controller parameters or, more generally, to switch among a given pool of candidate controllers. A good adaptive control algorithm must have the ability to reliably detect when an active controller is failing to meet stability and performance goals, else the algorithm cannot be guaranteed to converge. Typically, adaptive theories achieve convergence objectives by restricting attention to plants assumed to satisfy assumptions, e.g., the well known but difficult-to-satisfy standard assumptions of adaptive control [11]. The use of standard assumptions has been widely criticized, and recent progress in adaptive control has focused on switching adaptive controller schemes that eliminate the most troublesome assumptions on the plant (e.g., [5], [6], [7], [8]). There are even some algorithms for which convergence can be assured with essentially *no assumptions* on the plant, including a stochastic trial-and-error switching method of Fu and Barmish [12] and a mixed-sensitivity unfalsified control algorithm of Tsao and Safonov [4]. These algorithms are data driven. They have the ability to experimentally detect controllers that fail to meet goals without prior knowledge of the plant. If a candidate controller is available that meets performance and stability goals, these data-driven switching algorithms reliably converge to a controller that meets stability and performance objectives.

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In this paper, we study the stability and convergence of data-driven adaptive control systems, with a view towards identifying and generalizing the properties that distinguish those adaptive algorithms that consistently and reliably identify controllers that achieve stability and performance goals. We shall develop a model-free criterion for cost-minimization based adaptive algorithms to converge to a controller that stabilizes the system and achieves specified performance goal whenever such controller exists in the candidate pool. We adopt the falsification paradigm proposed in [1] and advanced in [2], [4], [8] for deciding how the controllers are selected from the pool. A feature of these direct data driven methods is the introduction of the concept of a *fictitious reference signal* that plays a key role in eliminating the burden of exhaustive on-line search over the candidate controllers that was present in the earlier direct switching adaptive algorithms [12], [5]. In this regard, the fictitious reference signal is analogous to the plant-model identification error signal of multi-model adaptive switching methods like [6], [7].

The key idea behind the falsification paradigm as applied in [1], [2], [4] is to associate a data-driven *cost function* with each controller model in the candidate pool. In a sense, any adaptive algorithm can be associated with a cost function that it minimizes. The very fact that an algorithm chooses a controller based on data implies that the algorithms orders controllers based on data. The ordering itself defines such a cost function. For example, recently proposed switching adaptive schemes [7], [8], [12] associate candidate controllers with candidate plant models, and order these controllers according to how closely its associated plant model fits measured plant data. The measure of model closeness to data is the data-driven cost function with respect to which the multi-model adaptive control (MMAC) methods of [6], [7], [10] are optimal. Convergence for MMAC algorithms is assured by assuming the true plant is sufficiently close to the identified model so that it is within the robustness margin of its associated controller model. In the absence of the sufficient closeness assumption on the plant, these may not necessarily converge or even provide stabilization for the true plant. In the present paper, we shall derive plant-assumption-free conditions under which stability and performance are guaranteed.

The paper is organized as follows. In Section II, the fundamental notions related to the problem we deal with are introduced [1], [2], and some basic results from the stability theory of adaptive control are discussed. The problem formulation is presented in Section III. In Section IV, our main results are stated giving plant-model-free sufficient conditions on the cost function for stability and convergence of optimal data-driven adaptive methods. An example of an optimal MMAC switching algorithm is given in the Section V. A weakness of the MMAC algorithm in failing to identify and correct unstable behavior is demonstrated, and a proposed correction is produced based on our Proposition 1. The paper concludes with some remarks in Section VI.

II. PRELIMINARIES

A few notions from the behavioral theory of dynamical systems are recalled next [3]. We review some of the relevant notions for the problem of data-driven discovery of controllers that fit control goals, as outlined in [2], [3], [4] and [13]. A given phenomenon (plant, process) produces elements (outcomes) that reside in some set **Z** (universum). A subset **B** $\subseteq$ **Z** (behavior of the phenomenon) contains all *possible* outcomes. The mathematical model of the phenomenon is the pair (**Z**, **B**). Set **T** denotes an underlying set that describes the evolution of the outcomes in **B** (usually, the time axis). We distinguish between manifest variables ($z_{manifest} \in \mathbf{Z}$) that describe explicitly the behavior of the phenomenon, and latent (auxiliary) variables ($z_{latent} \in \mathbf{Z}$); e.g., plant input and output may serve as the manifest data ($(u,y) \in L_{2e} \times L_{2e} \subset \mathbf{Z}$).

In this context, we define the linear truncation operator P_τ: $\mathbf{Z} \to \mathbf{Z}_\tau$ as:

$$(P_\tau z)(t) = \begin{cases} z_{manifest}(\mathrm{t}) & t \le \tau \\ 0 & otherwise \end{cases}$$

This definition differs slightly from the usual definition of the truncation operator (cf. [14]) in that the truncation is performed with respect to both time and signal vector z.

Measured data set [1], [3] contains the observed (measured) samples of the manifest plant data:

$\{z_{data}\} = \{(y_{data}, u_{data})\} \subset \mathbf{B}_{\mathbf{P_{true}}}$, where $\mathbf{B}_{\mathbf{P_{true}}}$ is the behavior of the true plant . The actually available plant data at time τ is $\mathrm{P}_\tau(z_{\mathrm{data}}) \subset \mathrm{P}_\tau(\mathbf{B}_{\mathbf{P_{true}}})$.

Set **K** denotes a finite set of candidate controllers. The *fictitious reference signal* $\tilde r(K, P_\tau z_{data}, \tau)$ is the reference signal that would have exactly reproduced the measured signals $P_\tau(z_{data})$ had the controller K been in the loop when the data was collected.

Almost any adaptive control algorithm associates a suitably chosen cost function with a particular controller that minimizes this cost. In multiple-model/multiple-controller switching scheme, this function has a role of ordering candidate controllers according to the chosen criterion. A data-driven cost-minimization paradigm used here implies that the ordering of the controllers is based on the available plant data. Therefore, the cost (call it V) admits the following definition:

Definition 1. The *cost functional* V is a mapping:

$$V: \mathrm{P}_\tau \mathbf{Z} \times \mathbf{K} \times \mathbf{T} \to \mathbb{R}_+$$

for the given controller K, measured data $P_\tau z_{data} \in P_\tau \mathbf{Z}$ and $\tau \in \mathbf{T}$. ♦

The cost V represents the cost that would be incurred had the controller K been in the loop when data $P_\tau z_{data}$ was recorded.

Definition 2. The *true cost* $V_{true} : \mathbf{K} \to \mathbb{R}_+ \cup \{\infty\}$ is defined as:

$$V_{true}(K) := \sup_{z \in \mathbf{B}_{P_{true}}, \tau \in \mathbf{T}} V(K, P_\tau z, \tau)$$

♦

The true cost represents, for each K, the maximum cost that would be incurred if we had a chance to perform an infinite duration experiment, for all possible experimental data. Thus, V_{true} is an abstract notion, as it is not actually known at any finite time.

Note: This definition implies that at any time τ the current unfalsified cost V is *upper-bounded* by the true cost V_{true}. However, for some controller K both the true cost and the unfalsified cost can have infinite values (this is the case when K is destabilizing); thus, it should be understood that V may not have a finite-valued bound when K is not stabilizing and unstable behaviors are excited.

Let $[y_{data}, u_{data}]$ represent the output signals of the supervisory feedback adaptive system Σ: $L_{2e} \to L_{2e}$ in Fig. 2-1.

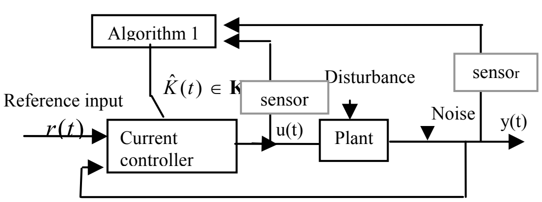

Figure 2-1: Supervisory feedback adaptive control system Σ

Throughout the paper, we make the assumption that all components of the system under consideration have zero input – zero output property.

Definition 3. (*Stability*): A system with input $\boldsymbol{w}$ and output $\boldsymbol{z}$ is said to be *stable* if $\forall \boldsymbol{w} \in L_{2e}, \boldsymbol{w} \neq \mathbf{0}$,

$\limsup_{\tau \to \infty} \|z\|_\tau / \|\boldsymbol{w}\|_\tau < \infty$; if, in addition,

$\sup_{w \in L_{2e}, w \neq 0} \left(\|z\|_\tau / \|w\|_\tau \right) < \infty$, the system is said to be *finite-gain stable*; otherwise, it is said to be *unstable*. ♦

Specializing to the system in Fig. 2-1, stability means: $\lim \sup \|[y,u]\|_\tau / \|r\|_\tau < \infty, \ \forall r \in L_{2e}, r \neq 0$.

Definition 4. A robustly stabilizing and performing controller K_{RSP} is a controller that *stabilizes* the given plant and *minimizes the true cost* V_{true}. ♦

Therefore, $K_{RSP} = \arg\min_{K \in \mathbf{K}} (V_{true}(K))$. Note that K_{RSP} is not necessarily unique.

III. Problem formulation

The problem we pursue in this study can be formulated as follows:

Derive the plant-assumption-free conditions under which stability of the adaptive system and convergence of the adaptive algorithm are guaranteed.

Definition 5. A *data-driven adaptive control law* is an algorithm that selects at each time τ a controller $\hat{K}_\tau$ dependent on experimental data. ♦

Note: There are different ways of actually choosing a controller (see e.g. [1], [4]). The selection algorithm used in this paper is the ε - *cost minimization algorithm* defined as follows. The algorithm outputs, at each time instant τ, a controller $\hat{K}_\tau$ which is the active controller in the loop.

Algorithm 1.

1. Initialize: Let t=0, τ=0; choose $\varepsilon > 0$.
   Let $\hat{K}_t \in \mathbf{K}$ be the first controller in the loop.
2. $\tau \leftarrow \tau+1$.
   If $V\left(\hat{K}_t, P_\tau z, \tau\right) > \min_{K \in \mathbf{K}} V\left(K, P_\tau z, \tau\right) + \varepsilon$
   then $t \leftarrow \tau$ and $\hat{K}_t \leftarrow \arg\min_{K \in \mathbf{K}} V\left(K, P_\tau z, \tau\right)$ (3-1);
3. $\hat{K}_\tau \leftarrow \hat{K}_t$; ***return*** $\hat{K}_\tau$;
4. ***go to*** 2.;

time instant t is the time of the last controller switch. The switch occurs only when the current unfalsified cost related to the currently active controller exceeds the minimum (over all K) of the current unfalsified cost by at least ε. Here, ε serves to limit the number of switches to a finite number, and so prevents the possibility of limit cycle type of instability that may occur when there is a continuous switching between two or more stabilizing controllers. It also ensures a non-zero dwell time between switches.

Throughout the rest of the paper we will have the following standing assumption. It is much less restrictive than the so-called ‘standard assumptions’ from the traditional adaptive literature (e.g. knowledge of the plant relative degree, high frequency gain, LTI minimum phase plants etc. [11]) or even the assumptions made in the recent works on supervisory switching MMAC methods [6], [7], [8] (assumption that the real plant is sufficiently close to a model in the assumed model set). In fact, the following assumption is inherently present in all other adaptive schemes, and it is ***minimal***, provided that we do not consider control laws such as dither control to be in the candidate set.

Assumption 1. The candidate controller set **K** contains at least one robustly stabilizing and performing controller. ♦

The performance cost functional V is chosen to have the following property:

Property 1. (Monotone non-decreasing cost property): For all τ_2, τ_1 such that $\tau_2 \geq \tau_1$:

$$\forall \mathrm{K} \in \mathbf{K}, \forall z_{data} \text{ with which K is consistent :}$$
$$V(K, P_{\tau 2} z, \tau_2) \geq V(K, P_{\tau 1} z, \tau_1)$$

Note: When V is monotonically non-decreasing in time, its optimal (minimal) value $\min_{K \in \mathbf{K}} V(\mathrm{K}, P_\tau z, \tau)$ is monotonically non-decreasing in time and uniformly bounded above for all $z \in \mathbf{Z}$ by $V_{true}(\mathrm{K}_{RSP})$:

$$\min_{K \in \mathbf{K}} V(\mathrm{K}, P_{\tau_1} z, \tau_1) \leq \min_{K \in \mathbf{K}} V(\mathrm{K}, P_{\tau_2} z, \tau_2), \forall \tau_2 > \tau_1$$

Definition 6. (*Unfalsified stability*): Given $K \in \mathbf{K}$ and measured data $\left[y_{data}, u_{data}\right]$ we say that the stability of the system given in Fig. 2-1 is *falsified* if

$$\exists \tilde{r}\left(K, z_{data}\right) \text{ such that } \limsup_{\tau \to \infty} \left(\frac{\left\|\left[y_{data}, u_{data}\right]\right\|_\tau}{\|\tilde{r}\|_\tau} \right) = \infty .$$

Otherwise, it is said to be unfalsified. ♦

Definition 7. A system is said to be *cost detectable* if, whenever stability of the system in Fig. 2-1 is falsified by data $z_{data} = \left[y_{data}, u_{data}\right]$, then

$$\lim_{\tau \to \infty} V\left(K, P_\tau(z_{data}), \tau\right) = \infty . \quad ♦$$

Note: The definition says that the unstable behavior associated with a non-stabilizing controller $K \in \mathbf{K}$ leads to unboundedness of the cost function V.

Definition 8. (Sufficient Richness): We say the *system input is sufficiently rich* if $\forall K$,

$$\lim_{\tau \to \infty} \max_{z \in \Sigma} V(K, P_\tau z, \tau) \geq (V_{true}(K_{RSP}) := \min_{K \in \mathbf{K}} V_{true}(K)) \quad ♦$$

Essentially, sufficient richness of the system input is necessary but not sufficient to ensure cost convergence of an adaptive control algorithm in the following sense:

$\lim_{\tau\to\infty} V_{true}(K, P_\tau z, \tau) = V_{true}(K_{RSP}),\ \forall K \in \mathbf{K}$. A sufficiently rich input contains enough frequencies to excite the unstable dynamics of the system and thus increase the unfalsified cost *V*.

IV. RESULTS

Let the Assumption 1 hold.

Proposition 1. Consider the feedback adaptive control system from Fig. 2-1. If the associated cost function has the properties of cost-detectability, monotone non-decrescence in time, and uniform boundedness from above by the true cost $V_{true} : \mathbf{K} \to \mathbb{R}^+ \cup \{\infty\}$ for all plant data, then the switching adaptive control Algorithm 1 will always converge with finitely many controller switches and yield *unfalsified stability* of the closed loop system satisfying $V\left(K, P_\tau(z_{data}), \tau\right) \le V_{true}(\mathrm{K}_{RSP})$ for all τ, where $z_{data} = \left[y_{data}, u_{data}\right]$. Moreover, if the system input is sufficiently rich, the sequence of optimal unfalsified costs $V\left(\hat{K}_\tau, \tau\right)$ will converge to $V_{true}(\mathrm{K}_{RSP}) \pm \varepsilon$.

Proof.

Let the current controller in the loop at time τ_0 be $\hat{K}$. Let $z_{data} = [r, u_{data}, y_{data}] \in \mathbf{B}_{\mathbf{P_{true}}}$. Suppose the stability of the closed-loop system with $\hat{K}$ in the loop is falsified by the data $\left[u_{data}, y_{data}\right]$ ($\exists \tilde{r}$ such that $\lim_{\tau\to\infty} \sup \left(\left\| \left[y_{data}, u_{data} \right] \right\|_\tau / \left\| \tilde{r} \right\|_\tau \right) = \infty$). Due to the cost-detectability property of *V*, $\lim_{\tau\to\infty} V\left(\hat{K}, P_\tau[, u_{data}, y_{data}]\right) = \infty$. In particular, for some $\tau_1 > \tau_0$,

$V\left(\hat{K}, P_{\tau_1}[u_{data} \quad y_{data}]\right) > V_{true}(K_{RSP}) + \varepsilon$ (due to (3-1) in Algorithm 1). Hence the controller $\hat{K}$ must switch before time τ_1 and the unfalsified cost $V\left(\hat{K}, \tau\right)$ must exceed $\min_{K\in\mathbf{K}} V\left(K, \tau\right)$ by at least ε by the time of the switch:

$$V\left(\hat{K}, P_{\tau_1}[u_{data} \quad y_{data}]\right) > \min_{K\in\mathbf{K}} V\left(K, P_{\tau_1}[u_{data} \quad y_{data}]\right) + \varepsilon .$$

If each controller is switched exactly 0 or 1 times, then we trivially have finite number of switches (since **K** is finite). If at least one controller is switched more than once (e.g. $\hat{K}$ switched at τ_0 and later, at τ_1), then due to Algorithm 1 the difference in the minimal cost between two consecutive switches must be greater than ε (recall monotonicity of the cost increase),

$$V\left(\hat{K}, P_{\tau_1}[u_{data} \quad y_{data}]\right) > V\left(\hat{K}, P_{\tau_0}[u_{data} \quad y_{data}]\right) + \varepsilon .$$

Since $\min_{K\in\mathbf{K}} V\left(K, \tau\right)$ is bounded above by $V_{true}\left(K_{RSP}\right)$, the number of switches to the same controller is upper-bounded by $V_{true}\left(K_{RSP}\right)/\varepsilon$, which is finite. Since $N \triangleq card(\mathbf{K}) < \infty$, the overall number of switches is upper-bounded by $(N+1) \cdot V_{true}\left(K_{RSP}\right)/\varepsilon$. ■

Note that at any time, a controller switched in the loop can remain there for an arbitrarily long time although it is different from K_{RSP}. However, if the system input is sufficiently rich so as to increase the cost more than ε above the level $V\left(\hat{K}, \tau_1\right)$ at the time of the last switch τ_1, a switch to a new controller that minimizes the current cost $V\left(\hat{K}_\tau, \tau\right)$ will eventually occur at some time $\tau_2 > \tau_1$. According to Property 1, the values of these cost minima at any time are monotone increasing and bounded above by $V_{true}(\mathrm{K}_{RSP})$. Sufficient richness will affect the cost to approach $V_{true}(\mathrm{K}_{RSP}) \pm \varepsilon$.

For finite ε, we *always* have guaranteed convergence to K_{RPS} after a finite number of steps. In practice, it may suffice to use ε=0 so that switching and adaptation can occur continuously. However, in this case the conditions of Proposition 1 are no longer satisfied and stability of the adaptive system is no longer guaranteed.

V. EXAMPLE AND DISCUSSION

Here, we present an example that shows how the adaptive control method using fixed multiple models [9], [6], [10] may fail to stabilize the plant if some of the conditions of Proposition 1 do not hold, even if there is a stabilizing controller among the candidate controllers. The switching Algorithm 1 with a cost function obeying the conditions of Proposition 1 succeeds in finding a stabilizing controller..

In adaptive control method using fixed multiple models, there is a group of N candidate plant models $P_i,\ i \in \{1,...N\}$, with corresponding candidate controllers $C_i\ ,i \in \{1,...N\}$, designed for the unknown plant $W_p(s)$. The C_i*'s* are designed so as to meet the control objective of the corresponding candidate plant models. The candidate plant model, which best represents the actual plant (has the least cost value), is identified at each instant and the corresponding controller is switched into the loop.

In the following example, the structure of plant models and controllers are the same as in [6] with parameters $\left(\beta_0, \beta_1^T, \alpha_0, \alpha_1^T\right)^T$ for plant models and $\left(k, \theta_1^T, \theta_0, \theta_2^T\right)^T$ for controllers. Two candidate plant models and their corresponding controllers are designed so that their parameters are far from those of the true plant P^* and its corresponding controller C^*. These parameters are listed in

Table 1. The controlled plant in feedback with the controller is shown in Fig. 5-3. The control specification is assigned via the reference model $W_m(s) = 1/(s+3)$, while the unknown plant is $W_p(s) = 1/(s+5)$. The input is a step signal. The simulations are carried out with a dwell time of 0.001 sec. All initial conditions are zero. The cost function $J(t)$ to be minimized is, as in [6]:

$$J(t) = e_{I_j}^2(t) + \int_0^t \exp^{-\lambda(t-\tau)} e_{I_j}^2(\tau)\,d\tau, \quad j = 1,2 \tag{5-1}$$

where $e_1(t)$ is the identification error and $\lambda = 0.05$ (λ is a non-negative forgetting factor that determines the weight of past data). Fig. 5-1 represents the on-line values of the cost function (5-1) for both identifiers, when either controller C_1 or C_2 is initially in the loop. C_2 is switched into the loop since it has smaller cost value than C_1 from the very beginning. However, C_2 is destabilizing, as can be confirmed by the analysis of the stability margins listed in Table 1, whereas C_1 is stabilizing. The adaptive control method in [6] based on minimizing the cost (5-1) fails to pick the stabilizing controller in this case. The cost (5-1) for both controllers quickly blows up regardless of which controller is in the loop initially.

To avoid choosing a destabilizing controller, we use the switching Algorithm 1 with the following cost function:

$$J(t) = \max_{l \in (0,t)} \left\{ \frac{\tilde{e}_i^2(l) + \int_0^l \exp(-\lambda(l-\tau)) \cdot \tilde{e}_i^2(\tau)d\tau}{\int_0^l \exp(-\lambda(l-\tau)) \cdot \tilde{r}_i^2(\tau)d\tau} \right\}, \quad i=1,2 \tag{5–2}$$

where $\int_0^{\ell} \exp^{-\lambda(\ell-\tau)} \tilde{r}_i^2(\tau)\,d\tau \neq 0 \cdot \tilde{r}_i, \tilde{e}_i$ are the fictitious reference signal and the fictitious error defined in [9]. The corresponding unfalsified cost can be calculated as shown in equation (5-3) at the end of the paper, where the controller K_i is given as $K_i = \left[1/k_i \quad -\theta_{0_i}/k_i\right]^T$ (i=1, 2), and ω_m is the impulse response for the reference model $W_m(s)$. The unfalsified cost (5-3) satisfies the conditions of the Proposition 1. We now use Algorithm 1 to simulate the adaptive system described above. At time t=0 one of the controllers was selected as the initial one and put in the loop. The stabilizing controller C_2 was quickly switched into the loop. The parameter ε is set to 0.001. Fig.5-2 shows the simulation result of the unfalsified cost for both controllers: the cost of C_1 is much smaller than that of C_2 (regardless of which controller is initially in the loop) and thus it will be switched into the loop. The stabilizing controller C_1 is successfully chosen.

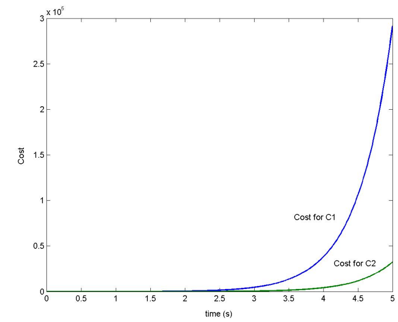

Figure 5-1: Cost (5-1) of C_1 and C_2 ; MMAC method

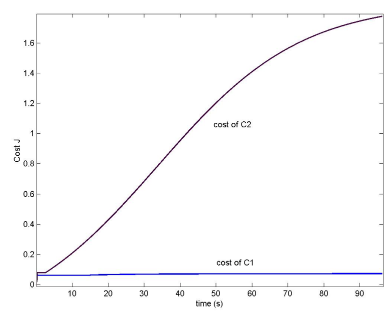

Figure 5-2: Cost (5-2) of C_1 and C_2 ; Algorithm 1

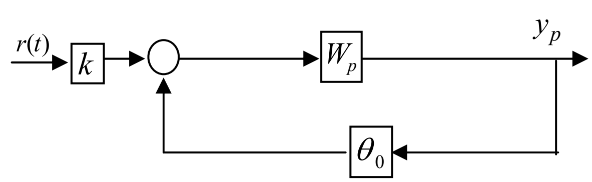

Figure 5-3: Feedback control system

VI. CONCLUSION

In this paper we studied the problem of stability and convergence in switching adaptive control. Noting that every adaptive scheme is optimal with respect to some data-driven controller-ordering cost function, we have examined the question of finding sufficient conditions on the cost function to ensure stability and convergence of the adaptive control system given the minimal assumption that there is at least one stabilizing controller in the candidate set.

Essentially our main conclusion is that if the **cost function** is selected so that its optimal value $\bar{V}(P_\tau z_{data}) \triangleq \min_{K \in \mathbf{K}} V(\mathrm{K}, P_\tau z_{data}, \tau)$ is *monotonically increasing*, *uniformly bounded above by* $V_{true}(\mathrm{K}_{RSP}) := \min_{K \in \mathbf{K}} V_{true}(\mathrm{K}) < \infty$, and the *cost detectability* holds, then the robust stability of the adaptive system is guaranteed whenever the candidate controller pool contains at least one stabilizing controller. If, in addition, system signals are sufficiently rich, convergence of the cost towards $V_{true}(\mathrm{K}_{\mathrm{RSP}})$ is guaranteed. An example showed how a typical MMAC switching adaptive scheme can fail to recognize and remove a destabilizing candidate controller from the feedback loop, and that this unstable behavior can be explained in terms of the failure of the model-error cost function associated with such MMAC schemes to satisfy the convergence conditions given by our stability and convergence results in Section IV. Based on these results, a modification MMAC cost function is proposed and demonstrated to remedy the MMAC instability problem.

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$$V(K_i, P_t\left[y_{data}, u_{data}\right], t) = \max_{l \in (0,t)} \left\{ \frac{\left(y_{data}(l) - \omega_m * \left(K_i^T \cdot \begin{bmatrix} y_{data}(l) \\ u_{data}(l) \end{bmatrix} \right) \right)^2 + \int_0^l \exp(-\lambda(l-\tau)) \cdot \left(y_{data}(\tau) - \omega_m * \left(K_i^T \cdot \begin{bmatrix} y_{data}(\tau) \\ u_{data}(\tau) \end{bmatrix} \right) \right)^2 d\tau}{\int_0^l \exp(-\lambda(l-\tau)) \cdot \left(K_i^T \cdot \begin{bmatrix} y_{data}(\tau) \\ u_{data}(\tau) \end{bmatrix} \right)^2 d\tau} \right\} \tag{5-3}$$

Table 1: Parameters of plant, models and controllers

	Parameters of Plant Models					Parameters of Controllers					Stability Analysis of the Closed Loop System		
	β_0	β_1	α_0	α_1		k	θ_1	θ_0	θ_2		Open Loop TF $(-W_p\theta_0)$	GM (dB)	PM (deg)
P^*	1	0	-2	0	C^*	1	0	2	0	Sys^*	-2/(s+5)	7.96	Inf
P_1	2	0	4	0	C_1	0.5	0	-2	0	Sys_1	2/(s+5)	Inf	Inf
P_2	1	0	-6	0	C_2	1	0	6	0	Sys_2	-6/(s+5)	-1.58	-33.5