

Electro-Hydraulic Actuator Trajectory Tracking

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Abstract— This paper combines block control, sliding mode control and integral control techniques to design a controller, which is able to force an electro-hydraulic actuator driven by a servovalve to track a given chaotic reference trajectory. This approach enables to compensate the inherent nonlinearities of the actuator and to reject external constant disturbances. A friction model incorporating Karnopp's stick-slip model and the Stribeck effect is used for the plant model. Simulations illustrate the approach applicability.

Keywords— Electro-Hydraulic actuators, Servo-Valves, Block Control Technique, Sliding Mode Control, Integral Control.

I. INTRODUCTION

Nowadays electro-hydraulic actuators are very important tools for industrial processes. This is mainly due to their fast response and great power supply capacity with respect to their mass or their volume. These features are not easily matched for any other commercial technology used today in actuators construction. However, the control of electro-hydraulic systems can be a difficult problem since their dynamics are highly nonlinear. Therefore, the investigation of the position or force control for electro-hydraulic actuators should be of great interest from both academic and industrial perspectives. This paper is motivated by coffee harvest automation, where in order to shake the tree branches, electro-hydraulic actuators would be very useful. In particular, to produce a broad band spectrum shaking action is very attractive. Hence the motivation is two folds: academic and technological.

Many different control techniques have been used to control position or force for a hydraulic actuator driven by a servovalve, including traditional PID controllers [1] and [2], recursive Lyapunov designs [3], controllers based on adaptive neural networks [4], and controllers based on quantitative feedback theory (QFT) [5]. On the other hand, a fruitful and relatively simple approach, especially when dealing with nonlinear plants subjected to perturbations, is based on the use of Variable Structure Control (VSC) technique with sliding mode [7]. Note that a

sliding mode controller was proposed in [6] to control the electro-hydraulic actuator force; considering a relative degree equal to two.

In this paper, we consider position tracking with respect to a prescribed chaotic trajectory. The plant model is governed by a nonlinear system, which includes the dynamics of an external cylinder load (a spring and a damper in parallel), a friction model and an approximation of the servovalve dynamics. This plant model, although being a greatly simplified representation of the actual system, captures the key component of the real dynamics; the relative degree in this case is equal to four. Based on this nonlinear plant model which can be presented in the Nonlinear Block Controllable form, and using the combination of the VSC and block control [8] techniques, we design a robust controller to drive the position tracking error to zero. To design this controller, we assume that the system state vector is completely measurable. An integral control element is also introduced in order to reject an unknown constant disturbance. In order to test the applicability of the approach, simulation results are shown at the end of the paper.

The paper is organized as follows. Section 2 reviews the detailed 4-th order state space model of the hydraulic cylinder. In Section 3 the block control technique is applied to design a nonlinear sliding surface in such a way that the sliding mode dynamics is represented by a linear system with desired eigenvalues. For a given bound on the control signal, a discontinuous control strategy ensuring stability of the sliding mode is proposed. Section 4 presents simulation results. Finally, relevant conclusions are stated in section 5.

II. MATHEMATICAL MODEL

The mathematical model which describes the dynamic behaviour of the electro-hydraulic actuator consider the dynamics of the hydraulic actuator which is disturbed by an external load (modeled as a spring and a damper element in parallel attached

to the piston), and the dynamics of the servovalve.

For the space state formulation this model can be separated as the mechanical subsystem, the hydraulic subsystem, and the servovalve subsystem.

A. Mechanical Subsystem

The differential equations governing the mechanical dynamics, namely the dynamics of the piston with a load (see Fig.1) can be derived using the Newton's equation,

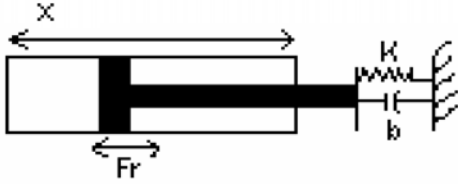


Fig. 1. piston with load

$$ma = \sum f_i = -k_s x_p - b_d v_p + \Lambda_a P_L - F_r \quad (1)$$

where x_p is the piston position, and $v_p = \frac{dx}{dt}$ is the piston velocity, a is the acceleration of the piston, $\sum f_i$ represents the forces acting on the system, P_L is the load pressure, F_r is the internal friction of the cylinder, m is the actuator mass, k_s is the load spring stiffness, b_d is the load viscous damping, and Λ_a is the piston area. Defining the state variables as $x_1 = x_p$, $x_2 = v_p$, and $x_3 = P_L$, and adding to (1) a constant unknown force M as a disturbance, we obtain the state space model as

$$\dot{x}_1 = x_2 \quad (2)$$

$$\dot{x}_2 = \frac{1}{m} (-k_s x_1 - b_d x_2 + \Lambda_a x_3 - F_r - M) \quad (3)$$

A.1 Friction Model

The used friction model (F_r) is a static one [3]. A typical velocity-friction plot of such friction model is shown in Fig. 2. This friction model includes Karnopp's stick-slip friction [11] and the Stribeck effect [12]. Regarding Karnopp's friction, there are two key points:

(1) A stick phase occurs when velocity is within a small critical velocity range, and

(2) there is a maximum value for friction when the mass under consideration sticks.

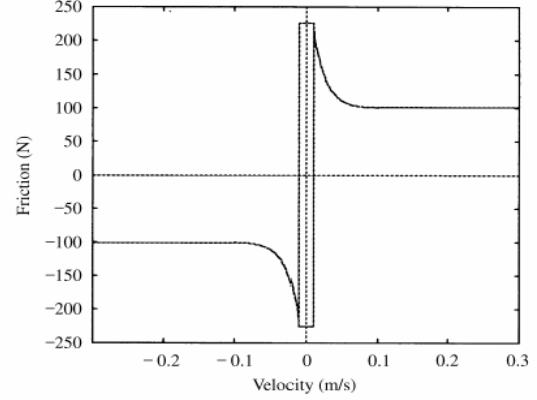


Fig. 2 Friction model used in the system model, including Karnopp and Stribeck models.

From this figure, it is possible to see that friction becomes a constant for high speed.

This function was programmed in a Simulink Matlab table function to be used for the plant model.

B. Hydraulic Subsystem

The dynamics of the cylinder are derived in [10] for a symmetric actuator. Defining the load pressure as the pressure across the actuator piston, its derivative is given by the total load flow through the actuator divided by the fluid capacitance:

$$\frac{V_t}{4\beta_e} \dot{P}_L = -\Lambda_a \dot{x}_p - C_{tm} P_L + Q_L \quad (4)$$

where V_t is the total actuator volume, β_e is the effective bulk modulus, C_{tm} is the coefficient of total leakage due to pressure and Q_L is the turbulent hydraulic fluid flow through an orifice. The relationship between the valve spool displacement x_v , and the load flow Q_L , is given as

$$Q_L = C_d w x_v \sqrt{\frac{P_s - \text{sgn}(x_v) P_L}{\rho}} \quad (5)$$

where C_d is the valve discharge coefficient, w is the valve spool area gradient, P_s is the supply pressure, and ρ is hydraulic fluid density. The spool area gradient for a cylindrical spool can be approximated simply as the circumference of the valve at each port. Combining (4) and (5), we obtain the load pressure state equation as

$$\begin{aligned} \dot{P}_L = & \frac{4\beta_e}{V_t} (-\Lambda_a v_p - C_{tm} P_L \\ & + C_d w x_v \sqrt{\frac{P_s - \text{sgn}(x_v) P_L}{\rho}}) \end{aligned}$$

or

$$\dot{x}_3 = -\alpha x_2 - \beta x_3 + \gamma x_v \sqrt{P_s - \text{sgn}(x_v) x_3} \quad (6)$$

with the following constant parameters:

$$\begin{aligned}\alpha &= (4\Lambda_a\beta_e/V_t) \\ \beta &= (4C_{tm}\beta_e/V_t) \\ \gamma &= (4C_d w\beta_e/V_t)\sqrt{1/\rho}\end{aligned}$$

Examining briefly the physical system of an hydraulic actuator, it can be readily seen that term in the square root in (6) can not take negative values because load pressure can not be larger than the supply one. However in practice, the term $\sqrt{Ps - \text{sgn}(x_v)x_3}$ could be seldom zero when the system is operating smoothly, due to the noise in the x_3 measurement. This implementation problem could be easily solved via software.

C. Servovalve Subsystem

Frequency response analysis of a servo valve dynamics was measured with an HP Digital Signal Analyzer in [9]. There, it was established that a second order linear model could match well for the measured frequency response. The model was found to be:

$$\frac{2.4315 \times 10^5}{s^2 + 6.2529 \times 10^2 s + 2.5676 \times 10^5} \quad (7)$$

This model can be approximated by a first order model, namely, $1/(\tau s + 1)$,

$$\frac{x_v(s)}{u(s)} = K_a \frac{1/\tau}{s + (1/\tau)}$$

where $\tau = 1/573 \text{ sec}^{-1}$ is the time constant, $K_a > 0$. Defining $x_4 = x_v$, the dynamics of the servovalve subsystem can be approximated as

$$\dot{x}_4 = -\frac{1}{\tau}x_4 + \frac{K_a}{\tau}u \quad (8)$$

where u is the input current to the servovalve.

Then using (2), (3), (6) and (8), we obtain the plant model as

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{m}(-k_s x_1 - b_d x_2 + \Lambda_a x_3 - F_r - M) \\ \dot{x}_3 &= -\alpha x_2 - \beta x_3 + (\gamma \sqrt{Ps - \text{sgn}(x_4)x_3})x_4 \\ \dot{x}_4 &= -\frac{1}{\tau}x_4 + \frac{K_a}{\tau}u.\end{aligned} \quad (9)$$

III. CONTROLLER DESIGN

The electro-hydraulic actuator model (9) can be presented as the Nonlinear Block Controllable form (NBC-form) consisting of four blocks see [13].

$$\begin{aligned}\dot{x}_1 &= f_1(x_1) + b_1(x_1)x_2 + d_1(x_1)w(t) \\ \dot{x}_i &= f_i(x_1, \dots, x_i) + b_i(x_1, \dots, x_i)x_{i+1} \\ &\quad + d_i(x_1, \dots, x_i)w(t) \\ \dot{x}_4 &= f_4(x) + b_4(x)u + d_4(x)w(t).\end{aligned} \quad (10)$$

with $i = 2, 3$ and with output

$$y = h(x) = x_1$$

where $x = (x_1, \dots, x_4)^T$.

For our system we have

$$\begin{aligned}f_1(\cdot) &= 0, \quad f_2(\cdot) = \frac{1}{m}(-k_s x_1 - b_d x_2), \quad f_3(\cdot) = \\ &= -\alpha x_2 - \beta x_3, \quad f_4(\cdot) = -\frac{1}{\tau}x_4, \quad b_1(\cdot) = 1, \quad b_2(\cdot) = \frac{\Lambda_a}{m}, \\ b_3(\cdot) &= \gamma \sqrt{Ps - \text{sgn}(x_4)x_3}, \quad b_4(\cdot) = \frac{K_a}{\tau}, \quad d_1(\cdot) = \\ 0, \quad d_2(\cdot) &= \frac{1}{m}(-F_r - M), \quad d_3(\cdot) = 0, \quad d_4(\cdot) = 0.\end{aligned}$$

The specific feature of this form, namely

$$b_i(\cdot) \neq 0, \quad i = 1, \dots, 4, \quad \forall x \in R^4. \quad (12)$$

allows to design a nonlinear sliding surface for a discontinuous control, using the block control feedback linearization technique [8]. As can be easily seen, the electro-hydraulic actuator fulfills condition (12). Due to space limitations the generic methodology given in [8] is not included.

Following this technique, first we define the position tracking error as a new variable z_1

$$z_1 = x_1 - r \quad (13)$$

where r is a reference signal. Then we introduce the new variable z_2 as

$$z_2 = x_2 - \dot{r} \quad (14)$$

Taking the derivative of (13) and (14), the first two transformed blocks of (10) can be represented in terms of the new variables z_1 and z_2 as

$$\dot{z}_1 = z_2 \quad (15)$$

$$\dot{z}_2 = -a_{21}z_1 - a_{22}z_2 + \bar{b}_2 x_3 + \bar{d}_2(t) + w \quad (16)$$

where $a_{21} = \frac{k_s}{m}$, $a_{22} = \frac{b_d}{m}$, $\bar{b}_2 = \frac{\Lambda_a}{m}$, $\bar{d}_2(t) = -\frac{k_s}{m}r(t) - \frac{b_d}{m}\dot{r}(t) - \ddot{r}(t)$, $w = \frac{M}{m}$.

Remark 1: For this transformation, the friction term F_r affecting \dot{x}_2 is neglected.

To reject the constant disturbance w in (16), we introduce an integral block as

$$\dot{z}_0 = z_1 \quad (17)$$

with new variable z_0 . Now we select the quasi control x_3 for (16) as

$$x_3 = -\bar{b}_2^{-1}(k_0 z_0 + k_1 z_1 + k_2 z_2 - z_3) - \bar{b}_2^{-1}\bar{d}_2(t) \quad (18)$$

where the gains k_0 , k_1 and k_2 are chosen such that the matrix A_s of the closed-loop system (15),(16)

and (17)with (18)

$$\begin{bmatrix} \dot{z}_0 \\ \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = A_s \begin{bmatrix} z_0 \\ z_1 \\ z_2 \end{bmatrix} + B_s z_3 + W_s.$$

$$A_s = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_0 & -(a_{21} + k_1) & -(a_{22} + k_2) \end{bmatrix}$$

$$B_s = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad W_s = \begin{bmatrix} 0 \\ 0 \\ w \end{bmatrix}$$

is Hurwitz.

The new variable z_3 can be obtained from (18) as

$$z_3 = \bar{b}_2 x_3 + k_0 z_0 + k_1 z_1 + k_2 z_2 + \bar{d}_2(t)$$

and then the differential equation for this variable takes the form

$$\dot{z}_3 = \bar{f}_3(z) + \bar{b}_3(x)x_4 + \bar{d}_3(t) \quad (19)$$

where $z = (z_1, \dots, z_4)^T$, and

$$\begin{aligned} \bar{f}_3(z) &= -z_0(-\beta k_0) - z_1(-\beta k_1 - k_0 \\ &\quad + k_2(a_{s21} + k_1)) \\ &\quad - z_2(a_{23}\alpha - \beta k_2 - k_1 + k_2(a_{22} + k_2)) \\ &\quad - z_3(\beta - k_2) \\ \bar{b}_3(x) &= \bar{b}_2 b_3(x) = \bar{b}_2 \gamma \sqrt{Ps - \text{sgn}(x_4)x_3}, \\ \bar{d}_3(t) &= -a_{21}\beta r - \bar{b}_2 \alpha \dot{r} - a_{22}\beta \dot{r} - a_{21}\dot{r} \\ &\quad - \beta \ddot{r} - a_{22}\ddot{r} - \ddot{r}. \end{aligned}$$

To eliminate $\bar{f}_3(z)$ and the known disturbance $\bar{d}_3(t)$ terms in (19), we choose the quasi control x_4 as

$$x_4 = -\bar{b}_3^{-1}(x)[\bar{f}_3(z) + \bar{d}_3(t)] + \bar{b}_3^{-1}(x)(-k_3 z_3 + z_4) \quad (20)$$

where z_4 is the new variable, and $k_3 > 0$. Substituting (20) in (19) results in the following linearized equation:

$$\dot{z}_3 = -k_3 z_3 + z_4.$$

From (20), we obtain the new variable z_4 as

$$z_4 = \bar{f}_3(z) + k_3 z_3 + \bar{d}_3(t) + \bar{b}_3(x)x_4 \quad (21)$$

Using this expression, we select the sliding variable s , which takes the value of z_4^+ or z_4^- depending of $x_4 > 0$, or $x_4 < 0$, respectively, that is

$$s = \begin{cases} z_4^+ & \text{if } x_4 > 0 \\ z_4^- & \text{if } x_4 < 0 \end{cases} \quad (22)$$

where

$$\begin{aligned} z_4^+ &= \bar{f}_3(z) + k_3 z_3 + \bar{d}_3(t) + \bar{b}_3^+(x)x_4 \\ \text{if } x_4 > 0, \bar{b}_3^+(z) &= \bar{b}_2 \gamma \sqrt{Ps - x_3(z)} \\ z_4^- &= \bar{f}_3(z) + k_3 z_3 + \bar{d}_3(t) + \bar{b}_3^-(x)x_4 \\ \text{if } x_4 < 0, \bar{b}_3^-(z) &= \bar{b}_2 \gamma \sqrt{Ps + x_3(z)} \end{aligned}$$

and $x_3(z) = -\bar{b}_2^{-1}(k_0 z_0 + k_1 z_1 + k_2 z_2 - z_3) - \bar{b}_2^{-1} \bar{d}_2(t)$.

Taking the derivative of s (21) results in

$$\dot{s} = \begin{cases} \text{if } x_4 > 0 & \text{with } \bar{b}_4^+(z) = \frac{\gamma \bar{b}_2 K a}{\tau} \sqrt{Ps - x_3(z)} \\ \text{if } x_4 < 0 & \text{with } \bar{b}_4^-(z) = \frac{\gamma \bar{b}_2 K a}{\tau} \sqrt{Ps + x_3(z)} \end{cases} \quad (23)$$

where $\bar{f}_4(z)$ and $\bar{d}_4(t) = \frac{d(\bar{d}_3(t))}{dt}$ are bounded functions.

Then the discontinuous control law is defined as

$$u = \begin{cases} -k_4 (\bar{b}_4^+(z))^{-1} \text{sign}(z_4^+) & \text{if } x_4 > 0 \\ -k_4 (\bar{b}_4^-(z))^{-1} \text{sign}(z_4^-) & \text{if } x_4 < 0 \end{cases} \quad (24)$$

To analyze the stability of the closed-loop system (23) and (24), we first consider $x_4 > 0$, and a Lyapunov function candidate [14] such as,

$$V^+ = \frac{1}{2}(z_4^+)^2$$

Then,

$$\begin{aligned} \dot{V}^+ &= z_4^+ [\bar{f}_4(z) - k_4 \text{sign}(z_4^+) + \bar{d}_4(t)] \\ &\leq -|z_4^+| (k_4 - |\bar{f}_4(z) + \bar{d}_4(t)|) \end{aligned}$$

For $x_4 < 0$, we have

$$V^- = \frac{1}{2}(z_4^-)^2$$

and

$$\begin{aligned} \dot{V}^- &= z_4^- [\bar{f}_4(z) - k_4 \text{sign}(z_4^-) + \bar{d}_4(t)] \\ &\leq -|z_4^-| (k_4 - |\bar{f}_4(z) + \bar{d}_4(t)|) \end{aligned}$$

Hence, under the following condition for the constant k_4 :

$$k_4 > |\bar{f}_4(z) + \bar{d}_4(t)|. \quad (25)$$

the state vector of the closed-loop system reach the sliding surface $z_4 = 0$ in a finite time. Since $\bar{f}_4(z)$ and $\bar{d}_4(t)$ are bounded functions, it is possible to find such a constant.

The sliding mode motion is governed by the following linear system .

$$\begin{aligned} \dot{z}_0 &= z_1 \\ \dot{z}_1 &= -z_2 \\ \dot{z}_2 &= -k_0 z_0 - (a_{21} + k_1) z_1 - (a_{22} + k_2) z_2 + z_3 \\ \dot{z}_3 &= -k_3 z_3 \end{aligned} \quad (26)$$

It is easy to see that are k_0, k_1, k_2 and k_3 such that the matrix

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -k_0 & -(a_{s21} - k_1) & -(a_{s22} + k_2) & 1 \\ 0 & 0 & 0 & -k_3 \end{bmatrix}$$

is Hurwitz, and in the steady state we have $z_1 = 0$, as well $z_2 = 0, z_3 = 0$ and $z_0 = \frac{M}{k_0}$.

That means that the output (position) will track the reference and the integral control variable will reject the constant disturbance M acting in the velocity state, as described in (9). Indeed, this result stablish, under condition (25), that asymptotic stability for the tracking error is achieved.

IV. SIMULATION RESULTS

Even if for the control law design, we use a fourth order model (9), in order to simulate the closed loop, we consider a servovalve model as a second order transfer function (7). Hence, the plant state space is given as:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{m} (-k_s x_1 - b_d x_2 + \Lambda_a x_3 - F_r - M) \\ \dot{x}_3 &= -\alpha x_2 - \beta x_3 + (\gamma \sqrt{Ps - \text{sgn}(x_4) x_3}) x_4 \\ \dot{x}_4 &= x_5 \\ \dot{x}_5 &= -c_n x_4 - b_n x_5 + a_n u \end{aligned} \quad (27)$$

where a_n, b_n and c_n must be suitable constants to be in agreement with (7).

The values of the parameters used for simulations are

Plant Parameters and Controller Parameters

m	24kg	desired eig.	-27,-31,-1
k_s	1610 N/m	k_0	837
b_d	310N/(m/s)	k_1	1.76e3
Ps	1.03e07pa	k_2	72.0833
Λ_a	3.26e-4m ²	k_3	100
α	1.51e10N/m ³	k_r	-96000000
β	1(1/s)	τ	.0017
γ	7.28e8kg ^{0.5} /m ^{1.5} s ²	ka	.947
M	300N		
a_n	2.4315e5		
b_n	6.2529e2		
c_n	2.5676e5		

The Chen chaotic attractor [15] is used to generate the reference trajectory. This system is given by

$$\begin{aligned} \dot{x} &= a(y - x) \\ \dot{y} &= (c - a)x - xz + cy \\ \dot{z} &= xy - bz \end{aligned} \quad (28)$$

with $a = 35, b = 3, c = 28$

The states x, y, z of (28) are multiplied for a suitable constant (.002) in order to fulfill the actuator amplitude limits.

Simulations results are presented in Fig.3, Fig.4, and Fig.5. As can be seen, tracking is achieved satisfactorily, using any of the three states of (28) as the reference.

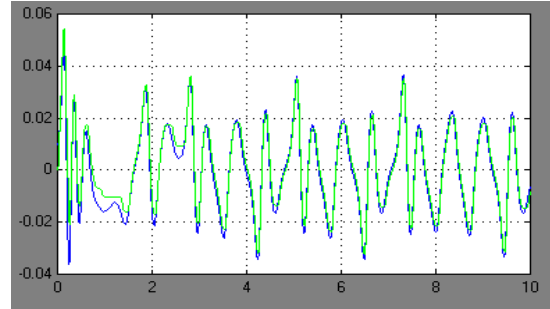


Fig. 3 - x tracking

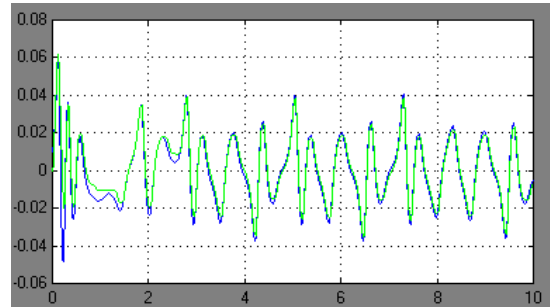


Fig. 4 - y tracking

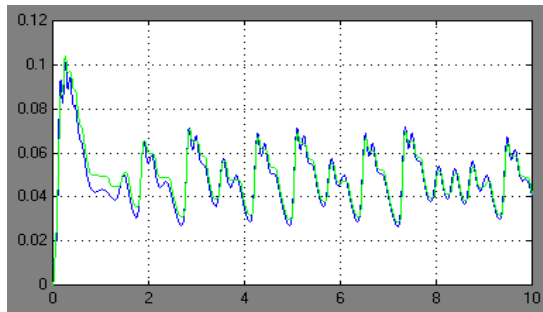


Fig. 5 z tracking

V. CONCLUSIONS

A robust controller was designed which is capable to force the electro-hydraulic actuator to track approximately desired chaotic position reference trajectories. This capability difference our approach from existing ones, which do not track a so-complex trajectory. This controller was developed on the basis of sliding modes and block control methodologies. The controller also reject constant disturbances by means of integral control. Note that a similar design could be done for force tracking.

Research is being pursued in order to extend the presented approach for considering disturbances, which do not fulfill the matched condition, and to include estimation, by nonlinear observers, for the case of no measured states.

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REFERENCES

- [1] A. Alleyne and R. Liu, "On the limitations of force tracking control for hydraulic active suspensions", *ASME Journal of Dynamic Systems Measurements and Control*, 121(2),184-190.
- [2] N. Niksefat and N. Sepehri, "Robust Force Controller Design for a Hydraulic Actuator based on Experimental input-output Data", *Proceedings of the American Control Conference*, San Diego, CA, 1999, (pp. 3718-3722).
- [3] A. Alleyne and R. Liu, "A simplified approach to force control for electro-hydraulic systems", *Control Engineering Practice*, 2000, 8 ,1347,1356.
- [4] B. Daachi, A. Benallegue and N.K.M' Sirdi, "Adaptive Neural Force Controller For a Hydraulic Actuator ", *Proceedings of the Control Conference* (2001).
- [5] N. Niksefat and N. Sepehri, "Design and Experimental Evaluation of a Robust Force Controller for an Electro-Hydraulic Actuator via Quantitative Feedback Theory", *Control Engineering Practice*, 2000, pp. 3718-3722.
- [6] M. Jerouane and F. Lamnabhi , "A new Robust Sliding Mode Controller for Hydraulic Actuator", *Proceedings of the 40th IEEE Conference on Decision and Control*, Orlando, Florida USA,2001.
- [7] Vadim Utkin, Jurgen Guldner and Jingxin Shi, *Sliding Mode Control in Electromechanical Systems*, Taylor & Francis Inc., Philadelphia,PA 19106, USA; 1999.
- [8] A. G. Loukianov, "Nonlinear Block Control with Sliding Mode", *Automation and Remote Control*, Vol.59, No.7, 1998, pp. 916-933.
- [9] Liu Rui, "Nonlinear Control of Electro-Hydraulic Servosystems Theory and Experiment", B. Engr., Tsinghua University,1994.
- [10] H.E. Merritt, "*Hydraulic Control Systems*", Wiley, New York, USA, 1967.
- [11] D. Karnopp, "Computer Simulation of Stick-Slip Friction in Mechanical Dynamic Systems", *ASME Journal of Dynamic Systems, Measurement and Control*,107(1),1985, 100-103.
- [12] Armstrong-B. Helouvry, P. Dupont and C. Canudas de Wit, "A Survey of Analysis Tools and Compensation Methods for the Control of Machines with Friction", *Automatica*, 30, 1994, pp1083-1138.
- [13] O. Serrano Verdugo, A. Loukianov and J.M.Cañedo Castañeda, "Sliding Modes Adaptive Control for Induction Motors", in Spanish, *Internal Report*, CINVESTAV, Unidad Guadalajara, Mexico.
- [14] H.K. Khalil, *Nonlinear Systems*, (2nd ed.)Prentice Hall, Upper Saddle River,NJ, USA; 1996.
- [15] G. Chen and T. Ueta, "Yet Another Chaotic Attractor", *Intl. J. of Bifurcation and Chaos*, 9, 1999, pp.1465-1466.