Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

A Model-Based Control Method Applicable to Unstable, Non-Minimum-Phase, Nonlinear Processes

Chanin Panjapornpon, Masoud Soroush and Warren D. Seider

***Abstract*— This paper presents a nonlinear control system that is applicable to stable and unstable processes, whether non-minimum- or minimum-phase. The closed-loop stability is ensured by forcing every process state variable to follow a desired linear response. This approach results in a nonlinear state feedback that induces approximately linear responses to the state variables. The control system includes the nonlinear state feedback and a reduced-order nonlinear state observer. The application and performance of the control system are shown by implementing it on a chemical reactor with multiple steady states. The control system is used to operate the reactor at one of the steady states, which is unstable and non-minimum-phase. The simulation results show that the closed-loop system is asymptotically stable for all physically-meaningful initial conditions.**

I. INTRODUCTION

During the past 20 years, many advances have been made in nonlinear model-based control, mainly in the frameworks of model-predictive control, differential-geometric control, and Lyapunov-based control. In model-predictive control, the controller action is the solution to a constrained optimization problem that is solved on-line. In contrast, differential-geometric control is a direct synthesis approach in which the controller is derived by requesting a desired closed-loop response in the absence of input constraints. In other words, model-predictive control involves numerical model inversion, while differential-geometric control involves analytical model inversion. In model-predictive control, non-minimum-phase behavior is handled simply by increasing prediction horizons, but in differential geometric control, special treatment is needed. In Lyapunov-based control, the asymptotic decay of a norm of the state variables is ensured by the use of a proper Lyapunov function in the controller design.

Differential-geometric controllers were initially developed for unconstrained, minimum-phase (MP) processes. During the past two decades, these controllers were extended to unconstrained, non-minimum-phase (NMP), nonlinear processes. A detailed review of these methods can be found in Kanter et al. (2002); for brevity these methods are not reviewed here.

The authors gratefully acknowledge financial support from the National Science Foundation.

M. Soroush is with Department of Chemical Engineering, Drexel University, Philadelphia, PA 19104, USA masoud.soroush@drexel.edu

W.D. Seider is with the Department of Chemical Engineering, University of Pennsylvania, Philadelphia, PA 19104, U.S.A. *seider@seas.upenn.edu*

This work essentially uses the same concept of shortest-prediction-horizon continuous-time model predictive control that was employed in (Soroush and Soroush, 1997) and (Kanter et al., 2002). However, it is different in several aspects. In (Soroush and Soroush, 1997), a nonlinear state feedback was derived by minimizing a function norm of the deviations of *controlled outputs* from linear reference trajectories with orders *equal to the (output) relative orders*. The function norm was defined over very short time horizons. The resulting state feedback can be used for operating processes at *minimum-phase steady states* and is input-output linearizing in the absence of constraints. In (Kanter et al., 2001), a nonlinear state feedback that could be used for operating processes at *stable non-minimum-phase and minimum-phase steady states,* was developed. It was derived by minimizing a function norm of the deviations of *controlled outputs* from linear reference trajectories with orders *higher than the (output) relative orders*. The function norm was again defined over very short time horizons. The resulting state feedback is *approximately, input-output linearizing* in the absence of constraints when the desired steady state is non-minimum-phase.

The state feedback presented in this paper can be used to operate processes at *stable and unstable steady states, whether minimum- or non-minimum-phase*. It is derived by minimizing the sum of the squared deviations of the *state variables* from their desired linear responses that have orders *higher than the state-variable relative orders*. The resulting state feedback *approximately induces linear responses to the state variables*, in the absence of constraints. The control system includes the nonlinear state feedback and a reduced-order nonlinear state observer.

This paper is organized as follows. The scope of the study and some mathematical preliminaries are given in Section 2. Section 3 presents the nonlinear feedback control method. The application and performance of the control method are illustrated by numerical simulation of a chemical reactor with multiple steady states in Section 4.

II. SCOPE AND MATHEMATICAL PRELIMINARIES

Consider general class of multivariable processes with a mathematical model in the form:

$$\left.\begin{array}{rcl} \dfrac{dx}{dt} & = & f(x,u), \qquad x(0)=x_0 \\ y & = & h(x) \end{array}\right\} \tag{1}$$

where $x = [x_1 \cdots x_n]^T \in \Re^n$ is the vector of state variables, $u = [u_1 \cdots u_m]^T \in \Re^m$ is the vector of manipulated inputs, $y = [y_1 \cdots y_m]^T \in \Re^m$ is the vector

0-7803-8335-4/04/$17.00 ©2004 AACC

of controlled outputs, $f(x,u) = [f_1(x,u)\cdots f_n(x,u)]^T$ and $h(x) = [h_1(x)\cdots h_m(x)]^T$ are smooth. The relative order (degree) of a state x_i, is denoted by r_i, where r_i is the smallest integer for which $\partial[d^{r_i}x_i/dt^{r_i}]/\partial u \neq 0$. The following assumptions are made: (a) the relative orders $r_1,\cdots,r_n$ are finite and (b) the process is controllable and observable locally (around the nominal steady state).

For a given setpoint value, y_{sp}, the corresponing steady state values of the state variables and manipulated inputs satisfy:

$$\begin{aligned} 0 &= f(x_{ss}, u_{ss}) \\ y_{sp} &= h(x_{ss}) \end{aligned}$$

These relations are used to describe the dependence of a nominal steady state, x_{ss_N}, on the setpoint: $x_{ss_N} = F(y_{sp})$.

Let $H(x) = x$, and define the following notation:

$$\begin{aligned} H_i^1(x) &= \frac{dx_i}{dt} \\ &\vdots \\ H_i^{r_i-1}(x) &= \frac{d^{r_i-1}x_i}{dt^{r_i-1}} \\ H_i^{r_i}(x,u) &= \frac{d^{r_i}x_i}{dt^{r_i}} \\ H_i^{r_i+1}(x,u^{(0)},u^{(1)}) &= \frac{d^{r_i+1}x_i}{dt^{r_i+1}} \\ &\vdots \\ H_i^{j}(x,u^{(0)},u^{(1)},\ldots,u^{(j-r_i)}) &= \frac{d^{j}x_i}{dt^{j}} \end{aligned} \quad (2)$$

where $j \geq r_i$ and $u^{(\ell)} = d^\ell u/dt^\ell$.

III. Nonlinear Control Method

A state feedback that induces approximately linear responses to the state variables, is first derived. A reduced-order state observer is then designed to reconstruct unmeasured state variables from output measurements. To add intgeral action to the state feedback-state observer system, a dynamic system is finally added.

A. State Feedback Design

Let us request a linear response of the following form for each of the state variables:

$$\begin{aligned} (\epsilon_1 D+1)^{p_1}x_1 &= x_{ss_{N_1}} \\ &\vdots \\ (\epsilon_n D+1)^{p_n}x_n &= x_{ss_{N_n}} \end{aligned} \quad (3)$$

where $D = d/dt$, and $\epsilon_1,\cdots,\epsilon_n$ are positive constants that set the speed of the state responses. The state responses in (3) can be achieved only when $m \geq n$. However, since in many processes $m < n$ (there are more state variables than manipulated inputs), the state responses in (3) can rarely be achieved. To relax the requirement of achieving the linear state responses, let us request state responses that are as close as possible to the the linear ones described by (3). To derive a state feedback that can achieve the relaxed state response requirement, we solve the following constrained optimization problem at each time instant:

$$\min_u \sum_{i=1}^{n} w_i \left[(\epsilon_i D+1)^{p_i}x_i - x_{ss_{N_i}}\right]^2 \quad (4)$$

subject to:

$$u^{(\ell)} = 0, \quad \ell \geq 1,$$

where $w_1,\cdots,w_n$ are adjustable positive scalar weights whose values are set according to the relative importance of the state variables: the higher the value of w_i, the smaller the x_i response from the desired linear reaponse for x_i.

For a process in the from of (1), the optimization problem in (4) takes the form:

$$\min_u \sum_{i=1}^{n} w_i \Bigg[x_i + \sum_{\ell=1}^{r_i-1}\epsilon_i^\ell \binom{p_i}{\ell} H_i^\ell(x) + \sum_{\ell=r_i}^{p_i}\epsilon_i^\ell \binom{p_i}{\ell} H_i^\ell(x,u,0,\cdots,0) - x_{ss_{N_i}}\Bigg]^2 \quad (5)$$

In the case that $m \geq n$, the minimum of the performance index in (4) can be zero; in this case, the linear closed-loop state responses of (3) can be achieved. The preceding state feedback is represented in a compact form by:

$$u = \Psi(x, x_{ss_N}) \quad (6)$$

Example.

$$\begin{aligned} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= 10x_1 + 9x_2 + u \\ y &= x_2 - 2x_1 \end{aligned}$$

This process has a zero at 2 and poles at -1 and 10. Also, $r_1 = 2$ and $r_2 = 1$. For $\epsilon_1 = 0.8$ nad $\epsilon_2 = 0.01$, the closed-loop eigenvalues of the example process under the state feedback are given in Table 1.

B. Reduced-Order State Observer

In general, measurements of all state variables are not available. In such cases, estimates of the unmeasured state variables can be obtained from the output measurements. Here, we use a reduced-order nonlinear state observer to reconstruct the unmeasured state variables. The details and properties of this estimator can be found in (Soroush, 1997).

For a nonlinear process in the form of (1), the non-redundancy of the controlled outputs ensures the existence of a locally invertible state transformation of the form

$$\begin{bmatrix} \eta \\ y \end{bmatrix} = \mathcal{T}(x) = \begin{bmatrix} Px \\ h(x) \end{bmatrix}$$

where $\eta = [\eta_1,\cdots,\eta_{n-q}]^T$, and P is a constant $(n-q)\times n$ matrix which for the sake of simplicity, is chosen such that

TABLE I

CLOSED-LOOP EIGENVALUES OF FOR SEVERAL p_1 AND p_2 VALUES.

p_1	p_2	λ_1	λ_2
1	1	-100.00	0.000
1	2	-47.89	0.000
1	3	-30.59	0.000
1	4	-21.99	0.000
1	5	-16.86	0.000
2	1	-1.44	-1.090
2	2	-1.52	-1.020
2	3	-1.60	-0.970
2	4	-1.67	-0.930
2	5	-1.73	-0.900
3	1	-1.00	-0.150
3	2	-1.00	-0.153
3	3	-1.00	-0.153
3	4	-1.00	-0.153
3	5	-1.00	-0.153
4	1	-1.00	-0.017
4	2	-1.00	-0.017
4	3	-1.00	-0.017
4	4	-1.00	-0.017
4	5	-1.00	-0.017

(i) each row of P has only one nonzero term equal to one, and (ii) locally

$$rank\left\{\frac{\partial}{\partial x}\left[\begin{array}{c}Px\\ h(x)\end{array}\right]\right\}=n$$

The new variables $\eta_1,\cdots,\eta_{n-q}$ are simply $(n-q)$ state variables of the original model of (1), which satisfy the preceding rank condition, and thus the state transformation $[\eta\ \ y]^T=\mathcal{T}(x)$ is at least locally invertible. In many cases such as the process example considered in this article, the measurable outputs are some of the state variables. In such cases, the state transformation is linear and globally invertible.

The system of (1), in terms of the new state variables $\eta_1,\cdots,\eta_{n-q},y$, takes the form

$$\left\{\begin{array}{rcl}\dot{\eta}&=&F_\eta(\eta,y,u)\\ \dot{y}&=&F_y(\eta,y,u)\end{array}\right. \tag{7}$$

where

$$F_\eta(\eta,y,u)=Pf\left[\mathcal{T}^{-1}(\eta,y),u\right];$$

$$F_y(\eta,y,u)=\left.\frac{\partial h(x)}{\partial x}\right|_{x=\mathcal{T}^{-1}(\eta,y)}f\left[\mathcal{T}^{-1}(\eta,y),u\right]$$

One can then design a closed-loop, reduced-order observer of the form:

$$\begin{array}{rcl}\dot{z}&=&F_\eta(z+Ly,y,u)-LF_y(z+Ly,y,u)\\ \hat{x}&=&T^{-1}(z+Ly,y)\end{array} \tag{8}$$

where the constant $[(n-q)\times q]$ matrix L is the observer gain. The observer gain should be set such that the observer error dynamics are asymptotically stable (Soroush, 1997).

C. Integral Action

To ensure offset-free response of the closed-loop system in the presence of constant disturbances and model errors, the final control system should have integral action. The integral action can be added by using the dynamic system:

$$\begin{array}{rcl}(\epsilon_1 D+1)^{p_1}\xi_1&=&\phi_1(x,u)\\ &\vdots&\\ (\epsilon_n D+1)^{p_n}\xi_n&=&\phi_n(x,u)\end{array} \tag{9}$$

where

$$\phi_i(x,u)=$$
$$\sum_{\ell=0}^{r_i-1}\epsilon_i^\ell\binom{p_i}{\ell}H_i^\ell(x)+\sum_{\ell=r_i}^{p_i}\epsilon_i^\ell\binom{p_i}{\ell}H_i^\ell(x,u^{(0)},0,\cdots,0),$$
$$i=1,\cdots,m$$

D. Control System

Combing the equations in (6), (8) and (9) leads to the following control system that has intgeral action:

$$\begin{array}{rcl}\dot{z}&=&F_\eta(z+Ly,y,u)-LF_y(z+Ly,y,u)\\ \hat{x}&=&T^{-1}(z+Ly,y)\\ (\epsilon_1 D+1)^{p_1}\xi_1&=&\phi_1(\hat{x},u)\\ &\vdots&\\ (\epsilon_n D+1)^{p_n}\xi_n&=&\phi_n(\hat{x},u)\\ v&=&F(y_{sp})-\hat{x}+\xi\\ u&=&\Psi(\hat{x},v)\end{array} \tag{10}$$

The control system parameters $\epsilon_1,\cdots,\epsilon_n$ set the speed of the closed-loop state responses; the smaller the value ϵ_i, the faster the x_i response. The parameters $p_1,\cdots,p_n$ should be chosen such that $p_1=r_1,\cdots,p_n=r_n$ when the process is minimum-phase, and $p_1>r_1,\cdots,p_n>r_n$ when the process is non-minimum-phase.

IV. APPLICATION TO A CHEMICAL REACTOR

Consider a constant-volume, non-isothermal, continuous-stirred-tank reactor, in which the reaction $A\rightarrow B$ takes place in liquid phase. The reactor dynamics are represented by the following model:

$$\begin{array}{rcl}\dot{C}_A&=&-kC_A+(C_{A_i}-C_A)u/V\\ \dot{T}&=&\gamma kC_A+(T_i-T)u/V+q\\ y&=&T\end{array} \tag{11}$$

where $k=5.0\times10^8\exp(-8100/T)\quad s^{-1}$, $\gamma=3.9\ m^3\ K\ kmol^{-1}$, $q=-2.519\times10^{-2}\ K.s^{-1}$, $C_{A_i}=12\ kmol\ m^{-3}$, $T_i=300\ K$, and $V=0.1\ m^3$.

The control system of (10) is applied to the reactor, and the resulting controller is used to operate the reactor at the unstable, non-minimum-phase steady state $(6.319\ \ kmol.m^{-3},\ \ 302.0\ \ K)$. The following controller parameter values are used: $\epsilon_1=360\ s$, $\epsilon_2=360\ s$, $p_1=2$, $p_2=2$, $w_1=1$, $w_2=1$, and $L=0.5$

For the two sets of initial conditions, $[C_A(0),\ T(0)]$= $[3.0,\ 320]$ and $[10.0,\ 290]$, the peformance of the controller is shown in Figures 1–3. Responses such as those in Figures 1 and 2 showed that the controller is capable of operating the process at the desired steady state, regardless of the initial conditions of the process.

NOTATION

A= Reactant
B = Product
C_{A_i}= Inlet concentration of the reactant, $kmol\ m^{-3}$.
C_A= Outlet concentration of the reactant, $kmol\ m^{-3}$.
D = Differential operator, $D = d/dt$.
k = Reaction rate constant, s^{-1}.
m= Number of manipulated inputs and controlled outputs.
n = Process order.
r_i = Relative order of state variable x_i.
t = Time, s.
T = Reactor outlet temperature, K.
T_i = Reactor inlet temperature, K.
u = Process input vector.
V = Reactor volume, m^3.
x = Vector of state variables.
y = Vector of controlled outputs.
y_{sp} = Vector of set-points.

Greek

$\epsilon_1, \cdots, \epsilon_n$ = adjustable parameters of controller.
$\xi_1, \cdots, \xi_n$= State variables of the controller.
γ = Reactor model parameter, $K\ m^3\ kmol^{-1}$.

REFERENCES

[1] Kanter, J. M., M. Soroush, and W. D. Seider, “Nonlinear Controller Design for Input-Constrained, Multivariable Processes,” *Ind. Eng. Chem. Research*, **41**, 3735–3744 (2002).

[2] Soroush, M., “Nonlinear State-Observer Design with Application to Reactors,” *Chem. Eng. Sci.*, **52(3)**, 387–404 (1997).

[3] Soroush, M., and H. M. Soroush, “Input-Output Linearizing Nonlinear Model Predictive Control,” *International J. of Contr.*, **68(6)**, 1449–1473 (1997).

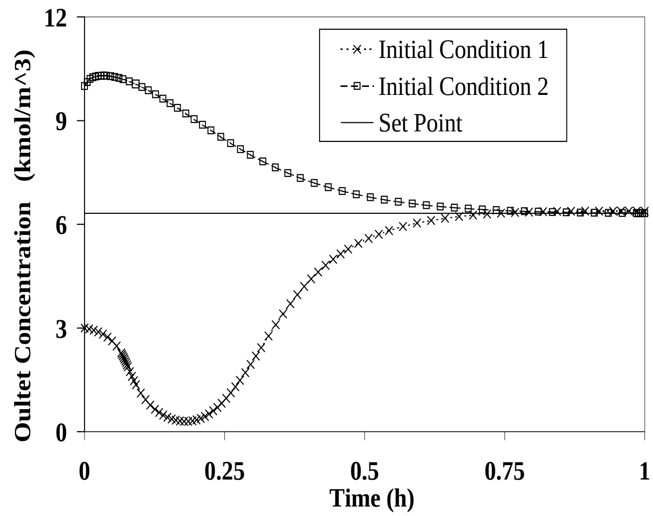

Fig. 1. Closed-loop response of the reactant outlet concentration for different initial conditions.

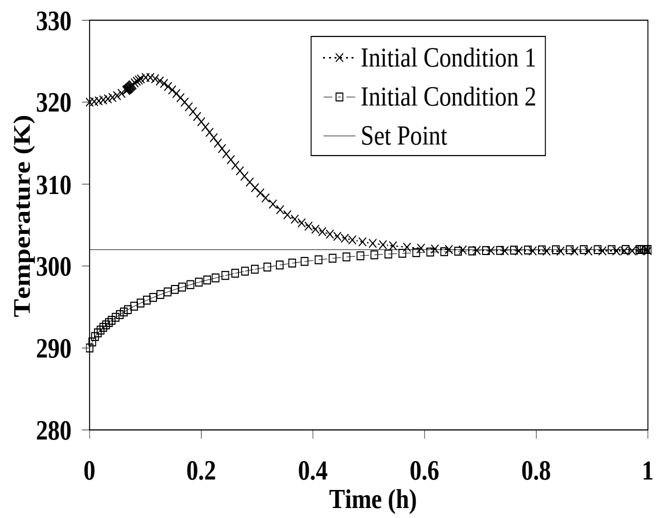

Fig. 2. Closed-loop response of the outlet stream temperature corresponding to Figure 1.

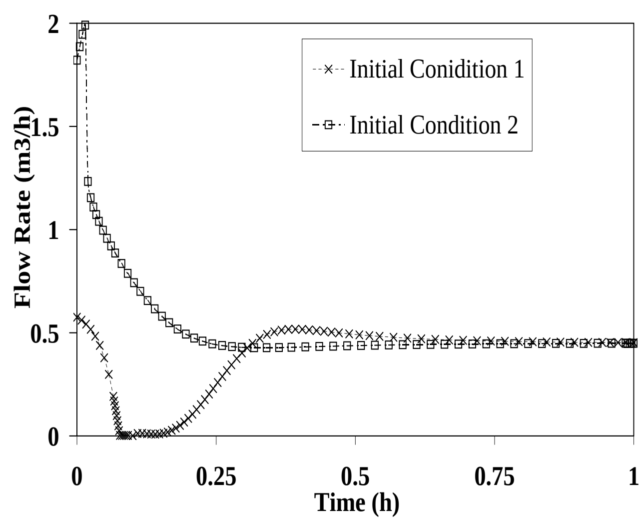

Fig. 3. Manipulated input profiles corresponding to Figures 2 and 3.