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Nonlinear Flow Control for Network Traffic Management with Capacity Constraints

Yi Fan and Zhong-Ping Jiang

Department of Electrical and Computer Engineering
Polytechnic University
Six Metrotech Center
Brooklyn, 11201, U.S.A

yfan01@utopia.poly.edu, zjiang@control.poly.edu

***Abstract*— In this paper we present our nonlinear flow control schemes based on a buffer management model with physical constraints. It extends previous result of Pitsillides, Ioannou and Rossides in [10] by improving the buffer management of a network to better serve a class of traffic. Extension to decentralized control of a large scale network is also considered. The proposed discontinuous controller asymptotically regulates the buffer queue length at the output port of a router/switch to a constant value, in the face of unknown interfering traffics and control input constraint. Its continuous approximation achieves practical regulation with an ultimate bound on the regulation error tunable by a design parameter.**

Keywords Congestion control, Capacity constraints, Buffer management, Asymptotic regulation

I. Introduction

Flow control is an important aspect of network traffic management. It has been heavily investigated in different environments such as ATM, TCP/IP, wireless network and mobile ad hoc network. While heuristic and emulation/experiment based approaches are popular among engineers and researchers, model based schemes have also been largely explored. For example, linear and nonlinear analysis and control design tools have been proven effective in ABR traffic control of ATM networks [6], congestion control in TCP [4][8], network performance analysis with time delay [1], and many other issues and references cited in recent literature. How control theory can be systematically used to address new challenges in networks is of great interest.

We focus our attention on the application of nonlinear control theory to the networking problems. Among the many publications in this area, we discuss some results that are closely related to the topic of our paper. In [10], the authors proposed a nonlinear congestion controller for a buffer management model. The control objective is constant buffer queue length regulation. Using feedback linearization and robust adaptive control ideas, the authors achieved bounded regulation due to unknown interfering traffics.

Our work is in part inspired by the above discussion with particular interest to improve the regulation when facing disturbances and physical constraints. Instead of only considering a single network node as in [10], we are also interested in designing controller for a large scale network.

When considering the controller design for a single node in the network, we base our work on the same model as in [10] and modify their control law. When extending our controller design to a large scale network composed of many nodes, instead of only considering the disturbance traffic bounded by a constant, we also address the case when the bound on the disturbance traffic is time varying, which is not addressed in [10]. In particular, we handle the disturbance traffic whose upper bound depends on the states of other interconnected nodes. We use sliding mode control to achieve asymptotic queue length regulation under certain assumptions. To eliminate possible undesirable controller behaviors such as "chattering" due to the controller discontinuity, a continuous approximation of the discontinuous controller is given along with stability analysis. Practical regulation is achieved with the continuous controller where the ultimate bound on the queue length is determined by a design parameter.

Our result and its comparison with the work in [10] are shown through theoretic analysis and simulations. One contribution of this paper is that we achieve asymptotic regulation as opposed to the bounded regulation in [10]. The same type of control law can be applied to a large scale system to achieve asymptotic regulation in a partially decentralized manner, with different design parameters tuned at each node.

The physical constraints on the control input and state variables is an important issue in many control systems. Many results have been established on the stabilization of linear systems with control input saturation constraint [5], while less work is known for nonlinear systems. Another contribution of our paper is that we specify the sufficient conditions under which asymptotic regulation is achieved under the physical constraints caused by limited capacity and link buffer size.

The rest of the paper is organized as follows: in Section 2, we introduce a differential equation model that follows from previous work on this subject. Design objectives are given with practical limitation in mind. The controller design for scalar systems is addressed in Section 3. The control law is further extended to an interconnected network in Section 4. Our conclusion in summarized in Section 5.

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II. Problem Formulation and Control Objectives

The following model uses the conservation law to establish buffer queue length dynamic equation at the output port of a node in the network. By "node", we refer to a router/switch in the network for the rest of the paper. The operating condition of the differential equation matches to the steady state of a M/M/1 queue; see [10] for more details. It should be noted that certain normalization is used for model simplification and unit match.

$$\dot{x}(t) = -\frac{x(t)}{1+x(t)} \cdot C(t) + \lambda(t) \quad (1)$$

$$x(t) \in [0, x_{buffer}] \quad (2)$$

$$C(t) \in [0, C_{server}] \quad (3)$$

Queue length x is taken as the state variable. C represents the capacity, it is chosen as the control input. These parameters are subject to physical constraints such that they are non-negative and upper bounded. λ is the incoming traffic rate. It is a disturbance input. With practical considerations, we assume that $\lambda(t)$ is essentially upper bounded and such that $\sup_{t\geq 0} \lambda(t) < C_{buffer}$. The model does not depend on a particular type of network such as TCP/IP or ATM network since no other conditions are assumed about the type of the incoming traffic.

x_{ref} is introduced as the reference queue length chosen by the designer. It should be chosen such that the switch/router is sufficiently utilized while preserving certain capability to handle additional traffic bursts. In practice, an empty or extremely small steady state queue generally leads to link under utilization and is thus undesirable. We choose the reference value according to

$$1 \leq x_{ref} \leq x_{buffer}. \quad (4)$$

The lower bound could be an arbitrary positive value. We choose "1" for simplicity[1]. $\bar{x} := x - x_{ref}$ is introduced to represent the regulation error between the queue state and the reference value.

The general objective of network flow control is to suppress congestion and to meet certain performance requirements. As introduced in [10], the choice for x_{ref} relates to these requirements, including fairness among traffics, sufficient bandwidth utilization and bounded delay, etc. The design objective in our paper, as well as in [10] is to accomplish regulation task such that $\bar{x} \to 0$ $(x \to x_{ref})$ under the constraint $0 \leq C \leq C_{server}$, while unknown but bounded disturbance λ is present.

III. Controller design for scalar systems

In this section, we use the following controller to achieve the control objective. The same type of controller will be applied to interconnected systems in next section. We choose appropriate controller parameters in different cases.

$$C = \begin{cases} 0 & x \leq x_{ref} \\ C_{server} \cdot sat\Big\{\frac{\widetilde{C}(x)}{C_{server}}\Big\} & otherwise \end{cases} \quad (5)$$

[1]If the reference value is less than "1", the control laws proposed in this paper can be modified to achieve asymptotic regulation when the interference traffic satisfies a certain bounded condition. We omit the discussion due to space limitations.

where

$$\begin{aligned} \widetilde{C}(x) &= \tfrac{1+x}{x}\big(\alpha\bar{x} + \beta sgn(\bar{x})\big) \\ &= \tfrac{1+x}{x}\big[\alpha(x - x_{ref}) + \beta\big], \end{aligned} \quad (6)$$

$$sat(y) = min\{|\, y\,|, 1\}sgn(y), \quad (7)$$

$$sgn(z) = \begin{cases} 1 & \text{if } z > 0 \\ 0 & \text{if } z = 0. \\ -1 & \text{if } z < 0 \end{cases} \quad (8)$$

The choice of the above controller will be clear from the analysis and synthesis shown below. α, β are design parameters to be determined.

Assumption 1: $\int_{t_0}^{\infty} \lambda(t)dt > x_{ref}$ when $x(t_0) < x_{ref}$.[2] For all $t \geq t_0, 0 \leq \lambda(t) \leq b < \bar{b}$ with $\bar{b}$ defined as follows:

$$\bar{b} := \frac{C_{server}}{\Big[\frac{(x_{buffer}-x_{ref})(x_{buffer}+1)}{x_{ref}^2+x_{ref}} + 1\Big]\frac{1}{x_{buffer}} + 1}. \quad (9)$$

Theorem 1: Consider the system defined in (1)-(4). Suppose λ satisfies Assumption 1. For all initial queue length $x(t_0) \in [0, x_{buffer}]$ at $t = t_0 \geq 0$, x will be asymptotically regulated by control law (5)-(7) to the reference value x_{ref} in (4) if choosing α and β according to:

$$\frac{b}{x_{ref}^2 + x_{ref}} \leq \alpha < \frac{C_{server}\cdot x_{buffer}}{(1+x_{bufer})(x_{buffer}-x_{ref})} - \frac{b}{x_{buffer}-x_{ref}}, \quad (10)$$

$$b < \beta \leq \min\Big\{\alpha(x_{ref}^2 + x_{ref}), \frac{C_{server}\cdot x_{buffer}}{1+x_{buffer}} - \alpha(x_{buffer} - x_{ref})\Big\}. \quad (11)$$

Proof: From (9), we can verify that there exists α satisfying (10). It follows

$$\min\Big\{\alpha(x_{ref}^2 + x_{ref}), \frac{C_{server}\cdot x_{buffer}}{1+x_{buffer}} - \alpha(x_{buffer} - x_{ref})\Big\} > b.$$

Thus the choices of α and β are valid. From (9) and (11)

$$\begin{aligned} \tfrac{d\widetilde{C}}{dx} &= \alpha - (\beta - \alpha x_{ref}) \cdot \tfrac{1}{x^2} \\ &= \alpha - \tfrac{\beta}{x^2} + \tfrac{\alpha\cdot x_{ref}}{x^2} \\ &\geq \alpha + \tfrac{\alpha\cdot x_{ref}}{x^2} - \tfrac{\alpha(x_{ref}^2+x_{ref})}{x^2} \\ &= \alpha(1 - \tfrac{x_{ref}^2}{x^2}) > 0. \end{aligned}$$

Thus $\widetilde{C}$ is a monotonically increasing function of x on $(x_{ref}, x_{buffer}]$. The maximum value of $\widetilde{C}$ is obtained by equating x to x_{buffer}.
$\widetilde{C}|_{x=x_{buffer}} = C_{max}$ where C_{max} is as follows:

$$C_{max} := \frac{1+x_{buffer}}{x_{buffer}}\big[\alpha(x_{buffer} - x_{ref}) + \beta\big]. \quad (12)$$

From (6),(12) and (11)

[2]$\int_{t_0}^{\infty} \lambda(t)dt > x_{ref}$ is a "persistent excitation"(PE) requirement. It assumes that there is enough traffic to utilize the network, such that queue x will be regulated to the reference value by incoming traffic when $x(t_0) < x_{ref}$.

$$\begin{aligned}\widetilde{C} &\leq C_{max} = \tfrac{1+x_{buffer}}{x_{buffer}}\left[\alpha(x_{buffer}-x_{ref})+\beta\right] \\ &\leq C_{server}\end{aligned} \quad (13)$$

for all $x \in [x_{ref}, x_{buffer}]$. Thus $\tilde{C}$ will not reach the capacity constraint C_{server}. We then analyze the regulation performance of our control law.

a) When $x(t_0) > x_{ref}$

Consider function $V(x) = \frac{1}{2}\bar{x}^2$. We calculate the derivative of V along the trajectory of the controlled system using $\dot{\bar{x}} = \dot{x}$, plant dynamics (1) and control law (5),(6) with parameter choices (10) and (11).

$$\dot{V}(t,x) \leq -\alpha \mid \bar{x} \mid^2 -(\beta - b) \mid \bar{x} \mid \leq 0. \quad (14)$$

Thus $\bar{x}(t)$ is bounded if $x(t_0) \in (x_{ref}, x_{buffer}]$. Denote by $W(\bar{x}(t)) := \alpha \mid \bar{x} \mid^2 + (\beta - b) \mid \bar{x} \mid$, using Barbalat's lemma, it can be shown that $W(\bar{x}(t)) \to 0$, thus $|\bar{x}(t)| \to 0$ as $t \to \infty$. (14) ensures that once the trajectory happens to be at the $\bar{x} = 0$, it will be confined at $\bar{x} = 0$ for all future time.

b) When $x(t_0) < x_{ref}$

$\bar{x}(t_0) < 0, C(t) = 0$ as long as $x(t) < x_{ref}$. The dynamic equation is simply $\dot{\bar{x}} = \lambda(t)$. Thus

$$x(t) = x(t_0) + \int_{t_0}^{t} \lambda(\tau) d\tau.$$

With the PE condition for the incoming traffic such that that $\int_{t_0}^{\infty} \lambda(t)dt > x_{ref}$, there exists $t_0 < T < \infty$ such that $x(T) = x_{ref}$ for any $x(t_0) < x_{ref}$. Thus the controller achieves asymptotic regulation of $\bar{x}$ to 0, namely x converges to x_{ref}. ∎

From the above proof, we can also see that queue state x reaches its reference value in finite time and remains there, due to the property of sliding mode controller (5) and (6) . The control law C is an increasing function of x on $[x_{ref}, x_{buffer}]$.

The simulation in Figure 2 showed that the buffer queue is regulated asymptotically to the desired reference value from full buffer size without control saturation when $0 \leq \lambda \leq b < \bar{b}$. As a comparison, bounded regulation is shown in Figure 3 where the control law in [10] is used. The simulation parameters for both figures are: $C_{server} = 5$, $x(t_0) = 25$, $x_{ref} = 5$, $x_{buffer} = 30$, $\alpha = 0.1$, $\beta = 2.1$, $b = 2$.

Remark 1: The choice of discontinuous control law is natural in face of the physical property of this problem. The choice of $C = 0$ when $x \leq x_{ref}$ is due to the fact that it is not necessary to assign further capacity when the buffer is under utilized. The assignment of capacity (control input) we design is an monotonic increasing function of the queue length, since longer queue length represents that the network is in higher congestion level thus more capacity is needed.

Remark 2: (9-11) reveals the tradeoffs among performance (convergence rate), regulation objective (queue reference) and traffic volume (bounds on disturbance). This tradeoff shows that the performance of the control system is subject to capacity constraint of the node (control input saturation).

Remark 3: The above proposed control law is discontinuous (sliding mode control) at $x = x_{ref}$. This discontinuity raises theoretical as well as practical difficulties. We refer readers to [3] concerning the existence and uniqueness of solutions for differential equation with discontinuous right hand side. As for practical issues, instead of staying at $x = x_{ref}$ when the trajectory reaches $\bar{x} = 0$, chattering occurs due to imperfect switching and delay, which is a known phenomenon in sliding mode control as shown by Figure 4. It may excite un-modelled high frequency dynamics and cause instability [7]. We use a continuous approximation of the discontinuous control law to overcome this phenomenon. Both the discontinuous control law and the continuous approximation are schematically shown in Figure 1.

Proposition 1: Consider the systems defined in (1)-(3). Suppose $\lambda(t)$ satisfies Assumption 1. For all $x(t_0) \in [0, x_{buffer}]$ at $t = t_0 \geq 0$, the trajectory $x(t)$ is bounded for all $t \geq t_0$ and is ultimately confined to

$$\left\{x_{ref} \leq x < x_{ref} + \epsilon\right\} \quad (15)$$

if the following control law is used

$$C = \begin{cases} 0 & x \leq x_{ref} \\ \widetilde{C} & \text{otherwise,} \end{cases} \quad (16)$$

$$\text{where} \quad \widetilde{C} = \frac{1+x}{x}[\alpha\bar{x} + \beta sat(\frac{\bar{x}}{\epsilon})] \quad (17)$$

where α and β are chosen the same with discontinuous design in (10),(11). ϵ is a design parameter which determines the ultimate bound on queue state x. It is chosen to satisfy

$$0 < \epsilon \leq x_{buffer} - x_{ref} \quad (18)$$

and is chosen to be small in practice for a good approximation of the discontinuous control law.

Proof: a) when $x(t_0) \geq x_{ref} + \epsilon$, ϵ satisfies (18)

A Lyapunov function candidate $V(\bar{x}) = \frac{1}{2}\bar{x}^2$ satisfies

$$\dot{V}(t,x) \leq -\alpha\bar{x}^2 - (\beta - b)\bar{x} < 0.$$

Thus $\mid \bar{x}(t) \mid$ will be strictly decreasing as long as $x(t) \geq x_{ref} + \epsilon$.

b) when $x(t_0) < x_{ref}$, due to the PE condition for the incoming traffic in Assumption 1, there exist $t_0 < T < \infty$ such that $x(T) = x_{ref}$ with the same reasoning as the case for discontinuous control law.

Thus the trajectory $x(t)$ reaches the boundary layer (15) in finite time and remains there. ∎

Remark 4: A similar analysis with the proof in Theorem 1 reveals that control input is not saturated. We omit this analysis due to space limitation.

IV. CONTROLLER DESIGN FOR INTERCONNECTED SYSTEMS

It is natural to first model the interconnected network as composed by scalar systems:

$$\dot{\bar{x}}_i = -\frac{x_i}{1+x_i} \cdot C_i + \lambda_i, \quad i = 1, ..., n \quad (19)$$

where notations have the same meaning as introduced in model (1-3) except for subscript i denoting the ith subsystem.

We will use subscripts i, j in this sense for the rest of the paper without further explanation. The model is valid for:

$$x_i \in [0, x_{buffer}^{[i]}], \quad (20)$$

$$C_i(t) \in \big[0, C_{server}^{[i]}\big], \quad (21)$$

$$x_{ref}^{[i]} \in \big[1, x_{buffer}^{[i]}\big], \quad (22)$$

with $x_{buffer}^{[i]}$ and $C_{server}^{[i]}$ being the physical constraints of the ith subsystem. $x_{ref}^{[i]}$ denote the reference value for the ith subsystem.

The following notations are introduced for convenience, there meanings are clear according to the context.

$$X := \big[x_1 ... x_n\big]^T, \qquad \bar{X} := \big[|\bar{x}_1| ... |\bar{x}_n|\big]^T.$$

λ_i is a nonlinear function denoting the incoming traffic to node i. It can be expressed as:

$$\lambda_i = \sum_{j=1, j\neq i}^{n} \lambda_{ij}\big(t, x_j\big) + \upsilon_i\big(t, X\big)$$

where $\lambda_{ij},\ \upsilon_i$ are unknown functions. $\lambda_{ij} : \big[0, \infty\big) \times \big[0, x_{buffer}^{[j]}\big] \to \mathcal{R}^+$ denotes the rate of traffic between two nodes. $\upsilon_i : \big[0, \infty\big) \times \mathcal{R}_+^n \to \mathcal{R}^+$ denotes all other background noise traffic. We use φ_i to represents its upper bound, namely $0 \le \upsilon_i(t, X) \le \varphi_i, \forall t \ge t_0 \ge 0, X \in \mathcal{R}_+^n$.

We assume the queue state x_i is an indicator of the activity level of each node. The interference between any two nodes is constrained by the activity of each other and factors such as position of the node, distance between them and power constraint. These constraints are represented by γ_{ij}. Thus we assume λ_i satisfies:

$$0 \le \lambda_i \le \sum_{j=1, j\neq i}^{n} \gamma_{ij} x_j + \varphi_i \quad (23)$$

$$= \sum_{j=1, j\neq i}^{n} \gamma_{ij} \bar{x}_j + \sum_{j=1, j\neq i}^{n} \gamma_{ij} x_{ref}^{[j]} + \varphi_i,$$

In the above bound, the first term($\sum_{j=1, j\neq i}^{n} \gamma_{ij} x_j$) emphasizes that the major interference between any two nodes is constrained by each other's activity level and factors such as distance between them and power constraint. Constant γ_{ij} represents these physical constraints. $\varphi_i > 0$ is a constant representing the upper bound for all other background noise traffic which doesn't satisfy the first bound. The equality is obtained by $x_j = \bar{x}_j + x_{ref}^{[j]}$.

We first define the control law we will use. The reason for choosing this control law will be revealed later.

$$C_d^{[i]} = \begin{cases} 0 & x_i \le x_{ref}^{[i]} \\ C_{server}^{[i]} \cdot sat\Big\{\frac{\widetilde{C}_d^{[i]}}{C_{server}}\Big\} & \text{otherwise} \end{cases} \quad (24)$$

where $\widetilde{C}_d^{[i]}$ is defined by:

$$\widetilde{C}_d^{[i]} = \frac{1 + x_i}{x_i}[\alpha_i \bar{x}_i + \beta_i] \quad (25)$$

with subscript "d" denoting "discontinuous control law". $\alpha_i,\ \beta_i$ are controller parameters whose choices are to be determined. The following lemma is useful for the rest of the paper.

Lemma 1: Define a $n \times n$ matrix S with its elements being:

$$s_{ij} = \begin{cases} \alpha_i & i = j \\ -\gamma_{ij} & i \neq j \end{cases} \quad (26)$$

where $\alpha_i, \gamma_{ij},\ i, j = 1, ..., n$ are nonnegative constants. S is an M matrix[3] if $\alpha_i,\ \gamma_{ij}$ satisfies

$$\alpha_i > \sum_{j=1, j\neq i}^{n} \gamma_{ij}. \quad (27)$$

Proof: Denote by $\sigma(S)$ the set of all eigenvalues of the square matrix S. For $\forall \lambda \in \sigma(S)$, according to the Gerschgorin disk theorem [2], there exists $i \in [1, ..., n]$, such that $\lambda \ge \alpha_i - \sum_{j=1, j\neq i}^{n} \gamma_{ij}$. From (27), $\alpha_i - \sum_{j=1, j\neq i}^{n} \gamma_{ij} > 0,\ \forall i = 1, ..., n$. Thus all eigenvalues of matrix S are positive, namely S is an M matrix. ■

Theorem 2: Consider the interconnected system defined by (19)-(22). Suppose the incoming traffic for each subsystem satisfies:

1) PE condition, i.e. $\int_{t_0}^{\infty} \lambda_i(t) dt > x_{ref}^{[i]}$ when $x(t_0) < x_{ref}$.
2) (23) and the following inequalities:

$$\sum_{j=1, j\neq i}^{n} \gamma_{ij} x_{ref}^{[j]} + \varphi_i < \frac{C_{server}^{[i]}}{\Big[\frac{(x_{buffer}^{[i]} - x_{ref}^{[i]})(x_{buffer}^{[i]} + 1)}{{x_{ref}^{[i]}}^2 + x_{ref}^{[i]}} + 1\Big] \frac{1}{x_{buffer}^{[i]}} + 1}, \quad (28)$$

$$\big(x_{buffer}^{[i]} - x_{ref}^{[i]}\big) \sum_{j=1, j\neq i}^{n} \gamma_{ij} + \sum_{j=1, j\neq i}^{n} \gamma_{ij} x_{ref}^{[j]} + \varphi_i < \frac{C_{server}^{[i]} \cdot x_{buffer}^{[i]}}{(1 + x_{buffer}^{[i]})}. \quad (29)$$

Choose the control law (24-25) where α_i, β_i satisfy

$$\max\left\{\frac{\sum_{j=1, j\neq i}^{n} \gamma_{ij} x_{ref}^{[j]} + \varphi_i}{{x_{ref}^{[i]}}^2 + x_{ref}^{[i]}}, \sum_{j=1, j\neq i}^{n} \gamma_{ij}\right\} < \alpha_i \quad (30)$$

$$< \frac{C_{server}^{[i]} \cdot x_{buffer}^{[i]}}{(1 + x_{buffer}^{[i]})(x_{buffer}^{[i]} - x_{ref}^{[i]})} - \frac{\sum_{j=1, j\neq i}^{n} \gamma_{ij} x_{ref}^{[j]} + \varphi_i}{x_{buffer}^{[i]} - x_{ref}^{[i]}},$$

$$\sum_{j=1, j\neq i}^{n} \gamma_{ij} x_{ref}^{[j]} + \varphi_i < \beta_i \le \min\Big\{\alpha_i\big({x_{ref}^{[i]}}^2 + x_{ref}^{[i]}\big), \frac{C_{server}^{[i]} \cdot x_{buffer}^{[i]}}{1 + x_{buffer}^{[i]}} - \alpha_i\big(x_{buffer}^{[i]} - x_{ref}^{[i]}\big)\Big\}. \quad (31)$$

For any $X(t_0) \in \Big\{X \in \mathcal{R}^n \mid 0 \le x_i \le x_{buffer}^{[i]}, \forall i = 1, ..., n\Big\}$, the above defined discontinuous controller achieves asymptotic regulation of the output queue length x_i to $x_{ref}^{[i]}$ for every subsystem.

[3] Please refer to [9] for the definition and test techniques about M matrix.

Proof: From inequality (28) and (29), we can easily verify the choice of α in (30) is valid. With α satisfying (30), it can be shown that

$$\sum_{j=1,j\neq i}^{n}\gamma_{ij}x_{ref}^{[j]}+\varphi_i<\min\Big\{\alpha_i\big({x_{ref}^{[i]}}^2+x_{ref}^{[i]}\big),\ \frac{C_{server}^{[i]}\cdot x_{buffer}^{[i]}}{1+x_{buffer}^{[i]}}-\alpha_i\big(x_{buffer}^{[i]}-x_{ref}^{[i]}\big)\Big\}.$$

Thus the choice of β_i in (31) is valid. It follows that $\beta_i\le\alpha_i({x_{ref}^{[i]}}^2+x_{ref})$.

Using this inequality and by differentiating $\widetilde{C}_d^{[i]}$ with respect to x_i on $(x_{ref}^{[i]},x_{buffer}^{[i]}]$, we have

$$\begin{aligned}\frac{d\widetilde{C}_d^{[i]}}{dx_i}&=\alpha_i-(\beta_i-\alpha_i x_{ref}^{[i]})\cdot\frac{1}{x_i^2}\\&\ge\alpha_i+\frac{\alpha_i\cdot x_{ref}^{[i]}}{x_i^2}-\frac{\alpha_i({x_{ref}^{[i]}}^2+x_{ref}^{[i]})}{x_i^2}\\&=\alpha(1-\frac{x_{ref}^2}{x^2})>0.\end{aligned}$$

Thus $\widetilde{C}_d^{[i]}$ is a increasing function of x_i on $(x_{ref},x_{buffer}]$. Denote by

$$C_{max}^{[i]}:=\frac{1+x_{buffer}^{[i]}}{x_{buffer}^{[i]}}\Big[\alpha_i(x_{buffer}^{[i]}-x_{ref}^{[i]})+\beta_i\Big].\quad(32)$$

Since

$$\beta_i\le\frac{C_{server}^{[i]}\cdot x_{buffer}^{[i]}}{1+x_{buffer}^{[i]}}-\alpha_i\big(x_{buffer}^{[i]}-x_{ref}^{[i]}\big),$$

it follows that

$$\widetilde{C}_d^{[i]}\le C_{max}^{[i]}\le C_{server}^{[i]}\quad(33)$$

for all $x_i\in(x_{ref}^{[i]},x_{buffer}^{[i]}]$. Thus $\widetilde{C}_d^{[i]}$ will not reach the capacity constraint $C_{server}^{[i]}$, $i=1,...,n$.

We then prove that asymptotic regulation is achieved for every subsystem. Since $\lambda_i(t)$ satisfies the PE condition, according to the discussion of scalar case, we can assume without loss of generality that $x_i(t_0)>x_{ref}^{[i]}$, $\forall i=1,...,n$. Consider function

$$V=\sum_{i=1}^{n}d_iV_i(x_i)$$

where $V_i(x_i)=\frac{1}{2}\bar{x}_i^2$. $d_i>0$ are positive constants to be determined. The following inequality holds naturally:

$$\frac{1}{2}d_{min}||\ \bar{X}\ ||^2\le V\le\frac{1}{2}d_{max}||\ \bar{X}\ ||^2,\quad(34)$$

$$\text{where}\left\{\begin{array}{lcl}d_{max}&=&max\{d_1,...,d_n\},\\d_{min}&=&min\{d_1,...,d_n\}.\end{array}\right.\quad(35)$$

Along the trajectory of the closed loop system,

$$\dot{V}(t,X)\le\sum_{i=1}^{n}d_i\Big\{-\alpha\bar{x}_i^2-\beta_i\bar{x}_i+\Big[\sum_{j=1,j\neq i}^{n}\gamma_{ij}\bar{x}_j\Big]\bar{x}_i+\Big[\sum_{j=1,j\neq i}^{n}\gamma_{ij}x_{ref}^{[j]}+\varphi_i\Big]\bar{x}_i\Big\}.\quad(36)$$

Since α_i satisfies (30), we can verify from Lemma 1 that matrix S defined in (26) is an M matrix. According to Lemma 9.7 in [7], S being an M matrix guarantees the existence of matrix $D=diag(d_1,...,d_n)>0$ such that $DS+S^TD$ is a positive definite matrix. Also due to that $\beta_i>\sum_{j=1,j\neq i}^{n}\gamma_{ij}x_{ref}^{[j]}+\varphi_i$ from (31), it follows from (36) that

$$\begin{aligned}&\dot{V}(t,X)\le-\tfrac{1}{2}\bar{X}^T(DS+S^TD)\bar{X}-\\&\sum_{i=1}^{n}\underbrace{\Big\{d_i\Big[\beta_i-\Big(\sum_{j=1,j\neq i}^{n}\gamma_{ij}x_{ref}^{[j]}+\varphi_i\Big)\Big]\Big\}}_{\nu_i>0}\bar{x}_i\\&\le-\tfrac{1}{2}\lambda_{min}||\ \bar{X}\ ||_2^2-\sum_{i=1}^{n}\nu_i\ |\ \bar{x}_i\ |\le 0\end{aligned}\quad(37)$$

where λ_{min} is the minimum eigenvalue of positive definite matrix $DS+S^TD$.

Using Barbalat's lemma, it can be shown from and (34) and (37) that $\lim_{t\to\infty}\sum_{i=1}^{n}\ |\ \bar{x}_i(t)\ |\to 0$ when $x_i(t_0)\in(x_{ref}^{[i]},x_{buffer}^{[i]}]$. Combining the "PE" condition for $\lambda_i(t)$ when $x_i(t_0)<x_{ref}^{[i]}$, we conclude that system trajectory converges to $\Big\{X\in\mathcal{R}^n\ |\ x_i=x_{ref}^{[i]},\forall i=1,...,n\Big\}$ asymptotically for all $x_i(t_0)\in[0,x_{buffer}^{[i]}]$. ■

Simulation in Figure 5 showed the case when two nodes are connected and interference between them satisfies (23). The two nodes have different buffer sizes and capacity constraints, different reference values and initial states while they both achieves asymptotic regulation without control saturation.

We then present our continuous approximation controller design for interconnected system.

Proposition 2: Consider the interconnected system defined by (19-22). Suppose $\lambda_i$satisfies (23) and inequalities (28) and (29). For any $x_i(t_0)\in[0,x_{buffer}]$, the system trajectory is bounded and will be confined to $\big[x_{ref}^{[i]},x_{ref}^{[i]}+\epsilon_i\big]$ ultimately, $i=1,...,n$, if the control law is chosen as follows:

$$C_c^{[i]}=\left\{\begin{array}{ll}0&x_i\le x_{ref}^{[i]}\\C_{server}^{[i]}sat\Big\{\frac{\widetilde{C}_c^{[i]}}{C_{server}^{[i]}}\Big\}&\text{otherwise,}\end{array}\right.\quad(38)$$

$$\widetilde{C}_c^{[i]}=\frac{1+x_i}{x_i}\Big[\alpha_i\bar{x}_i+\beta_i sat\big(\frac{\bar{x}_i}{\epsilon_i}\big)\Big],\quad(39)$$

α_i and β_i are chosen the same with discontinuous design in Theorem 2. ϵ_i and sat function have the same definitions in Proposition 1 for the scalar system. ϵ_is are design parameters chosen according to (18) for every subsystem. The subscript c denotes continuous control law.

The proof is omitted since it shares many features with the proof for the discontinuous control case of the interconnected system in Theorem 2 and the proof for the continuous approximation of the scalar system case in Proposition 1.

Remark 5: The scalability of this control scheme is evidenced by comparing the form of controller in (5)(6) and (24)(25). The control law proposed for isolated system is

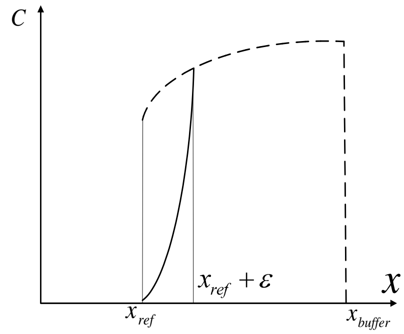

Fig. 1. Control (capacity) vs. state (queue)

scalable to large scale system, with changing of design parameters α_is and β_is.

Remark 6: The above designs use robust control ideas and requires that $\gamma_{ij}, \varphi_i, x_{ref}^{[j]}$ are known. They are partially decentralized control schemes. When $\gamma_{ij}, \varphi_i, x_{ref}^{[j]}$ are unknown or not locally available, adaptors may be built to supplement the above control designs. Using decentralized adaptive control techniques, we study totally decentralized controller designs for a large scale network modelled by (19-22) in another paper.

V. Conclusions

Through theoretic analysis and simulations, we show that our sliding mode control law improves the queue regulation result in [10] by achieving asymptotic regulation. Physical constraints on control input and state variable are handled. The same type of controller can be applied to large scale networks in a partially decentralized manner. The typical shapes of a discontinuous control law and of a continuous approximation are both shown in Figure 1. ϵ is a designing parameter which sets the ultimate bound of the regulation error.

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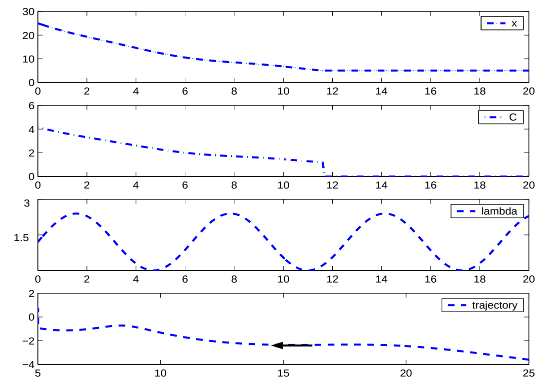

Fig. 2. Asymptotic regulation for scalar system

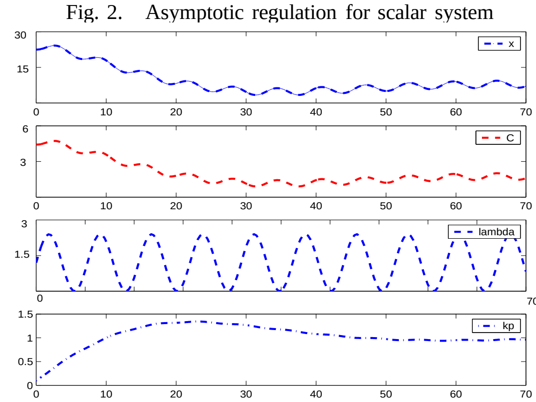

Fig. 3. Bounded regulation

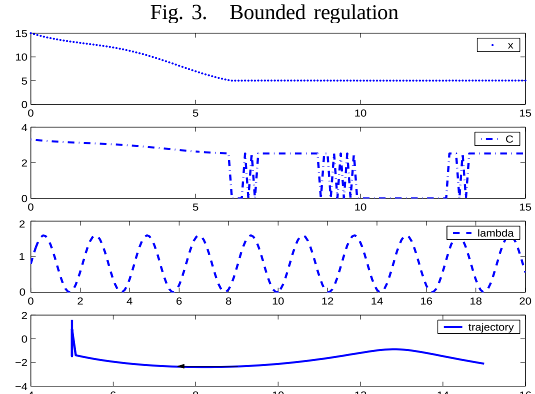

Fig. 4. Chattering behavior with imperfect switching

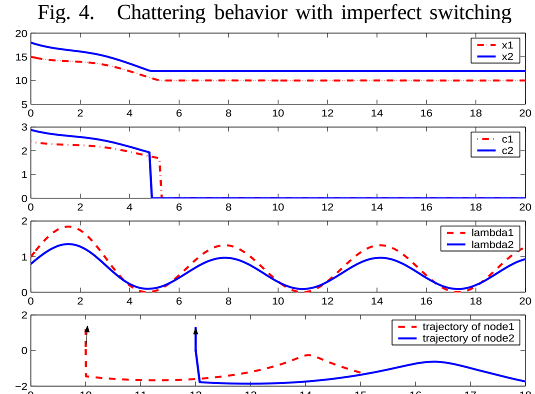

Fig. 5. Asymptotic regulation for two nodes interconnected