Controller tuning via minimization of time weighted absolute error

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Abstract— **The parameters of a fixed controller structure are tuned so that the output signal of an unknown system responds as quickly as possible to a set point change. The criterion that is minimized with respect to the controller parameters is a function of the absolute value of the output error, where zero weight is put on the transient part. Estimations of the gradient of the loss function are provided by the iterative feedback tuning method and these are used in a trust-region method in the minimization. A thorough discussion on how to choose the length of the zero time weight interval is given and the advantages of the proposed method are illustrated.**

I. INTRODUCTION

Consider the problem of tuning the parameters of some controller structure so that the output signal responds as quickly as possible to a set point change. A successful solution may be to minimize a modified loss function in which zero weight is put on the transient part of the output error. The reason for this is that most often it is of no interest how the new set point is reached as long as a large overshoot and an oscillatory behavior is avoided. This means that there is no need for a compromise between reaching the new set point and following a desired transient response that might not be natural for the closed loop system. Therefore, all effort is put on achieving the fastest possible settling time.

The approach of using zero time weights is applied to a cost function which contains the absolute value of the error between the actual and the desired output of the controlled system. The output error is a function of the controller parameters, and the criterion function is minimized with respect to the parameters in order to find the optimal values. This cost function is particularly suited to batch processing [1], where the state is reset at the start of each run. Batch processing is an important part of the rapidly growing pharmaceutical and chemical industries [2].

In the minimization, a subspace trust-region method is used. This method needs information about the gradient, and gradient estimations are here provided by the iterative feedback tuning (IFT) approach [3]. In the IFT method, estimates of the gradient are given by performing some simple experiments on the closed-loop system, without having to know the true open-loop system.

The idea of using zero time weights in combination with IFT is applied in [4] and [5] for squared error cost functions, whereas IFT and absolute error cost functions is the topic of [6]. The main contribution of this paper is to combine these ideas, i.e., to study the minimization of zero time weighted absolute error cost functions by utilizing the IFT technique for gradient estimation. In addition, the estimates of the gradient given by the IFT method is used in a subspace trust-region method, which is not the case in the original and complete IFT technique. Moreover, a general discussion on the choice of the length of the zero time weight span is given together with some guidelines. The technique described in the paper is applicable to any linear time-invariant controller and explicit examples that illustrate the advantages with zero time weights are given for PID-controllers.

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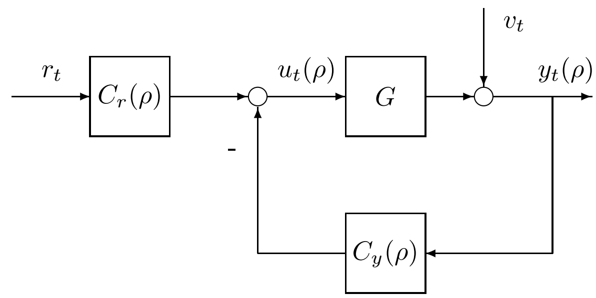

Fig. 1. The closed-loop control system.

II. PRELIMINARIES

Consider the closed-loop system in Fig. 1, where G is the system that is controlled by the fixed controller structure $C_r(\rho)$ and $C_y(\rho)$ with parameter vector ρ, and where r_t, $u_t(\rho)$, $y_t(\rho)$ and v_t are the reference signal, the control signal, the output signal and a disturbance signal, respectively. For the closed-loop system it holds that

$$\begin{aligned} y_t &= \frac{GC_r(\rho)}{1+GC_y} r_t + \frac{1}{1+GC_y(\rho)} v_t \\ &= G_{ry}(\rho) r_t + S(\rho) v_t, \end{aligned} \tag{1}$$

where $G_{ry}(\rho)$ denotes the closed-loop transfer function and $S(\rho)$ the output sensitivity function. The controller parameters are to be tuned so that the average of the absolute error

$$J(\rho) = \frac{1}{N} \sum_{t=t_0}^{N} |e_t(\rho)| = \frac{1}{N} \sum_{t=t_0}^{N} |y_t(\rho) - r_t| \tag{2}$$

is minimized. Assume that a set point change occurs at time $t = t_{spc} \geq 0$. Often, the choice $t_0 = t_{spc}$ is made in (2). If, on the other hand, a choice where $t_0 > t_{spc}$ is made, it can be interpreted as if zero weights are put on the transient part of the output error. When it is desired to tune the controller parameters in such a way that the output signal responds to a set point change as quickly as possible, the choice $t_0 > t_{spc}$ might be fruitful, as argued

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in Section I. From now on, without loss of generality, it is assumed that $t_{spc} = 0$.

When minimizing (2), the gradient $\nabla J(\rho)$ is certainly of great interest. It holds that

$$\nabla J(\rho) = \frac{1}{N} \sum_{t=t_0}^{N} \text{sgn}\{e_t(\rho)\} \nabla e_t(\rho), \tag{3}$$

where sgn denotes the sign function

$$\text{sgn}\{x\} = \begin{cases} -1, & x < 0, \\ 0, & x = 0, \\ 1, & x > 0 \end{cases} \tag{4}$$

and where

$$\begin{aligned} \nabla e_t(\rho) = & \frac{1}{C_r(\rho)} \nabla C_r(\rho) G_{ry}(\rho) r_t \\ & - \frac{1}{C_r(\rho)} \nabla C_y(\rho) G_{ry}(\rho) y_t(\rho), \end{aligned} \tag{5}$$

see [6] for a derivation. In this expression, the signals r_t and $y_t(\rho)$ are measurable, the gradients $\nabla C_r(\rho)$ and $\nabla C_y(\rho)$ are obtained from known functions, but the quantity $G_{ry}(\rho)$ is unknown unless the system G is known, which is not assumed to be the case here. The IFT algorithm presents a neat solution to this problem by performing experiments on the actual closed loop system, see [3].

The closed-loop responses to the signals r_t and $y_t(\rho)$ in (5), i.e., $G_{ry}(\rho) r_t$ and $G_{ry}(\rho) y_t(\rho)$, obtained when the disturbance signal is absent, can be estimated by performing two additional closed-loop experiments. The results from these two experiments are then combined with the result of the first closed-loop experiment to give an estimate of (3) as follows (see also [7]).

Algorithm 1.

1) In the first closed-loop experiment, use r_t as the reference signal and denote the output signal $y_t^{(1)}(\rho)$.
2) In the second closed-loop experiment, let $r_t = y_t^{(1)}(\rho)$ give the output signal $y_t^{(2)}(\rho) = G_{ry}(\rho) y_t^{(1)}(\rho)$.
3) In the third closed-loop experiment, use r_t as the reference signal to get the output signal $y_t^{(3)}(\rho) = G_{ry}(\rho) r_t$.
4) Take

$$\begin{aligned} \nabla \hat{e}_t(\rho) = & \frac{1}{C_r(\rho)} \nabla C_r(\rho) y_t^{(3)}(\rho) \\ & - \frac{1}{C_r(\rho)} \nabla C_y(\rho) y_t^{(2)}(\rho) \end{aligned} \tag{6}$$

as an estimate of (5).
5) Take

$$\hat{e}_t(\rho) = y_t^{(1)}(\rho) - r_t. \tag{7}$$

6) Use (6) and (7) to get the estimate

$$\nabla \hat{J}(\rho) = \frac{1}{N} \sum_{t=t_0}^{N} \text{sgn}\{\hat{e}_t(\rho)\} \nabla \hat{e}_t(\rho) \tag{8}$$

of (3).

The gradient estimate (8) is unbiased if the disturbances in the three experiments are uncorrelated. In the minimization of the loss function (2), the gradient estimate is used in a subspace trust-region method based on a Newton method, described next.

III. THE OPTIMIZATION PROBLEM

In the last section it was shown how to obtain a gradient estimate (8) of the loss function (2) that is to be minimized. This section describes how the gradient estimate is actually used in order to minimize the loss function.

The original and complete IFT algorithm gives, apart from a gradient estimate, the minimum value of the loss function. The minimum value is given by the iterative algorithm

$$\rho_{i+1} = \rho_i - \gamma_i R_i^{-1} \nabla J(\rho_i), \tag{9}$$

where ρ_i is the controller parameter vector, γ_i is a positive real scalar that determines the step size, R_i is a positive definite matrix, e.g., a Gauss-Newton approximation of the loss function, and the subscript i denotes the iteration index. As an alternative to this algorithm, a trust-region method is considered here. The trust-region method, briefly described next, is more computationally demanding but has faster convergence properties than (9).

In a trust-region method, the loss function $J(\rho)$ is approximated by the function $\mathcal{J}(\rho)$ in the trust-region $\mathcal{T}$ around ρ. The solution to the trust-region subproblem

$$\min_{\varrho} \{\mathcal{J}(\varrho)\}, \qquad \varrho \in \mathcal{T}, \tag{10}$$

gives the new point $\rho + \varrho$ if $J(\rho + \varrho) < J(\rho)$. Otherwise the minimization (10) is carried out again with a smaller $\mathcal{T}$. In practice, $\mathcal{J}$ is often choosen as the first two terms of the Taylor series expansion of J around ρ, giving the minimization problem

$$\min_{\varrho} \{\varrho^T \nabla J(\rho) + \frac{1}{2} \varrho^T \nabla^2 J(\rho) \varrho\}, \qquad \varrho \in \mathcal{T}. \tag{11}$$

To reduce the computational burden, the minimization problem (11) can be approximated by restricting it to a two-dimensional subspace, determined by a preconditioned conjugate gradient process, see [8]. The procedure outlined above is repeated until convergence.

IV. CHOICE OF ZERO TIME WEIGHT INTERVAL

An important question when tuning controller parameters according to a zero time weighted criterion is the choice of the length of the zero time weighted interval t_0. The choice is partcularly difficult to make when no model of the system that is to be controlled is available. If a model of the open-loop system was available, an idea would be to relate its poles and zeros to t_0. In practice, the unknown system operates in closed-loop and a possibility is to identify it using some closed-loop identification technique [9]. The easiest case is when both the input- and output signals are available. One can then proceed as in open-loop identification, using e.g. the prediction error method. If the input signal is not measurable, the closed-loop transfer function can be identified from measurements of the reference signal and the output signal. A model of the open-loop system can then be extracted from the closed-loop transfer function, since the controller structure and the controller parameters are known. However, it would be desirable to determine the length of the interval by studying the operating closed-loop system in some way, without the need for a model of the open-loop system.

In [4] and [5], the following scheme is used when searching for the global minimum of a squared error cost function. One starts with a large value for t_0, and a parameter vector that gives a very slow respons without an overshoot is used as initial value when minimizing the cost function. When the minimum value of this cost function is found, a second cost function is defined by reducing the value of t_0. The parameter vector that gives the minimum value of the first cost function is used as initial value when minimizing the second cost function. The procedure is repeated until a small overshoot starts to appear. This means that the last value of t_0 used is about the same as the settling time that can be achieved with minimal or no overshoot. It can be argued that minimizing a sequence of loss functions in this way reduces the risk of getting stuck in a local minimum. The most important thing is probably to use a parameter vector that gives a sluggish response without overshoot[1] as initial value in the minimization, meaning that the optimal value of t_0 should be used directly if it is known.

Another issue is the fact that the time domain response can be subject to constraints due to the locations of the open-loop poles and zeros. It is well known that these are related to rise time, settling time and overshoot, see e.g. [10]. If the overshoot strongly relates to the settling time, it means that the choice of t_0 is connected also to the overshoot. In this situation it would be natural to consider the minimization of (2) subject to the overshoot constraint, i.e.,

$$\min_{\rho} J(\rho) \tag{12}$$

subject to

$$\max \frac{y_t - y_\infty}{y_\infty} \leq a, \tag{13}$$

where y_∞ denotes the value of y_t when $t \to \infty$ and a is the maximum value of the relative overshoot. However, since (2) contains the absolute value of the error, it is not very likely that the parameter vector that yields the global minimum of (2) gives a large overshoot. If a large overshoot is present, it suggests that a local minimum is found.

As stated earlier, t_0 should be chosen as the settling time that is achievable with little or no overshoot, here denoted as t_s. In practice there are always limitations involved with the design of feedback control systems. For example, open-loop poles at the origin or in the right half-plane, or open-loop nonminimum phase zeros will put a constraint on the achievable settling time of the closed-loop system, see [11] and the recent paper [12] for interesting discussions. Another example is found in [13] where it is shown that slow stable zeros near the jω-axis imply a lower bound on the achievable settling time of the closed-loop system. In [14] the importance of knowing the existence of fundamental limitations before designing a control system to fulfil some specifications is illustrated. As an example, a design of a flight controller for the X-29 aircraft is described and several design methods were used in an attempt to reach the desired specifications. It is desirable to have a phase margin greater than $45°$ for all flight conditions. However, at one flight condition the model had an unstable pole in 6 and a nonminimum phase zero at 26. Under these conditions, see also [11], the desired phase margin is infeasible. A lot of effort could therefore have been saved by checking that, before applying any of the design methods. Most often, it is inevitable that when the open-loop system is unknown it is also very difficult to know t_s in advance.

For a specific choice of controller parameters ρ it is advisable to check the sensitivity function $S(\rho)$ and the complementary sensitivity function $T(\rho)$. Let

$$M_T = \max_{\omega} |T(\rho)| \tag{14}$$

denote the maximum value of the complementary sensitivity function

$$T(\rho) = 1 - S(\rho). \tag{15}$$

From the Nyquist diagram of the loop gain it can be shown that the phase margin φ_m of the open-loop gain and M_T are related through

$$\varphi_m \geq 2 \arcsin \frac{1}{2M_T}. \tag{16}$$

In most cases $\varphi_m \approx 45°$ (assuming it is feasible), which according to (16) corresponds to $M_T \approx 1.3$, gives a minimum value for the settling time. Since the overshoot will then most likely be too large, $\varphi_m \geq 45°$ and $M_T \leq 1.3$ are more reasonable choices which also give satisfactory stability margins.

Finally, the discussion in this section is also relevant for criteria other than the absolute value of the output error. Moreover, it is straightforward to use the technique described in this paper also for the criteria

$$J_1(\rho) = \frac{1}{N} \sum_{t=t_0}^{N} t|e_t(\rho)| \tag{17}$$

and

$$J_2(\rho) = \frac{1}{N} \sum_{t=t_0}^{N} t^2|e_t(\rho)|. \tag{18}$$

[1]Remark: It is risky to take the parameter vector that results from the well-known original version of the Ziegler-Nichols method as initial value, since it often gives an overshoot.

ρ	K	T_i	T_d
initial	1.5	70	5
$t_0 = 0$	9.35	63.04	4.01
$t_0 = 30$	2.33	40.83	6.02

TABLE I

THE INITIAL CONTROLLER PARAMETERS AND THE CONTROLLER PARAMETERS RESULTING FROM $t_0 = 0$ AND $t_0 = 30$ IN (2).

V. NUMERICAL STUDIES

The procedure described in the previous sections for tuning controller parameters via minimization of a zero time weighted absolute error criterion is illustrated numerically in this section. The technique is applicable to any controller of fixed structure and here the controller is chosen as a PID-controller without derivative action on the reference signal, i.e.,

$$C_r(\rho) = K\left(1 + \frac{1}{T_i s}\right), \tag{19}$$

$$C_y(\rho) = K\left(1 + \frac{1}{T_i s} + T_d s\right), \tag{20}$$

$$\rho = \begin{bmatrix} K & T_i & T_d \end{bmatrix}^T \tag{21}$$

in Fig. 1. The nonminimum phase system

$$G(s) = \frac{1 - 5s}{(1 + 10s)(1 + 20s)} \tag{22}$$

is considered in the simulations, where the sampling time is chosen as 0.01 s. A unit set point change occurs at $t = 0$ and Fig. 2 shows how the control system with controller (19)–(20) and plant (22) behaves for the three different sets of controller parameters (21) given in Table I. The first set consists of the initial parameter values that are used when minimizing (2). These parameters are chosen to give a very slow response with no overshoot. The second and third sets are the parameters that minimize (2) when $t_0 = 0$ and $t_0 = 30$, respectively. The parameters corresponding to $t_0 = 0$ give an output signal that reaches the new set point very quickly. It is of course inevitable that the derivative of the output signal is negative right after the set point change since the plant is a nonminimum phase system, but this controller is not useful since the output signal takes too large negative values. In addition, the input signal is way too large and the robustness is poor. On the other hand, the parameters corresponding to $t_0 = 30$ give a controller with much more desirable properties. The output signal does not take large negative values immediately after the set point change and the input signal is very limited. One way of reducing the large control action for the case with $t_0 = 0$ could be to add a penalty on the control effort in (2). However, this example clearly shows that the use of zero time weights can be advantageous.

An interesting observation from this and other simulations is that t_0 seems to be about the same as the equivalent time constant of the open-loop system, which for the system

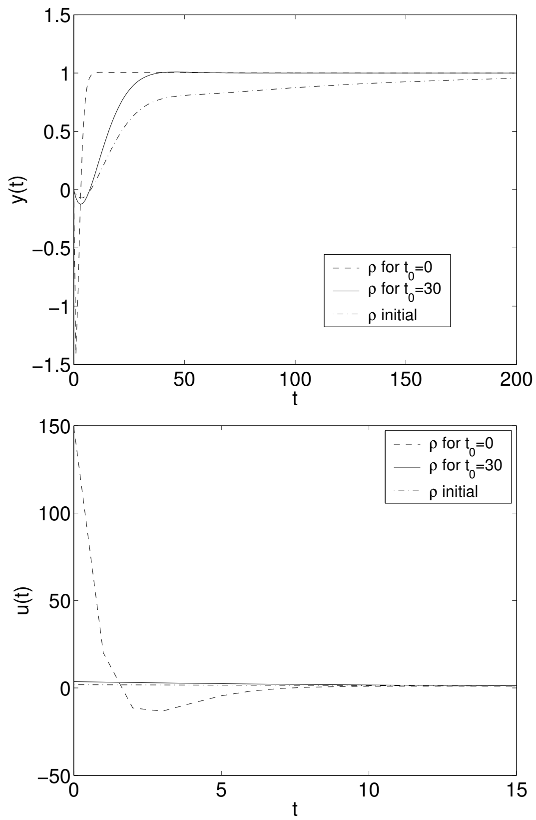

Fig. 2. Step responses (upper) and corresponding input signals (lower) for the closed-loop system with the controlled system given by (22) with initial controller parameters (dashdotted) and controller parameters resulting from $t_0 = 0$ (dashed) and $t_0 = 30$ (solid) in (2).

(22) is about 30. If this relation always holds it could be very useful for finding t_0. A simple step response from the open-loop system would then be all it takes to get an estimate of t_0.

From Fig. 2 it is seen that, since the system is nonminimum phase, both $t_0 = 30$ and $t_0 = 0$ lead to no overshoot. Therefore, the strategy of choosing t_0 as the settling time that can be achieved with minimal or no overshoot is not feasible for such a system.

VI. CONCLUSIONS

The problem of tuning controller parameters in a fixed controller structure so that the output signal of an unknown system quickly responds to a sudden set point change has been studied. The proposed solution is to consider a criterion which is a function of the absolute error between the actual and desired output signal. In the criterion, the transient part is time weighted with zeros. As a consequence, all effort can be put on reaching the new set point as quickly as possible without having to follow a trajectory that may be unnatural for the system. However, for future work it would be desirable to derive results that roughly describe the transient behaviour when $t_0 > t_{spc}$. It is of course important to at least be able to guarantee stability.

Since the system that is to be controlled is unknown, the gradient of the loss function can not be expressed explicitly. Here, information about the gradient have been given by the iterative feedback tuning method. The gradient information have then been used in a trust-region method when minimizing the loss function.

The length of the zero time weighted interval should be chosen about the same as the achievable settling time with little or no overshoot. Possible solutions for the case when the system that is to be controlled is unknown have been discussed together with limitations that are always involved when designing feedback control systems. This general discussion is also valid for other design criteria than the one studied here. Finally, the possible advantages with the proposed method have been illustrated numerically.

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