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Practical Adaptive Neural Control of Nonlinear Systems with Unknown Time Delays

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***Abstract*— In this paper, practical adaptive neural control is presented for a class of nonlinear systems with unknown time delays in strict-feedback form. Using appropriate Lyapunov-Krasovskii functionals, the uncertainties of unknown time delays are compensated for. Controller singularity problems are solved by employing practical neural network control based on decoupled backstepping design. It is proved that the proposed design method is able to guarantee semi-globally uniformly ultimate boundedness of all the signals in the closed-loop system and the tracking error is proven to converge to a small neighborhood of the origin. In addition, the residual set of each states in the closed-loop systems can be determined respectively.**

I. Introduction

Recent years have witnessed tremendous efforts in adaptive control of certain class of nonlinear systems. Adaptive control is well known for its great capability in compensating for linearly parameterized uncertainties. To overcome these uncertainties and obtain global stability, some restrictions have to be made to system nonlinearities such as matching conditions [1], extended matching conditions [2], or growth conditions [3]. To overcome these restrictions, a recursive and systematic backstepping design was developed in [2]. The overparametrization problem was then removed in [4] by introducing the concept of tuning function. Several adaptive approaches for nonlinear systems with triangular structures have been proposed in [5], [6]. Robust adaptive backstepping control has been studied for certain class of nonlinear systems whose uncertainties are not only from parametric ones but also from unknown nonlinear functions in [7], [8] and among others. For system $\dot{x} = f(x) + g(x)u$, the unknown system function $g(x)$ causes great design dif£culty in adaptive control. Based on feedback linearization, certainty equivalent control $u = [-\hat{f}(x)+v]/\hat{g}(x)$ is usually taken, where $\hat{f}(x)$ and $\hat{g}(x)$ are estimates of $f(x)$ and $g(x)$, and measures have to be taken to avoid controller singularity when $\hat{g}(x) = 0$. To avoid the singularity problem, stable neural network controllers have been constructed in [9] by augmenting a robustifying portion, in [10], [11] by estimating the derivation of the control Lyapunov function, and by introducing a family of integral Lyapunov function in [12] which do not require the estimate of the unknown function $g(x)$.

Robust control of systems with time delays has attracted much attention due to its mathematical challenge and application demand in real-time control. The existence of time delays may make the stabilization problem become much more dif£cult. Lyapunov-Krasovskii functionals [13] combined with the LMI technique [14] has been used to establish a framework for the stability and control of time-delay systems. Robust control of time-delay systems using the above-mentioned technique are also intensively investigated. However, for nonlinear systems with delay in the state, few results are reported. In [15], [16], the authors have studied a class of nonlinear time-delay systems in strict-feedback form and systematic and practical backstepping design has been presented. Under the mild assumption on the upper bound of the unknown time-delay, the proposed design based on the Lyapunov stability is delay-independent in the sense that the design is totally free of unknown delays. The controller singularity problem is solved by introducing the practical design and using integral Lyapunov function. However, due to the integral operation, the controller is very complicated to practical implementation. The derivation is also much involved due to coupling of the integrations and the time-delay terms. Motivated by the results [12], [17] in which the systems properties has been fully exploited such that rather simple control scheme has been developed without using integral-Lyapunov functions and singularity problems has been avoided as well, we present in this paper a direct NN controller for a class of time-delay systems in strict-feedback form. By making a simple assumption for the af£ne term $g_n(x)$ of control that $\partial g_n(x)/\partial x_n = 0$, the controller design can be simpli£ed using quadratic Lyapunov functions rather than integral-Lyapunov functions. The main contribution of the paper lies in: (i) the introduction of the practical control and the re-construction of compact sets, which effectively avoid the singularity problem and, at the same time guarantees the feasibility and validity of the neural networks approximation; and (ii) the employment of decoupled backstepping design, by which the stability analysis of the proposed practical control can be carried out in a nested manner to guarantee the closed-loop stability and and the residual set of each state in z_i coordinate can be iteratively individually determined.

II. Problem Formulation

Consider a class of single-input-single-output (SISO) nonlinear time-delay systems

$$\begin{aligned} &\dot{x}_i(t) = g_i(\bar{x}_i(t))x_{i+1}(t) + f_i(\bar{x}_i(t)) + h_i(\bar{x}_i(t-\tau_i)), \\ &\quad 1 \le i \le n-1 \\ &\dot{x}_n(t) = g_n(\bar{x}_n(t))u + f_n(\bar{x}_n(t)) + h_n(\bar{x}_n(t-\tau_n)) \quad (1) \end{aligned}$$

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where $\bar{x}_i = [x_1, x_2, ..., x_i]^T$, $x = [x_1, x_2, ..., x_n]^T \in R^n$ and $u \in R$ are the state variables and system input respectively, $g_i(\cdot)$, $f_i(\cdot)$ and $h_i(\cdot)$ are unknown smooth functions, and τ_i are unknown time delays of the states, $i = 1, ..., n$. The control objective is to design an adaptive controller for system (1) such that the state $x_1(t)$ follows a desired reference signal $y_d(t)$, while all signals in the closed-loop system are bounded. De£ne the desired trajectory $\bar{x}_{d(i+1)} = [y_d, \dot{y}_d, ..., y_d^{(i)}]^T$, $i = 1, ..., n-1$, which is a vector of y_d up to its ith time derivative $y_d^{(i)}$.

A1). The system states $x(t)$ and part of their time derivatives, $\dot{\bar{x}}_{n-1}(t)$, are all available for feedback.

A2). The signs of $g_i(\cdot)$ are known, and there exist constants $g_{\max} \geq g_{\min} > 0$ such that $g_{\min} \leq |g_i(\cdot)| \leq g_{\max}$, and $\partial g_n(x)/\partial x_n = 0$.

A3). The desired trajectory vectors $\bar{x}_{di}$, $i = 2, ..., n$ are continuous and available, and $\bar{x}_{di} \in \Omega_{di} \subset R^i$ with Ω_{di} known compact sets.

A4). The unknown smooth functions $h_i(\bar{x}_i(t))$ satisfy the following inequality $|h_i(\bar{x}_i(t))| \leq \sum_{j=1}^{i} |x_j(t)| \varrho_{ij}(\bar{x}_i(t))$ where $\varrho_{ij}(\cdot)$ are known smooth functions.

A5). The size of the unknown time delays is bounded by a known constant, i.e., $\tau_i \leq \tau_{\max}$, $i = 1, ..., n$.

The Assumption A2) implies that unknown constants g_i are strictly either positive or negative. Without losing generality, we shall only consider the case when $g_i(\cdot) > 0$. It should be emphasized that the bounds $g_{\min}$ and $g_{\max}$ are only required for analytical purposes, their true values are not necessarily known since they are not used for controller design. Note that the requirement for $\dot{\bar{x}}_{n-1}(t)$ is a constraint but realistic for many physical systems as we are not requiring $\dot{x}_n$ which is directly in¤uenced by the control. In addition, $\partial g_n(x)/\partial x_n = 0$ means that

$$\dot{g}_n(x) = \Big[\frac{\partial g_n(x)}{\partial x}\Big]^T \dot{x}(t) = \sum_{i=1}^{n-1} \frac{\partial g_n(x)}{\partial x_i} \dot{x}_{i+1}(t)$$

which is only dependent on the state x. Obviously, $\dot{g}_i(\bar{x}_i)$, $i = 1, ..., n-1$ is also dependent on the state x only. As $g_i(\cdot)$ are smooth function, we know that $\forall \bar{x}_i \in \Omega$ with Ω being a bounded compact set, there exist constants $g_{id} > 0$ such that $|\dot{g}_i(\cdot)| \leq g_{id}$. This nice property could be used to simplify the later controller design.

III. Preliminaries

In this paper, the Radial Basis Function (RBF) neural network (NN) as a kind of linearly parametrized neural networks (LPNNs) will be used as a function approximator to approximate the unknown nonlinear function $h(Z): R^q \to R$ as $h_{nn}(Z) = W^T S(Z)$ where the input vector $Z \in \Omega_Z \subset R^q$, weight vector $W = [w_1, w_2, \cdots, w_l]^T \in R^l$, the NN node number $l > 1$; and $S(Z) = [s_1(Z), \cdots, s_l(Z)]^T$, with $s_i(Z)$ being chosen as the commonly used Gaussian functions, which have the form $s_i(Z) = \exp[-(Z - \mu_i)^T (Z - \mu_i)/\eta_i^2]$, $i = 1, 2, \cdots, l$ with $\mu_i = [\mu_{i1}, \mu_{i2}, \cdots, \mu_{iq}]^T$ being the center of the receptive £eld and η_i being the width of the Gaussian function. Universal approximation results in [18], [19] indicate that, if l is chosen suf£ciently large, $W^T S(Z)$ can approximate any continuous function, $h(Z)$, to any desired accuracy over a compact set $\Omega_Z \subset R^q$ to arbitrary any accuracy in the form of $h(Z) = W^{*T} S(Z) + \epsilon(Z), \quad \forall Z \in \Omega_Z \subset R^q$ where W^* is the ideal constant weight vector, and $\epsilon(Z)$ is the approximation error which is bounded over the compact set, i.e., $|\epsilon(Z)| \leq \epsilon^*, \forall Z \in \Omega_Z$ where $\epsilon^* > 0$ is an unknown constant. The ideal weight vector W^* is an "arti£cial" quantity required for analytical purposes. W^* is de£ned as the value of W that minimizes $|\epsilon|$ for all $Z \in \Omega_Z \subset R^q$, i.e., $W^* := \arg\min_{W \in R^l} \{\sup_{Z \in \Omega_Z} |h(Z) - W^T S(Z)|\}$.

The stability results obtained in NN control literature are semi-global in the sense that, as long as the input variables Z of the NNs remain within some pre-£xed compact set, $\Omega_Z \subset R^q$, where the compact set Ω_Z can be made as large as desired, there exists controller(s) with suf£ciently large number of NN nodes such that all the signals in the closed-loop remain bounded.

Suppose that $x \in \Omega_Z$, where Ω_Z is a compact set. De£ne sets $\Omega_{c_z} \subset \Omega_Z$ and Ω_Z^0 as

$$\Omega_{c_z} := \{x \mid |x| < c_z\}, \quad \Omega_Z^0 := \Omega_Z - \Omega_{c_z} \tag{2}$$

where constant $c_z > 0$ and "$-$" is used to denote the complement of set B in set A as $A - B := \{x | x \in A \text{ and } x \notin B\}$.

The following lemma shows the compactness of set $\Omega_{z_1}^0$, which is useful to re-construct the compact domain of neural network approximation later.

Lemma 1: Set Ω_Z^0 is a compact set.

Proof: See [15], [16]. ■

IV. Direct Neural Network Control for First-order System

To illustrate the design methodology clearly, we £rst consider the tracking problem of a £rst-order system

$$\dot{x}_1(t) = g_1(x_1(t))u(t) + f_1(x_1(t)) + h_1(x_1(t - \tau_1)) \tag{3}$$

where $u(t)$ is the control input. De£ne the tracking error $z_1 = x_1 - y_d$, we have

$$\dot{z}_1(t) = g_1(x_1(t))u(t) + f_1(x_1(t)) + h_1(x_1(t - \tau_1)) - \dot{y}_d(t)$$

Based on feedback linearization, the certainty equivalent control is usually taken the form $u(t) = \frac{1}{g_1(x_1)}[-f_1(x_1) + v(t)]$. In the case that $g_1(\cdot)$ and $f_1(\cdot)$ are unknown, their estimates $\hat{g}_1$ and $\hat{f}_1$ shall be used instead to construct the controller and singularity problem may occur when $\hat{g}_1(x_1) = 0$. To avoid the singularity problem, we shall estimate the unknown term, e.g., $\frac{f_1(x_1)}{g_1(x_1)}$ as a whole rather than estimate the function $g_1(\cdot)$ and $f_1(\cdot)$ individually.

Another design dif£culty comes from the unknown time-delay τ_1, which can be compensated for by introducing the Lyapunov-Krasovskii functional in the form of

$$V_U(t) = \int_{t-\tau_1}^{t} U(x(t)) d\tau \tag{4}$$

with $U(\cdot) \geq 0$ being a properly chosen function. The time derivative of $V_U(t)$ is

$$\dot{V}_U(t) = U(x(t)) - U(x(t-\tau_1))$$

among which the term $U(x(t-\tau_1))$ can be used to compensate for the unknown time-delay terms related to τ_1, while the remaining term $U(x(t))$ does not introduce any uncertainties to the system.

Consider the scalar smooth function $V_{z_1} = \frac{1}{2g_1(x_1)} z_1^2(t)$ and the Lyapunov-Krasovskii functional V_{U_1} as

$$V_{U_1}(t) = \frac{1}{2g_{\min}} \int_{t-\tau_1}^{t} U_1(x_1(t)) d\tau \quad (5)$$

with $U_1(x_1(t)) = \frac{1}{2} x_1^2(t) \varrho_1(x_1(t)) \geq 0$. Noting Assumption A4), we have

$$\begin{aligned}
&\dot{V}_{z_1}(t) + \dot{V}_{U_1}(t) \\
&\leq z_1(t)\Big\{u(t) + \frac{1}{g_1}[f_1(x_1(t)) - \dot{y}_d(t)]\Big\} - \frac{\dot{g}_1}{2g_1^2} z_1^2(t) \\
&\quad + \frac{1}{g_1}|z_1(t)||x_1(t-\tau_1)|\varrho_1(t-\tau_1) \\
&\quad + \frac{1}{2g_{\min}}[U_1(x_1(t)) - U_1(x_1(t-\tau_1))] \quad (6)
\end{aligned}$$

The terms $z_1(t)$ and $|x_1(t-\tau_1)|\varrho_1(x_1(t-\tau_1))$, which are entangled in their present form, shall be separated such that the terms with unknown time delay can be dealt with separately. Using Young's inequality, (6) becomes

$$\begin{aligned}
&\dot{V}_{z_1}(t) + \dot{V}_{U_1}(t) \\
&\leq z_1(t)\Big\{u(t) + \frac{1}{g_1}[f_1(x_1(t)) - \dot{y}_d(t) + \frac{1}{2} z_1(t)]\Big\} \\
&\quad - \frac{\dot{g}_1}{2g_1^2} z_1^2(t) + \frac{1}{2g_1} x_1^2(t-\tau_1)\varrho_1^2(x_1(t-\tau_1)) \\
&\quad + \frac{1}{2g_{\min}}[x_1^2(t)\varrho_1^2(x_1(t)) - x_1^2(t-\tau_1)\varrho_1^2(x_1(t-\tau_1))] \quad (7)
\end{aligned}$$

As $g_1 \geq g_{\min}$, it follows that $\frac{1}{2g_1} x_1^2(t-\tau_1)\varrho_1^2(x_1(t-\tau_1)) \leq \frac{1}{2g_{\min}} x_1^2(t-\tau_1)\varrho_1^2(x_1(t-\tau_1))$. In addition, from Assumption A2), we have $-\frac{\dot{g}_1 z_1^2}{2g_1^2} \leq \frac{|\dot{g}_1| z_1^2}{2g_1^2} \leq \frac{g_{1d}}{2g_{\min}} z_1^2$. Thus, (7) becomes

$$\dot{V}_{z_1}(t) + \dot{V}_{U_1}(t) \leq z_1(t)[u(t) + Q_1(Z_1(t))] + \frac{g_{1d}}{2g_{\min}} z_1^2 \quad (8)$$

where

$$\begin{aligned}
Q_1(Z_1(t)) &= \frac{1}{g_1(x_1)}[f_1(x_1(t)) - \dot{y}_d(t) + \frac{1}{2} z_1(t)] \\
&\quad + \frac{1}{2g_{\min} z_1(t)} x_1^2(t)\varrho_1^2(x_1(t)) \quad (9)
\end{aligned}$$

with $Z_1 = [x_1, y_d, \dot{y}_d]^T \in \Omega_{Z_1} \subset R^3$ and $\Omega_{Z_1} := \{z_1, \bar{x}_{d2} | z_1 \in R, \bar{x}_{d2} \in \Omega_{d2}\}$.

From (8), it is found that the controller design is free from unknown time-delay τ_1 at present stage. For notation conciseness, we will omit the time variables t and after time-delay terms have been eliminated.

Since $f_1(\cdot)$ and $g_1(\cdot)$ are unknown smooth function, neural networks shall be used to approximate the function $Q_1(Z_1)$. According to the main result stated in [20], any real-valued continuous function can be arbitrarily closely approximated by a network of RBF type over a compact set. However, it is apparent that $Q_1(Z_1)$ is not continuous over the compact set Ω_{Z_1} as it is not well-de£ned at $z_1(t) = 0$. Therefore, we shall re-construct the compact set over which the neural network approximation is feasible and valid. To this end, let us de£ne sets $\Omega_{c_{z_1}} \subset \Omega_{Z_1}$ and $\Omega_{Z_1}^0$ as follows

$$\Omega_{c_{z_1}} := \{z_1 \big| |z_1| < c_{z_1}\}, \quad \Omega_{Z_1}^0 := \Omega_{Z_1} - \Omega_{c_{z_1}}$$

From Lemma 1, we know that $\Omega_{Z_1}^0$ is a compact set, over which function $Q_1(Z_1)$ is continuous and well-de£ned and can be approximated by neural networks to arbitrary any accuracy as follows

$$Q_1(Z_1) = W_1^{*T} S(Z_1) + \epsilon_1(Z_1) \quad (10)$$

where $\epsilon_1(Z_1)$ is the approximation error. Note that as the ideal weight W_1^* is unknown, we shall use its estimate $\hat{W}_1$ instead in the later controller design.

As can be seen from the previous discussion, the control effort will be activated only in the compact set $\Omega_{Z_1}^0$ so that we would like to relax our control objective to boundedness of states around the origin rather than the asymptotic convergence to origin. Accordingly, the following practical adaptive control is proposed

$$u(t) = \begin{cases} -k_1(t) z_1 - \hat{W}_1^T S(Z_1), & z_1 \in \Omega_{Z_1}^0 \\ 0, & z_1 \in \Omega_{c_{z_1}} \end{cases} \quad (11)$$

$$k_1(t) = k_{10} + k_{11} + \frac{\varepsilon_{10}}{z_1^2} \int_{t-\tau_{\max}}^{t} \frac{1}{2} x_1^2(\tau)\varrho_1^2(x_1(\tau)) d\tau \quad (12)$$

$$\dot{\hat{W}}_1 = \Gamma_1[S(Z_1) z_1 - \sigma_1(\hat{W}_1 - W_1^0)] \quad (13)$$

where $k_{10}^* \triangleq k_{10} - \frac{g_{1d}}{2g_{\min}} > 0$, $k_{11} > 0$, $\varepsilon_{10} > 0$, matrix $\Gamma_1 = \Gamma_1^T > 0$, σ_1 is a small constant to introduce the σ-modi£cation for the closed-loop system.

Theorem 1: Consider the closed-loop systems consisting of the £rst-order plant (3), the adaptive control (11)-(13), for bounded initial conditions $x_1(0)$ and $\hat{W}_1(0)$, all signals in the closed-loop systems are SGUUB, and the vector Z_1 remains in a compact set $\Omega_{Z_1}^0$ speci£ed by

$$\begin{aligned}
\Omega_{Z_1}^0 = \Big\{ Z_1 \Big| & z_1^2 \leq \mu_1, \|\hat{W}_1\|^2 \leq 2C_{01}/\lambda_{\min}(\Gamma_1^{-1}), \\
& \bar{x}_{d2} \in \Omega_{d2}, z_1 \notin \Omega_{c_{z_1}} \Big\} \quad (14)
\end{aligned}$$

whose size, $\mu_1 = \max\{\sqrt{2g_{\max} C_{01}}, c_{z_1}\}$, can be adjusted by appropriately choosing the design parameters.

Proof: Consider the Lyapunov function candidate $V_1(t)$ as

$$V_1(t) = V_{z_1}(t) + V_{U_1}(t) + \frac{1}{2}\tilde{W}_1^T(t)\Gamma_1^{-1}\tilde{W}_1(t) \quad (15)$$

with $(\tilde{\cdot}) = (\hat{\cdot}) - (\cdot)$. Its time derivative along (8) is

$$\dot{V}_1 \leq z_1[u + Q_1(Z_1)] + \frac{g_{1d}}{2g_{\min}} z_1^2 + \tilde{W}_1^T \Gamma_1^{-1} \dot{\hat{W}}_1 \quad (16)$$

Now, the stability analysis will be carried out in the following two regions: (i) $z_1 \in \Omega^0_{Z_1}$, and (ii) $z_1 \in \Omega_{c_{z_1}}$.

Region (i) $z_1 \in \Omega^0_{Z_1}$*:* In this region, the control is invoked. Substituting (10), (11)-(13) into (16) by noting the following inequalities

$$-k_{11}z_1^2 + z_1\epsilon_1(Z_1) \leq -k_{11}z_1^2 + |z_1|\epsilon^*_{z_1} \leq \frac{\epsilon^{*2}_{z_1}}{4k_{11}}$$

$$-\tilde{W}_1^T(\hat{W}_1 - W_1^0) \leq -\frac{1}{2}\|\tilde{W}_1\|^2 + \frac{1}{2}\|W_1^* - W_1^0\|^2$$

$$\int_{t-\tau_1}^{t} \frac{1}{2}x_1^2(\tau)\varrho_1^2(x_1(\tau))d\tau \leq \int_{t-\tau_{\max}}^{t} \frac{1}{2}x_1^2(\tau)\varrho_1^2(x_1(\tau))d\tau$$

we have

$$\dot{V}_1(t) \leq -c_1V_1(t) + \lambda_1 \tag{17}$$

with positive constants $\lambda_1 := \frac{1}{2}\sigma_1\|W_1^* - W_1^0\|^2 + \frac{\epsilon^{*2}_{z_1}}{4k_{11}}$ and $c_1 := \min\{2k^*_{10}g_{\min}, \varepsilon_{10}g_{\min}, \frac{\sigma_1}{\lambda_{\max}(\Gamma_1^{-1})}\}$. Let $\rho_1 := \lambda_1/c_1$, it follows that

$$0 \leq V_1(t) \leq \rho_1 + [V_1(0) - \rho_1]e^{-c_1t} \leq \rho_1 + V_1(0) \tag{18}$$

Region (ii) $z_1 \in \Omega_{c_{z_1}}$*:* In this region, $|z_1| < c_{z_1}$, i.e., z_1 is already bounded, and $\dot{\hat{W}}_1 = 0$. Since $z_1 = x_1 - y_d$ and y_d is bounded, x_1 is bounded. In addition, the adaptation for $\hat{W}_1$ has stopped and $\hat{W}_1$ is kept unchanged in bounded value.

From (15) and (18), we have

$$z_1^2 \leq 2g_{\max}C_{01}, \quad \|\tilde{W}_1\|^2 \leq 2C_{01}/\lambda_{\min}(\Gamma_1^{-1}) \tag{19}$$

with $C_{01} := \rho_1 + V_1(0)$. Noting that (19) holds for $|z_1| > c_{z_1}$, we readily have the compact set $\Omega^0_{Z_1}$ speci£ed in (14), over which the NN approximation is carried out with its feasibility being guaranteed. ■

V. Direct Neural Network Control for Nth-Order System

In this section, adaptive neural control is extended the higher-order system (1) using backstepping design and the stability results of the closed-loop system are presented. The n-step backstepping design procedure is based on the change of coordinates: $z_1 = x_1 - y_d$, $z_i = x_i - \alpha_{i-1}$, $i = 2, ..., n$, where $\alpha_i(t)$ is an intermediate control functions developed for the ith-subsystem based on an appropriate Lyapunov function $V_i(t)$. The control law $u(t)$ is designed in the last step. Note that the controller design based on such compact sets $\Omega^0_{Z_i}$ will render α_i not differentiable at points $|z_i| = c_{z_i}$. This problem can be easily £xed by simply setting the differentiation at these points to be any £nite value, say 0, and then every signal in the closed-loop system can be shown to be bounded. Theoretically speaking, by doing so, there is no much loss either as these points are isolated with £nite energy and can be ignored. For ease and clarity of presentation, we assume that all the control functions are differentiable throughout this Section. A modi£cation of the proposed design is provided in [21], in which the control functions strictly meet the differentiable condition required by backstepping design and much more involved stability analysis is also given.

For uniformity of notation, throughout this section, de£ne estimation errors $\tilde{W}_i = \hat{W}_i - W_i^*$, compact sets $\Omega_{c_{z_i}}$ and $\Omega^0_{Z_i}$ as $\Omega_{c_{z_i}} := \{z_i||z_i| < c_{z_i}\}$, $\Omega^0_{Z_i} := \Omega_{Z_i} - \Omega_{c_{z_i}}$ where constants $c_{z_i} > 0$, $\hat{W}_i \in R^{l_i}$ are the estimates of ideal NN weights $W_i^* \in R^{l_i}$, and the integral Lyapunov functions $V_{z_i}(t)$, the Lyapunov-Krasovskii functionals $V_{U_i}(t)$, and the Lyapunov function candidates $V_i(t)$ as

$$V_{z_i} = \frac{1}{2g_i(\bar{x}_i)}z_i^2 \tag{20}$$

$$V_{U_i} = \frac{1}{2g_{\min}}\int_{t-\tau_i}^{t} U_i(\bar{x}_i(\tau))d\tau \tag{21}$$

$$V_i = V_{z_i} + V_{U_i} + \frac{1}{2}\tilde{W}_i^T\Gamma_i^{-1}\tilde{W}_i \tag{22}$$

where positive functions $U_i(\bar{x}_i) = \sum_{j=1}^{i} x_j^2\varrho_{ij}^2(\bar{x}_i)$.

In the following steps, the unknown functions $Q_i(Z_i)$, $i = 1, ..., n$, which will be de£ned later, will be approximated by neural networks as

$$Q_i(Z_i) = W_i^{*T}S(Z_i) + \epsilon_i(Z_i), \forall Z_i \in \Omega^0_{Z_i} \tag{23}$$

where $\epsilon^*_{z_i}$ are the upper bounds of the NN approximation errors, i.e., $|\epsilon_i(Z_i)| \leq \epsilon^*_{z_i}$ with Z_i being the corresponding inputs to be de£ned later.

The following adaptive neural control laws are proposed

$$\alpha_i = \begin{cases} -k_i(t)z_i - \hat{W}_i^TS(Z_i), & z_i \in \Omega^0_{Z_i} \\ 0, & z_i \in \Omega_{c_{z_i}} \end{cases} \tag{24}$$

$$k_i(t) = k_{i0} + k_{i1} + \frac{\varepsilon_{i0}}{z_i^2}\int_{t-\tau_{\max}}^{t} \frac{1}{2}\sum_{j=1}^{i} x_j^2(\tau)\varrho_{ij}^2(\bar{x}_i(\tau))d\tau \tag{25}$$

$$\dot{\hat{W}}_i = \Gamma_i[S(Z_i)z_i - \sigma_i(\hat{W}_i - W_i^0)], i = 1, ..., n \tag{26}$$

with $\varepsilon_{i0} > 0$, matrix $\Gamma_i = \Gamma_i^T > 0$, W_i^0 being constant vector, constants $k_{i0}, k_{i1} > 0$ satisfying $k^*_{i0} \triangleq k_{i0} - g_{id}/(2g_{\min}) - 1/2 > 0$, constant $\sigma_i > 0$ to introduce σ-modi£cation to closed-loop systems (when $i = n$, $\alpha_n = u(t)$). De£ne positive constants $c_i := \min\{2k^*_{i0}g_{\min}, \epsilon_{10}g_{\min}, \frac{\sigma_i}{\lambda_{\max}(\Gamma_i^{-1})}\}$, $\lambda_i := \frac{1}{2}\sigma_i\|W_i^* - W_i^0\|^2 + \frac{\epsilon^{*2}_{z_i}}{4k_{i1}}$,

The unknown functions $Q_i(Z_i)$ is de£ned by

$$Q_i(Z_i) = \frac{1}{g_i(\bar{x}_i)}[f_i(\bar{x}_i) - \dot{\alpha}_{i-1} + \frac{1}{2}z_i] + \frac{1}{2g_{\min}z_i}\sum_{j=1}^{i} x_j^2(t)\varrho_{ij}^2(\bar{x}_i(t)) \tag{27}$$

with $Z_i = [\bar{x}_i, \dot{\bar{x}}_{i-1}, \frac{\partial\alpha_{i-1}}{\partial x_1}, \frac{\partial\alpha_{i-1}}{\partial x_2}, ..., \frac{\partial\alpha_{i-1}}{\partial x_{i-1}}, \omega_{i-1}] \in \Omega^0_{Z_i} \subset R^{3i-1}$, where

$$\dot{\alpha}_{i-1} = \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\dot{x}_j + \omega_{i-1}$$

$$\omega_{i-1} = \frac{\partial\alpha_{i-1}}{\partial\bar{x}_{di}}\dot{\bar{x}}_{di} + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{W}_j}\dot{\hat{W}}_j$$

Step 1*:* Let us £rstly consider the z_1-subsystem as $z_1 = x_1 - y_d$ and $z_2 = x_2 - \alpha_1$

$$\begin{aligned}\dot{z}_1(t) &= g_1(x_1(t))[z_2(t) + \alpha_1(t)] + f_1(x_1(t)) \\ &+h_1(x_1(t-\tau_1)) - \dot{y}_d(t) \end{aligned} \quad (28)$$

Consider the Lyapunov function candidate in (22). Following the same procedure as in Section IV by applying Assumption A4) and Young's inequality, we obtain

$$\begin{aligned}\dot{V}_1 &\le z_1[\alpha_1 + Q_1(Z_1)] + \frac{g_{1d}}{2g_{\min}} z_1^2 \\ &+z_1 z_2 + \tilde{W}_1^T \Gamma_1^{-1} \dot{\hat{W}}_1 \end{aligned} \quad (29)$$

Applying Young's inequality again for $z_1 z_2$, i.e., $z_1 z_2 \le \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2$, we have

$$\dot{V}_1 \le (\frac{g_{1d}}{2g_{\min}} + \frac{1}{2}) z_1^2 + \frac{1}{2} z_2^2 + z_1[\alpha_1 + Q_1(Z_1)] + \tilde{W}_1^T \Gamma_1^{-1} \dot{\hat{W}}_1 \quad (30)$$

where $Q_1(Z_1)$ is given in (9).

Considering the practical adaptive intermediate control given in (24)-(26), the stability analysis is carried out in the following two regions de£ned by the compact sets $\Omega^0_{Z_1}$ and $\Omega_{c_{z_1}}$ respectively.

Region 1: $z_1 \in \Omega^0_{Z_1}$. Substituting (24)-(26) into (30) yields $\dot{V}_1(t) \le -c_1 V_1(t) + \lambda_1 + \frac{1}{2} z_2^2$, from which we know that if z_2 can be regulated as bounded, the boundedness of $V_1(t)$, z_1, x_1 and $\hat{W}_1$ can be obtained as can be seen from Theorem 1. The regulation of z_2 will be conducted in the next steps.

Region 2: $z_1 \in \Omega_{c_{z_1}}$. In this region, $|z_1| < c_{z_1}$ is already bounded, and $\dot{\hat{W}}_1 = 0$. Hence, $x_1 = z_1 + y_d$ is bounded, and $\hat{W}_1$ is kept unchanged in bounded values. As $V_{z_1}(t)$ and $V_{U_1}(t)$ are smooth functions, for bounded x_1 and z_1, $V_{z_1}(t)$, $V_{U_1}(t)$ and$V_1(t)$ are bounded.

Step i *(*$2 \le i \le n-1$*):* Similar procedures are taken for $i = 2, \cdots, n-1$ as in Step 1.

The dynamics of z_i-subsystem is given by

$$\begin{aligned}\dot{z}_i(t) &= g_i(\bar{x}_i(t))[z_{i+1}(t) + \alpha_i(t)] + f_i(\bar{x}_i(t)) \\ &+h_i(\bar{x}_i(t-\tau_i)) - \dot{\alpha}_{i-1}(t)\end{aligned}$$

Consider the Lyapunov function candidate $V_i(t)$ in (22). Using Young's inequality and noting Assumption A4), the time derivative of $V_i(t)$ is

$$\begin{aligned}\dot{V}_i &\le -[\frac{\dot{g}_i(\bar{x}_i)}{2g_i^2(\bar{x}_i)} - \frac{1}{2}] z_i^2(t) + \frac{1}{2} z_{i+1}^2(t) \\ &+z_i[\alpha_i + Q_i(Z_i)] + \tilde{W}_i^T \Gamma_i^{-1} \dot{\hat{W}}_i \end{aligned} \quad (31)$$

where $Q_i(Z_i)$ is given in (27).

Consider the practical adaptive intermediate control given in (24)-(26). Similarly as in Step 1, the stability analysis is carried out in the two regions de£ned by the compact sets $\Omega^0_{z_i}$ and $\Omega_{c_{z_i}}$ respectively as follows.

For $z_i \in \Omega^0_{z_i}$, we have $\dot{V}_i(t) \le -c_i V_i(t) + \lambda_i + \frac{1}{2} z_{i+1}^2$, from which it can be seen that the stability of z_i-subsystem in this case is dependent on z_{i+1}, which will be dealt with in the next steps. For $z_i \in \Omega_{c_{z_i}}$, the boundedness of z_i, x_i and $\hat{W}_i$ directly follows.

Step n*:* This is the £nal step, since the actual control u appears in the dynamics of z_n-subsystem as given by

$$\dot{z}_n = g_n(\bar{x}_n(t))u + f_n(\bar{x}_n(t)) + h_n(\bar{x}_n(t-\tau_n)) - \dot{\alpha}_{n-1}(t)$$

Consider the Lyapunov function candidate $V_n(t)$ given in (22). The time derivative of $V_n(t)$ is

$$\dot{V}_n \le -\frac{\dot{g}_n(x)}{2g_n^2(x)} z_n^2(t) + z_n[\alpha_n + Q_n(Z_n)] + \tilde{W}_n^T \Gamma_n^{-1} \dot{\hat{W}}_n$$

where $Q_n(Z_n)$ is given in (27).

Considering the practical adaptive control given in (24)-(26). Similarly as in the previous steps, the stability analysis is carried out in the two regions de£ned by the compact sets $\Omega^0_{Z_n}$ and $\Omega_{c_{z_n}}$ respectively as follows. For $z_n \in \Omega^0_{z_n}$, the £nal control $u(t)$ is invoked and the time derivative of $V_n(t)$ is $\dot{V}_n(t) \le -c_n V_n(t) + \lambda_n$, from which we can conclude that $V_n(t)$ is bounded, hence z_n, $\hat{W}_n$ are bounded. For $z_n \in \Omega_{z_n}$, the boundedness of z_n directly follows. Hence, z_i, x_i and $\hat{W}_i$, $i = 1, ..., n-1$ are bounded. As $\dot{\hat{W}}_n = 0$, $\hat{W}_n$ is kept £xed in a bounded value.

Theorem 2: Consider the closed-loop system consisting of the plant (1) under Assumptions A1)-A5), the practical adaptive neural control (24)-(26). For bounded initial conditions, the following properties hold:

(i) all signals in the closed-loop system are semi-globally uniformly ultimately bounded and the vector $Z = [Z_1^T, ..., Z_n^T]^T$ remains in a compact set $\Omega^0_Z := \Omega^0_{Z_1} \cup ... \cup \Omega^0_{Z_n}$ speci£ed as

$$\begin{aligned}\Omega^0_Z = \Big\{ Z \big| \sum_{i=1}^n z_i^2 \le 2g_{\max} C_0, \sum_{i=1}^n \|\tilde{W}_i\|^2 \le \frac{2C_0}{\lambda_{\min}(\Gamma_i^{-1})}, \\ \bar{x}_{di} \in \Omega_{di}, i = 2, ..., n, z_i \notin \Omega_{c_{z_i}}, i = 1, ..., n \Big\}\end{aligned} \quad (32)$$

where $C_0 > 0$ is a constant whose size depends on the initial conditions (as will be de£ned later in the proof);

(ii) the closed-loop signal $z(t) = [z_1, ..., z_n]^T \in R^n$ will eventually converge to a compact set de£ned by

$$\Omega_S := \{ z \big| \|z\|^2 \le \mu \} \quad (33)$$

with $\mu > 0$ is a constant related to the design parameters and will be de£ned later in the proof, and Ω_S can be made as small as desired by an appropriate choice of the design parameters.

Proof: Consider the following Lyapunov function candidate

$$V(t) = \sum_{i=1}^n [V_{z_i}(t) + V_{U_i}(t) + \frac{1}{2} \tilde{W}_i^T \Gamma_i^{-1} \tilde{W}_i] \quad (34)$$

where $V_{z_i}(t)$ and $V_{U_i}(t)$ are de£ned in (20) and (21) respectively. The following three cases are considered.

Case 1): $z_i \in \Omega_{c_{z_i}}$, $i = 1, ..., n$. In this case, the controls $\alpha_i = 0$ and $\dot{\hat{W}}_i = 0$. Since $z_1 = x_1 - y_d$ and y_d is bounded, x_1 is bounded. For $i = 2, ..., n$, x_i is bounded as $x_i =$

$z_i+\alpha_{i-1}$ and $\alpha_{i-1}=0$. In addition, $\hat{W}_i$ is kept unchanged in a bounded value, $i=1,...,n$. Observing the de£nition for $V_{z_i}(t)$ and $V_{U_i}(t)$ and noting that $g_i(\cdot)$, $\varrho_{ij}(\cdot)$ are smooth functions, we know that for bounded x_i, z_i and $\hat{W}_i$, $V_{z_i}(t)$ and $V_{U_i}(t)$ are bounded, i.e., there exists a £nite C_B such that $V(t)\le C_B$.

Case 2): $z_i\in\Omega^0_{Z_i}$, $i=1,...,n$. In Step n, we have $\dot{V}_n(t)\le -c_nV_n(t)+\lambda_n$. Let $\rho_n=\lambda_n/c_n$, it follows that

$$0\le V_n(t)\le [V_n(0)-\rho_n]e^{-c_nt}+\rho_n\le V_n(0)+\rho_n \quad (35)$$

From (22), we have $z_n^2\le 2g_{\max}[V_n(0)+\rho_n]$, and $\|\tilde{W}_n\|^2\le 2[V_n(0)+\rho_n]/\lambda_{\min}(\Gamma_n^{-1})$. Similarly, we can conclude that for $i=1,\cdots,n-1$, $z_i^2\le 2g_{\max}(V_i(0)+\rho_i)$, $\|\tilde{W}_i\|^2\le 2(V_i(0)+\rho_i)/\lambda_{\min}(\Gamma_i^{-1})$ with $\rho_i=[\lambda_i+g_{\max}(V_{i-1}(0)+\rho_{i-1})]$.

Case 3): Some $z_i\in\Omega^0_{Z_i}$ and some $z_j\in\Omega_{c_{z_j}}$. In this case, the corresponding α_i or u and the adaptation law for $\hat{W}_i$ will be invoked for $z_i\in\Omega^0_{Z_i}$ while $\alpha_j=0$ or $u=0$ and $\dot{\hat{W}}_j=0$ for $z_j\in\Omega_{c_{z_j}}$. Let us de£ne $V_I(t)=\sum_i V_i$ and $V_J(t)=\sum_j V_j$. For $z_j\in\Omega_{c_{z_j}}$, we know that $V_J(t)$ is bounded, i.e., $V_J(t)\le C_J$ with C_J being £nite, and for $z_i\in\Omega^0_{Z_i}$, we obtain that $\dot{V}_i(t)\le -c_i^IV_i(t)+\lambda_i^I+\frac{1}{2}z_{i+1}^2$. Let us de£ne $\rho_i^I=[\lambda_i^I+\frac{1}{2}\max\{z_{i+1}^2\}]/c_i^I$, we have $V_I\le V_I(0)+\rho_I$ with $V_I(0)=\sum_i V_i(0)$ and $\rho_I=\sum_i\rho_i^I$. Therefore, it can be obtained that $V(t)=V_I(t)+V_J(t)\le V_I(0)+\rho_I+C_J$.

Thus, from Cases 1), 2) and 3), we can conclude that $V(t)\le C_0$ with $C_0=\max\{C_B,\sum_{i=1}^n(V_i(0)+\rho_i),V_I(0)+\rho_I+C_J\}$, from which we know that $V_i(t)$, z_i and $\hat{W}_i$, $i=1,...,n$, are bounded, and the boundedness of the systems' states x_i, $i=1,...,n$ directly follows.

Considering (34), we know that $\sum_{i=1}^n z_i^2\le 2g_{\max}V(t)$, $\sum_{i=1}^n\|\tilde{W}_i\|^2\le 2V(t)/\lambda_{\min}(\Gamma_1^{-1},...,\Gamma_n^{-1})$ from which, we readily have the compact set Ω_Z^0 de£ned in (32) over which the NN approximation is carried out with its feasibility being guaranteed.

In addition, in Case 1), as $z_i\in\Omega_{c_{z_i}}$, $i=1,...n$, we know that $\|z\|^2=\sum_{i=1}^n z_i^2\le\sum_{i=1}^n c_{z_i}^2$. In Case 2), we have that $\lim_{t\to\infty}\|z\|^2=2g_{\max}\sum_{i=1}^n\rho_i$. In Case 3), we have that $\lim_{t\to\infty}\sum_i z_i^2=2g_{\max}\rho_I$ and $\sum_j z_j^2\le\sum_j c_{z_j}^2$. Therefore as $t\to\infty$, we can conclude that $\|z\|^2\le\mu$ where $\mu=\max\{2g_{\max}\sum_{i=1}^n\rho_i,2g_{\max}\rho_I,\sum_{i=1}^n c_{z_i}^2\}$, i.e., the vector z will eventually converge to the compact set Ω_S de£ned in (33). This completes the proof. ∎

VI. Conclusion

Practical adaptive neural control has been addressed for a class of nonlinear systems with unknown time delays in strict-feedback form. The unknown time delays has been compensated for through the use of appropriate Lyapunov-Krasovskii functionals. Controller singularity problems have been solved by employing practical neural network control based on decoupled backstepping design. The proposed design has been proven to be able to guarantee semi-globally uniformly ultimate boundedness of all the signals in the closed-loop system and the tracking error is proven to converge to a small neighborhood of the origin. In addition, the residual set of each states in the closed-loop systems has been determined respectively.

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