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On the Discrete–Time Modelling and Control of Induction Motors with Sliding Modes

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***Abstract*— In this work a discrete–time controller for an induction motor is proposed. State feedback and diffeomorphism are applied to the plant dynamics in order to be finitely discretized. Then, on the base of the sampled dynamics, a discrete–time controller is derived, achieving speed and flux modulus tracking objectives. Finally, a reduced order observer is designed for rotor fluxes and load torque observation.**

***Index Terms*— Induction motors, sliding mode control, discrete–time systems, observer.**

I. Introduction

INDUCTION motors are among the most used actuators for industrial applications due to their reliability, ruggedness and relatively low cost. On the other hand, the control of induction motor is a challenging task since the dynamical system is multivariable, coupled, and highly nonlinear. Several control techniques have been developed for induction motors [1], [3], [13], [12], among which the sliding mode technique [14],[5]. Typically, when implemented on digital devices, the control law is approximated by using zero order holders. This approximation represents a clear disadvantage. Analogously to [2] and to what done in other applications such as in [6], [11] and [4], the alternative is to design a digital controller directly using a digital model of the motor [9]. Unfortunately, the sampled model of the induction motor is only approximated, since it is expressed as an infinite series. To bypass this difficulty, following [10] in this work we obtain an exact closed form of the sampled dynamics using a preliminary continuous feedback which ensures the finite discretizability. In the case of the induction motors such a closed form discretization can be obtained in a rather simple way. The advantage of working with a closed form discretization is clear, and in this respect the use of the sliding mode technique fits well with the design of the control law directly in the digital setting. After deriving the digital controller, we will design a reduced order observer for the estimation of the load torque and motor fluxes, in order to eliminate the need of the full state measurements.

The paper is organized as follows. In Section II the continuous–time induction motor model is briefly reviewed, and the exact sampled dynamics of this model are derived. In Section III a discrete–time sliding mode control for rotor angular velocity and square modulus of the rotor flux vector tracking is designed. To remove the hypothesis on rotor fluxes and load torque measurability, a discrete–time observer is proposed in Section IV. Section V shows the simulation of the closed-loop induction motor control system. Final comments conclude the paper.

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II. Sampled Dynamics of Induction Motors

In the following a sampled version of the dynamics of an induction motor will be derived. Under the assumptions of equal mutual inductance and linear magnetic circuit, a fifth–order induction motor model is written as follows [8]

$$\begin{array}{rcl} \dot{\Phi} & = & -\alpha\Phi + p\omega\Im\Phi + \alpha L_m I \\ \dot{I} & = & \alpha\beta\Phi - p\beta\omega\Im\Phi - \gamma I + \frac{1}{\sigma}u \\ \dot{\omega} & = & \mu I^T\Im\Phi - \frac{1}{J}T_L \\ \dot{\theta} & = & \omega \end{array} \tag{1}$$

where θ and ω are the rotor angular position and velocity respectively, $\Phi = (\phi_\alpha, \phi_\beta)^T$ is the rotor flux vector, $I = (i_\alpha, i_\beta)^T$ is the stator current vector, $u = (u_\alpha, u_\beta)^T$ is the control input voltage vector, T_L is the load torque, J is the rotor moment of inertia, $\Im = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $\alpha = \frac{R_r}{L_r}$, $\beta = \frac{L_m}{\sigma L_r}$, $\gamma = \frac{L_m^2 R_r}{\sigma L_r^2} + \frac{R_s}{\sigma}$, $\sigma = L_s - \frac{L_m^2}{L_r}$, $\mu = \frac{3}{2}\frac{L_m p}{J L_r}$, with L_s, L_r L_m being the stator, rotor and mutual inductances respectively, R_s and R_r are the stator and rotor resistances, and p is the number of pole pairs.

The following hypothesis will be instrumental for deriving the sampled model of the motor dynamics.

(H_1) *The load torque T_L can be approximated by a signal C_L which is constant over the sampling period δ.*

Hypothesis (H_1) is acceptable in all cases in which T_L varies slowly with respect to the system dynamics. In order to obtain a finite discretization of the system dynamics (1), in the spirit of [9], [10] let us consider first the following feedback

$$u = \sigma p\omega\Im(I + \beta\Phi) + e^{p\theta\Im}e^{-p\theta_k\Im}v. \tag{2}$$

Here θ_k indicates the value of θ at the time instant $k\delta$, with δ the sampling period and $k = 0, 1, 2, \cdots$. Note that the first term of (2) and the term $e^{p\theta\Im}$ have to be implemented via an analogical device, while the term $e^{-p\theta_k\Im}$ and the new control v (designed on the basis of the discrete time representation of the system) can be implemented via a digital device.

Hence, the following controlled dynamics are obtained

$$\begin{array}{rcl}\dot{\Phi} & = & -\alpha\Phi + p\omega\Im\Phi + \alpha L_m I \\ \dot{I} & = & \alpha\beta\Phi - \gamma I + p\omega\Im I + \frac{1}{\sigma}e^{p\theta\Im}e^{-p\theta_k\Im}v \\ \dot{\omega} & = & \mu I^T\Im\Phi - \frac{1}{J}T_L \\ \dot{\theta} & = & \omega.\end{array} \tag{3}$$

Then, the finite discretization will be obtained making use of the following globally defined change of coordinates and inputs

$$\begin{pmatrix}\tilde{\Phi}\\ \tilde{I}\\ \omega\\ \theta\end{pmatrix} = \begin{pmatrix} e^{-p\theta\Im}\Phi\\ e^{-p\theta\Im}I\\ \omega\\ \theta\end{pmatrix}, \quad \tilde{v} = e^{-p\theta_k\Im}v. \tag{4}$$

The transformed variables $\tilde{\Phi}$, $\tilde{I}$ in (4) are the flux and the current rotated according to the electrical rotor angular position $p\theta$. An analogous consideration holds for the new input $\tilde{v}$ in (4). Note that $\tilde{v}$ is constant over the sampling period when v is constant and equal to $v_k = v(k\delta)$.

In the new variables (4) and under hypothesis (H_1), the dynamics in (3) are expressed as follows

$$\begin{array}{rcl}\dot{\tilde{\Phi}} & = & -\alpha\tilde{\Phi} + \alpha L_m\tilde{I} \\ \dot{\tilde{I}} & = & \alpha\beta\tilde{\Phi} - \gamma\tilde{I} + \frac{1}{\sigma}\tilde{v} \\ \dot{\omega} & = & \mu\tilde{I}^T\Im\tilde{\Phi} - \frac{1}{J}C_L \\ \dot{\theta} & = & \omega\end{array} \tag{5}$$

since $\frac{d}{dt}e^{-p\theta\Im} = -p\omega\Im e^{-p\theta\Im}$. Note that equations (5) are nonlinear, but the closed form discretization is now easily obtained by noting that the dynamics for $\tilde{\Phi}$ and $\tilde{I}$ are linear, and the control $\tilde{v}$ will be designed to be constant over the sampling period δ. Denoting $\tilde{\Phi}_k = \tilde{\Phi}(k\delta)$, $\tilde{I}_k = \tilde{I}(k\delta)$, $\omega_k = \omega(k\delta)$, $C_{L,k} = C_L(k\delta)$, and $\tilde{v}_k = \tilde{v}(k\delta)$, long but trivial calculations provide the *exact closed form* discretization of the system (5)

$$\begin{array}{rcl}\begin{pmatrix}\tilde{\Phi}_{k+1}\\ \tilde{I}_{k+1}\end{pmatrix} & = & A_d\begin{pmatrix}\tilde{\Phi}_k\\ \tilde{I}_k\end{pmatrix} + B_d\tilde{v}_k \\ \omega_{k+1} & = & \omega_k + \eta_{1,k}\tilde{I}_k^T\Im\tilde{\Phi}_k - \dfrac{C_{L,k}}{J}\delta \\ & & +\left(\eta_{2,k}\tilde{\Phi}_k^T + \eta_{3,k}\tilde{I}_k^T\right)\Im\tilde{v}_k \\ \theta_{k+1} & = & \omega_k\delta + \kappa_{1,k}\tilde{I}_k^T\Im\tilde{\Phi}_k - \dfrac{C_{L,k}}{J}\dfrac{\delta^2}{2} \\ & & +\left(\kappa_{2,k}\tilde{\Phi}_k^T + \kappa_{3,k}\tilde{I}_k^T\right)\Im\tilde{v}_k + \theta_k\end{array}$$

with output

$$y_k = \begin{pmatrix}\omega_k\\ \tilde{\Phi}_k^T\tilde{\Phi}_k\end{pmatrix}$$

where $\eta_{1,k}$, $\eta_{2,k}$, $\eta_{3,k}$, $\kappa_{1,k}$, $\kappa_{2,k}$ and $\kappa_{3,k}$ are bounded functions,

$$A_d = e^{\delta A} = \begin{pmatrix} a_{11}I_{2x2} & a_{12}I_{2x2}\\ a_{21}I_{2x2} & a_{22}I_{2x2}\end{pmatrix},$$

$$B_d = \int_0^\delta e^{\xi A}B\,d\xi = \begin{pmatrix} b_1 I_{2x2}\\ b_2 I_{2x2}\end{pmatrix},$$

$$A = \begin{pmatrix}-\alpha I_{2x2} & \alpha L_m I_{2x2}\\ \alpha\beta I_{2x2} & -\gamma I_{2x2}\end{pmatrix}, \quad B = \begin{pmatrix}0_{2x2}\\ \frac{1}{\sigma}I_{2x2}\end{pmatrix}$$

a_{11}, a_{12}, a_{21}, a_{22}, b_1, b_2 are constants, and I_{2x2}, 0_{2x2} are the identity and zero matrices, respectively.

III. Discrete–time Control of Induction Motors

The controlled variables are the angular velocity and flux modulus tracking. The control aim is to track fixed references along with disturbance rejection. This will be realized by means of a discrete–time sliding mode control [14]. The hypothesis of full state and disturbance measurability, here used, will be removed in the next section.

Let us define the output tracking error

$$e_k = y_k - y_{r,k} \tag{6}$$

where $e_k = (e_{1,k} \quad e_{2,k})^T$, $y_{r,k} = (\omega_{r,k} \quad \tilde{\Phi}_{r,k})^T$ with $\omega_{r,k}$ and $\tilde{\Phi}_{r,k}$ the rotor angular velocity and the rotor flux square modulus references, respectively. Then, the system error dynamics are given by

$$e_{k+1} = \begin{pmatrix}\xi_{1,k} + \lambda_{1,k}^T\tilde{v}_k - \frac{\delta}{J}C_{L,k} - \omega_{r,k+1}\\ \xi_{2,k} + \lambda_{2,k}^T\tilde{v}_k + b_1^2\tilde{v}_k^T\tilde{v}_k - \tilde{\Phi}_{r,k+1}\end{pmatrix} \tag{7}$$

where

$$\begin{array}{rcl}\xi_{1,k} & = & \omega_k + \eta_{1,k}\tilde{I}_k^T\Im\tilde{\Phi}_k, \\ \lambda_{1,k}^T & = & \left(\eta_{2,k}\tilde{\Phi}_k^T + \eta_{3,k}\tilde{I}_k^T\right)\Im, \\ \xi_{2,k} & = & a_{11}^2\tilde{\Phi}_k^T\tilde{\Phi}_k + 2a_{11}a_{12}\tilde{\Phi}_k^T\tilde{I}_k + a_{12}^2\tilde{I}_k^T\tilde{I}_k, \\ \lambda_{2,k}^T & = & 2a_{11}b_1\tilde{\Phi}_k^T + 2a_{12}b_1\tilde{I}_k^T.\end{array}$$

The design of the control law is complicated by the fact that system (7) depends on quadratic control terms. In order to simplify the control design, the input $\tilde{v}_k$ is transformed into a new control w_k as follows

$$w_k = B_k\tilde{v}_k \tag{8}$$

where $B_k = (\lambda_{1,k}^T, \lambda_{2,k}^T)^T$, $w_k = (w_{\alpha,k}, w_{\beta,k})^T$ and $d_k = \det(B_k) \neq 0$. Due to (8), the difference equation for $e_{1,k+1}$ depends only on the input $w_{\alpha,k}$, and therefore, the control design is simplified. Now, replacing (8) in (7) we obtain the following equations

$$\begin{array}{rcl}e_{1,k+1} & = & \xi_{1,k} + w_{\alpha,k} - \dfrac{\delta}{J}C_{L,k} - \omega_{r,k+1} \\ e_{2,k+1} & = & \xi_{2,k} + w_{\beta,k} \\ & & +b_1^2 w_k^T(B_k^{-1})^T(B_k^{-1})w_k - \tilde{\Phi}_{r,k+1}.\end{array}$$

In discrete–time sliding mode control schemes [14], two steps design are performed. First, a sliding surface S_k is chosen and, second, a sliding control is designed. Error functions are natural choices as sliding surface functions. Therefore, we choose $S_k = (S_{1,k}, S_{2,k})^T = (e_{1,k}, e_{2,k})^T = 0$ as sliding surface.

A. Rotor Angular Velocity Control

As mentioned above, the control objective is to realize angular velocity tracking and disturbance rejection. For, an equivalent control $w_{eq_\alpha,k}$ is calculated from $S_{1,k+1} = 0$, obtaining

$$w_{eq_\alpha,k} = -\xi_{1,k} + \frac{\delta}{J} C_{L,k} + \omega_{r,k+1}.$$

Let us write $w_{eq_\alpha,k}$ and $S_{1,k+1}$ as follows

$$\begin{aligned} w_{eq_\alpha,k} &= -(S_{1,k} + \xi_{1,k} - \tfrac{\delta}{J} C_{L,k} \\ &\qquad -\omega_{r,k+1} - \omega_k + \omega_{r,k}) \\ S_{1,k+1} &= S_{1,k} + \xi_{1,k} - \tfrac{\delta}{J} C_{L,k} \\ &\qquad -\omega_{r,k+1} - \omega_k + \omega_{r,k} + w_{\alpha,k}. \end{aligned} \tag{9}$$

Then, we consider the following control

$$w_{\alpha,k} = \begin{cases} w_{eq_\alpha,k} & \text{if} \quad |w_{eq_\alpha,k}| \le w_{0,\alpha} \\ w_{0,\alpha} \frac{w_{eq_\alpha,k}}{|w_{eq_\alpha,k}|} & \text{if} \quad |w_{eq_\alpha,k}| > w_{0,\alpha} \end{cases}$$

with $w_{0,\alpha}$ a bound. Now, when $|w_{eq_\alpha,k}| \le w_{0,\alpha}$, one has $w_{\alpha,k} = w_{eq_\alpha,k}$, ensuring the evolution on the sliding manifold $S_{1,k} = 0$. When $|w_{eq_\alpha,k}| > w_{0,\alpha}$, the second equation of (9) yields

$$\begin{aligned} S_{1,k+1} &= S_{1,k} + \xi_{1,k} - \frac{\delta}{J} C_{L,k} \\ &\qquad -\omega_{r,k+1} - \omega_k + \omega_{r,k} + w_{0,\alpha} \frac{w_{eq_\alpha,k}}{|w_{eq_\alpha,k}|} \\ &= \Big(S_{1,k} + \xi_{1,k} - \frac{\delta}{J} C_{L,k} \\ &\qquad -\omega_{r,k+1} - \omega_k + \omega_{r,k} \Big) \left(1 - \frac{w_{0,\alpha}}{|w_{eq_\alpha,k}|} \right) \end{aligned}$$

and making use of absolute values we have that

$$\begin{aligned} |S_{1,k+1}| &= \Big| S_{1,k} + \xi_{1,k} - \frac{\delta}{J} C_{L,k} \\ &\qquad -\omega_{r,k+1} - \omega_k + \omega_{r,k} \Big| \left(1 - \frac{w_{0,\alpha}}{|w_{eq_\alpha,k}|} \right) \\ &= \Big| S_{1,k} + \xi_{1,k} - \frac{\delta}{J} C_{L,k} \\ &\qquad -\omega_{r,k+1} - \omega_k + \omega_{r,k} \Big| - w_{0,\alpha} \\ &\le |S_{1,k}| + \Big| \xi_{1,k} - \frac{\delta}{J} C_{L,k} \\ &\qquad -\omega_{r,k+1} - \omega_k + \omega_{r,k} \Big| - w_{0,\alpha}. \end{aligned}$$

If

$$\left| \xi_{1,k} - \frac{\delta}{J} C_{L,k} - \omega_{r,k+1} - \omega_k + \omega_{r,k} \right| < w_{0,\alpha}$$

then $|S_{1,k+1}| < |S_{1,k}|$ and therefore $|S_{1,k}|$ decreases monotonically and after a finite number of steps $|w_{eq_\alpha,k}| \le w_{0,\alpha}$ is achieved, so that ω_k tends asymptotically to $\omega_{r,k}$.

B. Square Modulus Rotor Flux Control

Now, let us turn to the design of $w_{\beta,k}$ in order to stabilize $S_{2,k}$. The dynamics for $S_{2,k+1}$ can be written as follows

$$S_{2,k+1} = a_k w_{\beta,k}^2 + b_k w_{\beta,k} + c_k \tag{10}$$

where

$$\begin{aligned} a_k &= \frac{b_1^2 \lambda_{1,k}^T \lambda_{1,k}}{d_k^2}, \qquad b_k \;=\; 1 - \frac{2 b_1^2 w_{\alpha,k} \lambda_{1,k}^T \lambda_{2,k}}{d_k^2} \\ c_k &= \xi_{2,k} - \tilde{\Phi}_{r,k+1} + \frac{b_1^2 w_{\alpha,k}^2 \lambda_{2,k}^T \lambda_{2,k}}{d_k^2}. \end{aligned}$$

Then, we calculate the equivalent control $w_{eq_\beta,k}$ as solution of $S_{2,k+1} = 0$:

$$w_{eq_\beta,k} = \frac{-b_k + \sqrt{b_k^2 - 4 a_k c_k}}{2 a_k}. \tag{11}$$

It can be checked that $a_k \neq 0, \forall\, k$. On the other hand, the equivalent control (11) is only valid when the discriminant is greater or equal to zero, i.e., $b_k^2 - 4 a_k c_k \ge 0$. When $b_k^2 - 4 a_k c_k < 0$, in order to overcome the mathematical difficulty, we consider $\tilde{w}_{eq_\beta,k} = -\frac{b_k}{a_k}$ as equivalent control, which is such that $S_{2,k+1} = c_k$. Therefore, we introduce the term

$$\tilde{w}_{\beta,k} = \begin{cases} w_{eq_\beta,k} & \text{if} \quad b_k^2 - 4 a_k c_k \ge 0 \\ \tilde{w}_{eq_\beta,k} & \text{if} \quad b_k^2 - 4 a_k c_k < 0 \end{cases} \tag{12}$$

and the following control

$$w_{\beta,k} = \begin{cases} \tilde{w}_{\beta,k} & \text{if} \quad |\tilde{w}_{\beta,k}| \le w_{0,\beta} \\ -w_{0,\beta} \frac{\tilde{w}_{\beta,k}}{|\tilde{w}_{\beta,k}|} & \text{if} \quad |\tilde{w}_{\beta,k}| > w_{0,\beta} \end{cases}$$

with $w_{0,\beta}$ an appropriate bound.

When $|\tilde{w}_{\beta,k}| \le w_{0,\beta}$ the applied control is $\tilde{w}_{eq_\beta,k}$, and one can verify that coherently condition $b_k^2 - 4 a_k c_k < 0$ will take place. In this case, $S_{2,k}$ tends to c_k, and since c_k tends asymptotically to zero there exists a critical time instant k_{cr} in which $b_k^2 - 4 a_k c_k \ge 0$, $\forall\, k \ge k_{cr}$, and $\tilde{w}_{\beta,k}$ will switch to $w_{eq_\beta,k}$, determining an evolution on the sliding manifold $S_{2,k} = 0$ from the time instant $k_{cr} + \delta$ on.

To complete the stability analysis, let us consider the case $|\tilde{w}_{\beta,k}| > w_{0,\beta}$. Correspondingly, (10) is represented in the following form

$$\begin{aligned} S_{2,k+1} &= S_{2,k} + a_k w_{\beta,k}^2 + b_k w_{\beta,k} + c_k \\ &\qquad -\tilde{\Phi}_k^T \tilde{\Phi}_k + \tilde{\Phi}_{r,k}. \end{aligned}$$

Hence

$$\begin{aligned} S_{2,k+1} &= S_{2,k} + a_k w_{0,\beta}^2 - b_k w_{0,\beta} \frac{\tilde{w}_{\beta,k}}{|\tilde{w}_{\beta,k}|} \\ &\qquad + c_k - \tilde{\Phi}_k^T \tilde{\Phi}_k + \tilde{\Phi}_{r,k} \end{aligned}$$

and making use of absolute values

$$\begin{aligned} |\, S_{2,k+1} \,| &\le |\, S_{2,k} + a_k w_{0,\beta}^2 - b_k w_{0,\beta} \frac{\tilde{w}_{\beta,k}}{|\tilde{w}_{\beta,k}|} \\ &\qquad + c_k - \tilde{\Phi}_k^T \tilde{\Phi}_k + \tilde{\Phi}_{r,k} \,| \,. \end{aligned}$$

Now, if

$$\left|\tilde{w}_{\beta,k} - \tilde{\Phi}_k^T\tilde{\Phi}_k + \tilde{\Phi}_{r,k}\right| < w_{0,\beta}$$

it can be checked that $|S_{2,k}|$ and $\tilde{w}_{\beta,k}$ decrease monotonically. Hence, when $|w_{eq_\beta,k}| \leq w_{0,\beta}$, the control will change from $-w_{0,\beta}\frac{\tilde{w}_{\beta,k}}{|\tilde{w}_{\beta,k}|}$ to (12).

IV. Discrete–Time Control From Measured Variables

In practical cases, the rotor flux and the load torque are not measurable. Hence, a discrete–time observer is proposed in the following.

For the load torque estimation we consider the following hypothesis.

(H_2) *The load torque dynamics are slow with respect to the electromagnetic ones, namely* $C_{L,k+1} = C_{L,k}$.

The flux observer is of the following form

$$\hat{\tilde{\Phi}}_{k+1} = a_{11}\hat{\tilde{\Phi}}_k + a_{12}\tilde{I}_k + b_1\tilde{v}_k$$

so that the dynamical error equation becomes

$$e_{\Phi,k+1} = a_{11}e_{\Phi,k}, \quad e_{\Phi,k} = \tilde{\Phi}_k - \hat{\tilde{\Phi}}_k.$$

It can be checked that $|a_{11}| < 1$. Hence, $\hat{\tilde{\Phi}}_k$ asymptotically converges to $\tilde{\Phi}_k$.

As far as the load torque estimation is concerned, let us consider the following estimator

$$\begin{aligned} \hat{\omega}_{k+1} &= \omega_k + \eta_{1,k}\tilde{I}_k^T\Im\hat{\tilde{\Phi}}_k \\ &\quad + \left(\eta_{2,k}\hat{\tilde{\Phi}}_k^T + \eta_{3,k}\tilde{I}_k^T\right)\Im\tilde{v}_k \\ &\quad - \frac{\delta}{J}\hat{C}_{L,k} + l_1(\omega_k - \hat{\omega}_k) \\ \hat{C}_{L,k+1} &= \hat{C}_{L,k} + l_2(\omega_k - \hat{\omega}_k). \end{aligned}$$

Setting $e_{\omega,k} = \omega_k - \hat{\omega}_k$, $e_{L,k} = C_{L,k} - \hat{C}_{L,k}$ as rotor angular velocity and load torque estimate errors, respectively, the dynamical error equations are

$$\begin{aligned} \begin{pmatrix} e_{\omega,k+1} \\ e_{L,k+1} \end{pmatrix} &= \begin{pmatrix} -l_1 & -\frac{\delta}{J} \\ -l_2 & 1 \end{pmatrix}\begin{pmatrix} e_{\omega,k} \\ e_{L,k} \end{pmatrix} \\ &\quad + \eta_{1,k}\tilde{I}_k^T\Im e_{\Phi,k} + \eta_{2,k}e_{\Phi,k}^T\Im\tilde{v}_k \end{aligned} \quad (13)$$

Since $e_{\Phi,k}$ tends asymptotically to zero and $\eta_{1,k}\tilde{I}_k^T\Im$ and $\eta_{2,k}\Im\tilde{v}_k$ are bounded terms, choosing l_1 and l_2 such that the dynamical matrix in (13) is Hurwitz, then $\hat{w}_k$, $\hat{C}_{L,k}$ asymptotically converge to w_k, $C_{L,k}$.

V. Simulation Results

The results of the above sections are simulated considering a three–phase, two pole induction motor with parameters values defined as follows: $R_s = 14\ \Omega$, $L_s = 400$ mH, $L_m = 377$ mH, $R_r = 10.1\ \Omega$, $L_r = 412.8$ mH, $J = 0.01$ Kg m^2 and $\delta = 0.0001$ s.

The output tracking simulations results are shown in figure 1. The rotor angular velocity reference has a sinusoidal

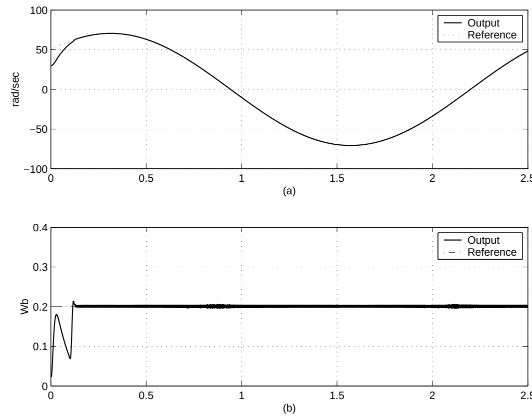

Fig. 1. (a) Rotor angular velocity tracking (b) Flux modulus tracking.

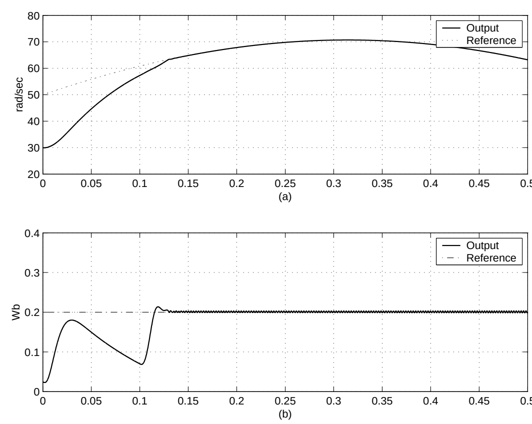

Fig. 2. (a) Transient angular velocity response (b) Transient flux modulus response

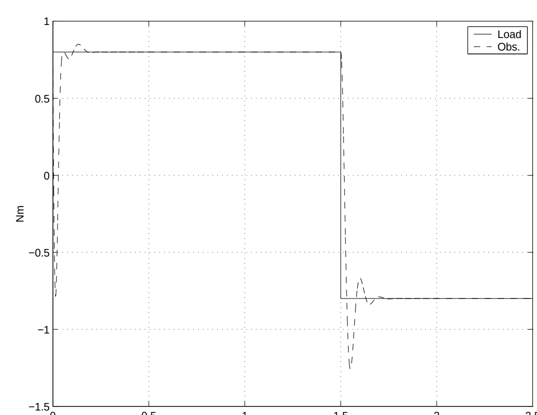

Fig. 3. Load estimate

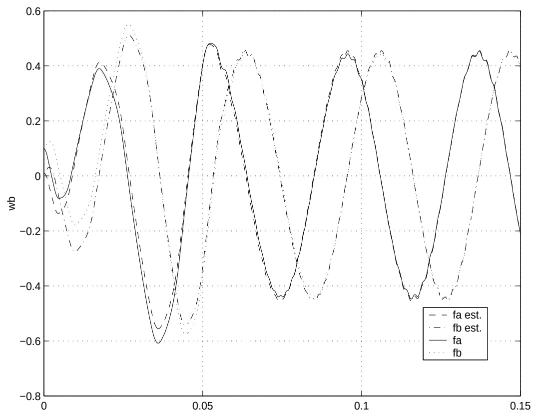

Fig. 4. Estimates of the rotor fluxes

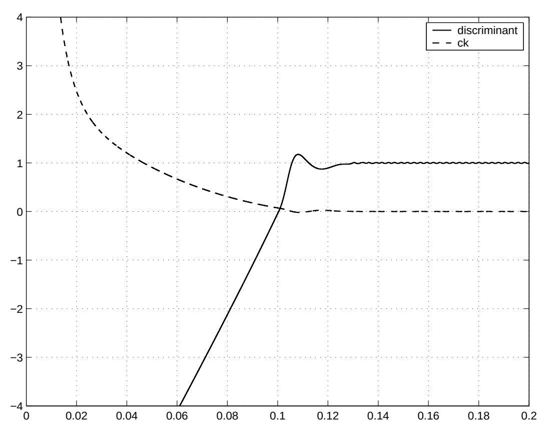

Fig. 5. Graphical depiction of the discriminant and c_k

shape and the flux modulus reference is constant at 0.2 Wb. Figure 2 is a zoomed image of 1 in order to observe the transient response. The unknown load torque is supposed square–shape and the observer behavior is shown in figure 3, where the observer gains are $l_1 = 50$ and $l_2 = -45.78$.

The estimate of the rotor fluxes is shown in figure 4. It is worth to mention that the continuous–time induction motor model is simulated with the discrete–time controller, and as can be appreciated, the results predict that the control strategy here presented performs well.

Now, we show by simulations the facts presented in Section III about the discriminant $(b^2 - 4a_k c_k)$ and the value c_k. Figure 5 shows these variables. It can be appreciated that the discriminant starts with a negative value but, as c_k asymptotically decays to zero, the discriminant approaches zero and finally reaches the steady–state positive value equal to one. All initial assumptions are satisfied by any initial conditions and any value of the plant load torque.

VI. Conclusions and Future Work

In this work we have used some results on finite discretizability of nonlinear continuous–time systems [9], [10] to determine an exact sampled–data representation of induction motors. Using the model so determined, we have designed a hybrid observer–based controller to solve the tracking problem on output velocity and flux modulus, in presence of an unknown load torque. Open problems remain, among which the implementation with discrete devices of the continuous part of the controller, and the study of the robustness versus parameters uncertainties.

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