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Investigation of different techniques for determining the road uphill gradient and the pitch angle of vehicles

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***Abstract*-- This report presents different approaches to estimate the road uphill gradient and the vehicle pitch angle with the help of the vertical accelerometer, the longitudinal accelerometer and the wheel speed sensor, which are usually installed in modern vehicles. As the first attempt, only straight forward driving is considered here. All these approaches are developed based on mathematical models. The practicability of these techniques is investigated by car tests. In order to extend the operation area, a strategy to combine the different techniques is developed.**

Keywords: vehicle dynamic model, longitudinal dynamics, road uphill gradient, vehicle pitch angle, identification, least squares method, recursive least squares method.

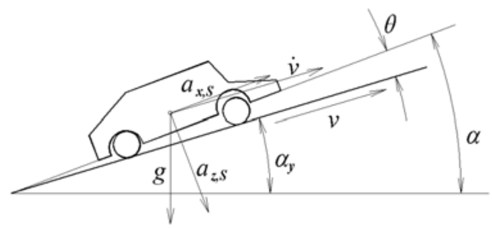
I. INTRODUCTION

The knowledge of the vehicle loading state is very important for the dynamic behavior description. As well-known, the load of a vehicle changes the lateral and longitudinal dynamic behaviors. The first attempts to estimate the loading state, unfortunately, do not yield satisfying results [2], since the road uphill gradient and the pitch angle are unknown changing values. The pitch angle depends on the loading state, the road uphill gradient and the vehicle acceleration, while the road uphill gradient depends on the road construction.

The subject of this work is to describe different techniques for determining these two angles. The information from the vertical accelerometer, the longitudinal accelerometer and the four wheel speed sensors is necessary. The first two techniques presented here use the least squares method [3] and [4]. The other two techniques are analytic solutions derived directly from the mathematical models. All these techniques are investigated via a virtual vehicle. Based on this, a method to combine these different techniques is developed, which increases the applicability.

II. MATHEMATICAL MODELS

As mentioned above, estimating the road uphill gradient angle α_y and the vehicle pitch angle θ is carried out in case of straight forward driving. Fig. 1 shows schematically a vehicle on a road. It is assumed that the uphill gradient is constant.



g:	acceleration of gravity;
θ:	vehicle pitch angle;
α_y:	road uphill gradient angle;
α:	vehicle inclination angle;
v:	vehicle velocity;
$\dot{v}$:	vehicle acceleration;
$a_{x,S}$:	longitudinal accelerometer signal;
$a_{z,S}$:	vertical accelerometer signal.

Fig. 1: Schematic of the vehicle during straight forward driving

The accelerometers should be installed in a sensor cluster close to the center of gravity of the car. The longitudinal accelerometer measures the x-component $a_{x,S}$ and the vertical accelerometer the z-component $a_{z,S}$ of the vehicle acceleration in respect of the car coordinate system. The direction of the vehicle velocity v is parallel to the road surface.

The vehicle inclination angle α can be divided into the vehicle pitch angle θ and the road uphill gradient angle α_y:

$$\alpha = \alpha_y + \theta \,. \tag{1}$$

In order to determine the two unknown angles, two mathematical models are needed. The first model uses the vertical accelerometer signal and the second model the longitudinal accelerometer signal. Both of these models need the vehicle acceleration $\dot{v}$. The calculation of $\dot{v}$ and the derivation of the two models are described in the following sections.

2.1. *Calculation of the vehicle acceleration*

The four wheel speeds are usually measurable. In case of small wheel slip, the sensor signals can be used to calculate the vehicle acceleration:

$$\dot{v} = \frac{d\left(\dfrac{v_{fr} + v_{fl} + v_{rr} + v_{rl}}{4}\right)}{dt}, \tag{2}$$

where the new symbols are defined as follows:

- $\dot{v}$: derivation of the velocity;
- v_{fr}: wheel speed front right;
- v_{fl}: wheel speed front left;
- v_{rr}: wheel speed rear right;
- v_{rl}: wheel speed rear left.

In other case, it is possible to calculate the vehicle acceleration by using only the wheel speeds, where the wheel slip is small.

2.2. *Model using the vertical accelerometer*

Assuming that the vehicle motion is straight forward and the road uphill gradient is constant, the vertical accelerometer measures the signal $a_{z,S}$, which satisfies the following equation:

$$a_{z,S} = a_z + g\cos\alpha \,, \tag{3}$$

where:

- a_z: vehicle acceleration in relation to the z-component of the vehicle coordinate system.

It is also assumed that the average value of the disturbances associated with the road, like potholes and bumps, is equal to zero. Neglecting this influence on the estimation, the vertical acceleration a_z can be described as:

$$a_z = \dot{v}\sin\theta \,. \tag{4}$$

Applying (1) and (4) to (3), the following model is obtained:

$$\begin{aligned} a_{z,S} &= \dot{v}\sin\theta + g\cos(\alpha_y + \theta) \\ &= \begin{bmatrix} \dot{v} & g \end{bmatrix} \begin{bmatrix} \sin\theta \\ \cos(\alpha_y + \theta) \end{bmatrix}. \end{aligned} \tag{5}$$

In order to reduce the on-line implementation effort, the model is simplified for small angles and rewritten in vector form:

$$a_{z,S} - g = \begin{bmatrix} \dot{v} & -0.5g \end{bmatrix} \begin{bmatrix} \theta \\ (\alpha_y + \theta)^2 \end{bmatrix}. \tag{6}$$

This model serves as the basis for the estimation of the pitch angle and the road uphill gradient. It can be formulated by a linear vector equation:

$$\underline{a}_1^T \, \underline{x}_1 = y_1 \,, \tag{7}$$

where:

$$\underline{a}_1^T = \begin{bmatrix} a_{11} & a_{12} \end{bmatrix} = \begin{bmatrix} \dot{v} & -0.5g \end{bmatrix};$$

$$\underline{x}_1 = \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} = \begin{bmatrix} \theta \\ (\alpha_y + \theta)^2 \end{bmatrix};$$

$$y_1 = a_{z,S} - g \,.$$

From (7) we recognize that just the square of the angle $\alpha_y + \theta$ can be identified. To achieve the unique solution of the angle, a further model is needed. This will be described in the following.

2.3. *Model using the longitudinal accelerometer*

Under the same assumptions as in subsection 2.2, the longitudinal accelerometer delivers the signal $a_{x,S}$, which satisfies the following equation:

$$a_{x,S} = a_x - g\sin\alpha \,. \tag{8}$$

The new symbol a_x occurred in (8) stands for the vehicle acceleration in the x-direction relating to the vehicle coordinate system and it is given as:

$$a_x = \dot{v}\cos\theta \,. \tag{9}$$

Similarly to the derivation of (5), (6) and (7) one obtains the equations (10), (11) and (12) for the acceleration in the x-direction:

$$\begin{aligned} a_{x,S} &= \dot{v}\cos\theta - g\sin(\alpha_y + \theta) \\ &= \begin{bmatrix} \dot{v} & -g \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin(\alpha_y + \theta) \end{bmatrix}; \end{aligned} \tag{10}$$

$$a_{x,S} - \dot{v} = \begin{bmatrix} -0.5\dot{v} & -g \end{bmatrix} \begin{bmatrix} \theta^2 \\ (\alpha_y + \theta) \end{bmatrix}; \tag{11}$$

$$\underline{a}_2^T \, \underline{x}_2 = y_2 \,, \tag{12}$$

where:

$$\underline{a}_2^T = \begin{bmatrix} a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} -0.5\dot{v} & -g \end{bmatrix};$$

$$\underline{x}_2 = \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} = \begin{bmatrix} \theta^2 \\ \alpha_y + \theta \end{bmatrix};$$

$$y_2 = a_{x,S} - \dot{v}\ .$$

III. Description of the techniques

In the following the identification and the analytic techniques are presented. Two of these techniques are based on the least squares method [3] and [4]. The third technique is an analytic method. Using this analytic method, a unique solution for the pitch angle and the road uphill gradient can be achieved. The fourth technique is a further analytic method, which leads to two possible solutions for the pitch angle. Analyzing the two solutions, the correct solution for the pitch angle and then the road uphill gradient can be determined.

3.1. *The least squares method*

The starting point of this section is (7) or (12). These are static equations. The advantage of static models in contrast to dynamic models is that the computation effort of the parameter identification is much smaller. Fig. 2 shows the identification using the least squares method schematically.

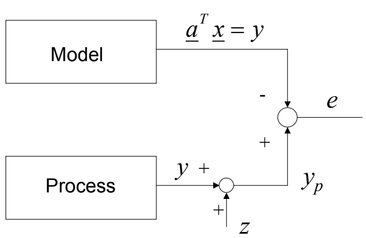


Fig. 2: Schematic of the least squares method

The error $e(k)$ is formed by using the model output $y(k)$ and the process output $y_p(k)$:

$$e(k) = y_p(k) - y(k)\ . \qquad (13)$$

The process output $y_p(k)$ is interfered by the disturbing signal $z(k)$ resulting from noise and other unknown influences, where k is the loop index.

If the convergence conditions described in [3] are fulfilled, the parameter vector $\underline{x}_1$ in (7) and the parameter vector $\underline{x}_2$ in (12) can be estimated by using the following formula:

$$\hat{\underline{x}} = (\underline{A}^T \underline{A})^{-1} \underline{A}^T \underline{y}_p\ , \qquad (14)$$

where

$$\underline{y}_p = \begin{bmatrix} y_p(0) \\ y_p(1) \\ \vdots \\ y_p(N-1) \end{bmatrix},$$

$$\underline{A} = \begin{bmatrix} a_{11}(0) & a_{12}(0) \\ a_{11}(1) & a_{12}(1) \\ \vdots & \vdots \\ a_{11}(N-1) & a_{12}(N-1) \end{bmatrix}. \qquad (15)$$

The vehicle pitch angle θ and the road uphill gradient angle α_y can be determined as follows:

$$\theta = \hat{x}_{11} \quad \text{and} \quad \alpha_y = \pm\sqrt{\hat{x}_{12}} - \theta\ .$$

To determine the sign of α_y, the identification result based on the longitudinal accelerometer signal is used.

Since this identification algorithm is developed originally for static processes, however, the two angles to be identified vary, it is necessary to form a suitable time window, which contains N loops and moves from loop to loop. The time window used here begins at the current time $t = t_0$ und ends at $t = t_0 - 1.2\,\text{sec}$. So, the time window is 1.2sec. long. The sample time is one millisecond, so that the recent 1200 measured values are applied to the identification in every loop. The measured values located outside of the time window are not considered. This method has the weakness that a large number of measured values must be stored. So, memory space or operation time is to be demanded. For this reason, an alternative technique will be presented in the next section that replaces the time window with an exponential memory weighting factor. In addition to this, further improvements of the identification can be achieved.

3.2. *Recursive least squares method with an exponentially diminishing memory*

In [3] a recursive least squares procedure is described. Using this recursive procedure, the result of every loop only depends on the result of the previous loop, hence the storage of a large number of prior measured data as mentioned before is not necessary. In this method a memory weighting factor is so defined that the prior data are less weighted than the actual data. Thus, changing parameters can be identified.

According to [3], the following equations are used:

$$\underline{\gamma}_W(k) = \frac{\underline{P}_W(k)\,\underline{a}(k+1)}{\underline{a}^T(k+1)\,\underline{P}_W(k)\,\underline{a}(k+1) + \lambda}, \qquad (16)$$

$$\underline{\hat{x}}(k+1) = \underline{\hat{x}}(k) + \underline{\gamma}_W(k)\cdot(y_p(k+1) - \underline{a}^T(k+1)\cdot\underline{\hat{x}}(k)), \tag{17}$$

$$\underline{P}_W(k+1) = \frac{[\underline{I} - \underline{\gamma}_W(k)\,\underline{a}^T(k+1)]}{\lambda}\underline{P}_W(k), \tag{18}$$

where:

$\underline{\gamma}_W(k)$:	correction vector;
$\underline{a}(k+1)$:	actual process input vector;
$\underline{P}_W(k)$:	weighted matrix in the loop k;
λ :	memory weighting factor;
$\underline{\hat{x}}(k)$:	estimated vector in the loop k;
$y_p(k+1)$:	actual process output value.

In the following, the memory weighting factor is set equal to 0.997.

Now, it is interesting to compare the identification methods presented above with the analytic solutions. In the following subsections, two analytic methods to estimate the angles will be developed.

3.3. *Analytic solution by using the trigonometric identities*

Now, the equations (5) and (10) are used. Squaring and adding both of these equations, the following equation arises:

$$a_{z,S}^2 + a_{x,S}^2 = g^2 + \dot{v}^2 - 2g\dot{v}\sin\alpha_y. \tag{19}$$

For small angles, the sinα_y can be replaced with α_y. So, the angle α_y is computed as follows:

$$\alpha_y = -\frac{a_{z,S}^2 + a_{x,S}^2 - g^2 - \dot{v}^2}{2g\dot{v}}. \tag{20}$$

Eq. (20) can be used to determine the road uphill gradient directly.

Rewriting Eq. (10) as follows:

$$a_{x,S} = \dot{v}\cos\theta - g(\sin\alpha_y\cos\theta + \cos\alpha_y\sin\theta), \tag{21}$$

setting cosθ to one and replacing sinθ with θ due to the small pitch angle θ, Eq. (22) results:

$$\theta = \frac{g\alpha_y - \dot{v} + a_{x,S}}{-g(1-0.5\alpha_y^2)}. \tag{22}$$

To increase robustness against disturbances such as noise and brief fluctuations of the inputs, averaging of the last 600 values arithmetically is accomplished. In the next section, an alternative to this analytic method will be presented.

3.4. *Analytic solution of a fourth order polynomial*

This analytic solution results from the equations (6) and (11). Reforming Eq. (11):

$$(\alpha_y + \theta) = \frac{\dot{v}(1-0.5\theta^2) - a_{x,S}}{g} \tag{23}$$

and replacing the term $(\alpha_y + \theta)$ of Eq. (6) with Eq. (23), one obtains the following polynomial:

$$\theta^4 + 4(\frac{a_{x,S}}{\dot{v}} - 1)\theta^2 - \frac{8g}{\dot{v}}\theta + \frac{4(\dot{v} - a_{x,S})^2 - 8g^2 + 8ga_{z,S}}{\dot{v}^2} = 0. \tag{24}$$

Since the pitch angle is very small, the fourth order can be neglected. Then, this polynomial is reduced to a second order polynomial:

$$\theta^2 - \frac{2g}{(a_{x,S} - \dot{v})}\theta + \frac{(\dot{v} - a_{x,S})^2 - 2g^2 + 2ga_{z,S}}{(a_{x,S}\dot{v} - \dot{v}^2)} = 0. \tag{25}$$

The roots of this equation are:

$$\theta_{1/2} = \frac{g}{(a_{x,S} - \dot{v})} \pm \sqrt{(\frac{g}{(a_{x,S} - \dot{v})})^2 - \frac{(\dot{v} - a_{x,S})^2 - 2g^2 + 2ga_{z,S}}{(a_{x,S}\dot{v} - \dot{v}^2)}}. \tag{26}$$

Analyzing both of these roots, it is obvious that the root with the negative sign is the correct pitch angle.

Using Eq. (11) again, the road uphill gradient can be obtained:

$$\alpha_y = \frac{\dot{v}\left(1-0.5\theta^2\right) - a_{x,S}}{g} - \theta. \tag{27}$$

Also here, averaging the last 600 values arithmetically is carried out.

IV. TEST RESULTS

In section III different techniques to determine the unknown angles are described. Now, the practicability of these techniques are investigated. A virtual vehicle is used, which is the professional vehicle simulation software CARSIM [1]. The techniques developed above are implemented in MATLAB. During simulation, data exchange between MATLAB and CARSIM is carried out.

Two driving maneuvers are selected for the test. The first one is driving with constant velocity, the other one is an acceleration process. Both of these maneuvers are straight forward driving on roads with constant road uphill gradients. In the simulation results presented here, the road uphill gradient is 10%.

4.1. *Quasi steady driving*

The pre-requisite to apply these techniques is that the acceleration of the vehicle is not constant zero, which corresponds to practical driving situations. Therefore the vehicle velocity in the simulation is defined by an oscillation function with the amplitude of 1 km/h and the offset of 40 km/h. The oscillation frequency is 0.1592Hz. The mathematical formulation of the defined vehicle velocity is shown as follows:

$$v = 40\text{km/h} + 1\text{km/h}\sin(2\pi f t)\ .$$

Fig. 3 shows the input variables used here. They are the longitudinal accelerometer signal, the vertical accelerometer signal, the vehicle velocity and the derivative of the vehicle velocity computed by Eq. (2).

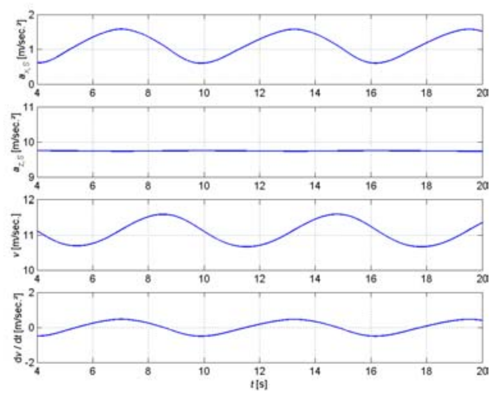


Fig. 3: Input variables during quasi steady driving

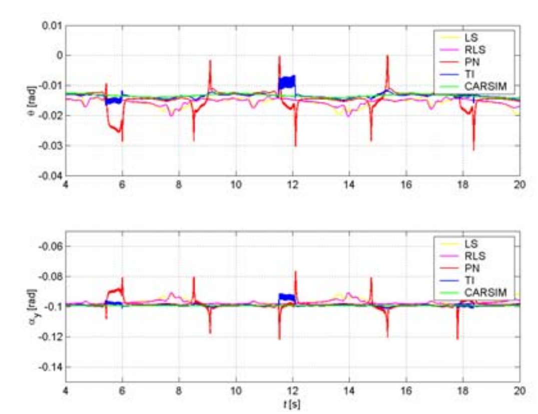


Fig. 4: Simulation results during quasi steady driving

Fig. 4 shows the simulation results of the techniques described in section III. Green lines show the desired values delivered by CARSIM. Yellow lines result from the identification by using the non-recursive least squares method (LS), magenta lines the recursive least squares method (RLS), red lines the analytic solution using the trigonometric identities (TI) and blue lines the analytic solution of the fourth order polynomial (PN).

Analyzing Fig. 3 and Fig. 4, it can be said that the analytic methods work well in the intervals, in which the absolute value of the acceleration exceeds a certain limit. On the other hand the analytic techniques work less well within the intervals, in which the absolute value of the acceleration is about zero. This statement can be proven mathematically by using Eq. (20) and Eq. (24). A pole exists if the acceleration is equal to zero.

From Fig. 3 and Fig. 4, one can see that the two identification methods behave similarly. Significant deviations of the identification results to the true values appear, if the derivation of the acceleration is close to zero. This fact can also be proven mathematically. The acceleration of gravity and the acceleration of the vehicle are strongly correlated in these intervals. This means, that the convergence conditions in these intervals are not satisfied according to [3]. On the other side the identification methods deliver good results in the intervals, where the acceleration changes quickly.

4.2. *Acceleration process*

The vehicle velocity for the simulation is defined by the following equation:

$$v = 4\frac{\text{km/h}}{\text{sec.}}t + 5\text{km/h}\ .$$

During the acceleration process the gear is shifted up every 5sec..

Fig. 5 illustrates the input variables, which are the same variables as in Fig. 3.

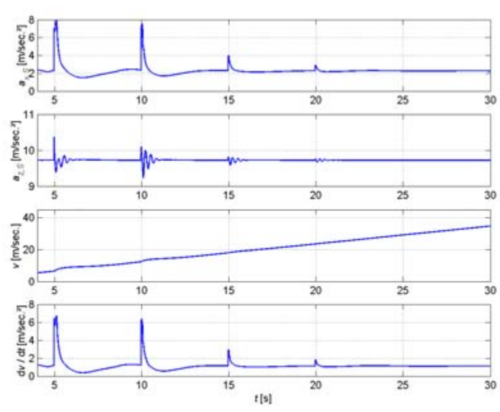


Fig. 5: Input variables during acceleration process

Fig. 6 shows the simulation results in a similar manner as Fig. 4.

Looking at Fig. 6 it is obvious that the analytic techniques have less deviations from the correct value. Both analytic methods yield nearly identical results. Large deviations occur among the identification methods.

Obviously the recursive method works better than the non-recursive one. Apart from larger deviations, the non-recursive method seems to have a time delay.

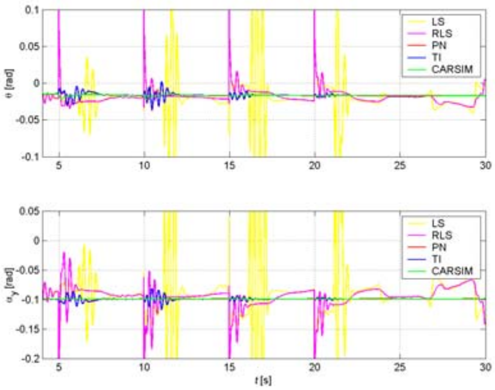


Fig. 6: Simulation results during an acceleration process

4.3. *Summary of the simulation results*

The simulation results show that especially the analytic techniques make the determination of the unknown angles possible. In wide area, the two methods yield nearly identical values. The advantage of the analytic solution using trigonometric identities is that the method has a unique solution. This makes the on-line evaluation simpler. The weakness of the analytic methods presented here is the need of the wheel speed sensor signals, both of the acceleration sensor signals and the square of these sensor signals. Since sensor signals are generally interfered with disturbances, using an increasing number of such signals, the number of error sources increases, too. This can cause more uncertainties.

Unlike the analytic methods, the identification methods have the advantage that the equation consists only of a single accelerometer signal and the wheel speed sensor signals. Comparing the two identification methods, the recursive procedure is to be preferred, since, apart from the fundamental strength mentioned above, the memory-usage and the operation time are less.

Since the identification techniques work better in time intervals, when the analytic techniques malfunction, a combination of different techniques is considered in the following.

4.4. *Combination of the techniques*

The analytic technique using the trigonometric identities and the recursive least squares method are used for the combination. Both of these techniques run simultaneously. A priority factor w is introduced, which weights the two techniques differently. This priority factor depends on the acceleration $\dot{v}$. If the absolute value of the acceleration exceeds a certain value, the priority factor is set equal to one and the analytic method operates. If the absolute value of the acceleration is about zero, the priority factor is set equal to zero and the identification operates. In the interval between $|\dot{v}| = 0$ and $|\dot{v}| = 0.3\,\mathrm{m}/s^2$, the priority factor rises constantly. This can also be seen in Fig. 7.

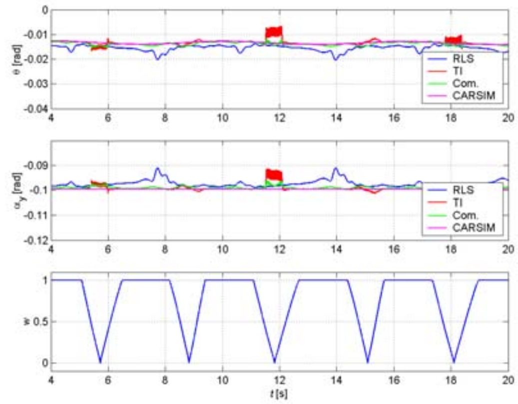


Fig. 7: Combination technique during quasi steady driving

During the quasi steady driving, the change of the two techniques occurs continuously at every zero crossing of the acceleration. From the green lines in Fig. 7 one can see that the two angles determined here are close to the desired values. During the acceleration process the priority factor is always set to one. So, one can take the corresponding results from Fig. 6. In summary it can be said that the estimation result is improved by the combination strategy.

V. Conclusion

The different possible techniques to determine the pitch angle and the road uphill gradient with the help of the sensors installed in series-vehicles are presented. By combining these techniques the area of applicability is extended. As the simulation shows, the technique works very well on roads, whose road uphill gradients are quasi-constant. In order to determine the pitch angle and the road uphill gradient on roads with changing road uphill gradients, further investigations are necessary. For example, the models used here have to be extended to dynamic models.

VI. Acknowledgement

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VII. Literature

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