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Nonlinear Analysis of RED—A Comparative Study

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***Abstract*— Random Early Detection (RED) is an active queue management (AQM) mechanism for routers on the Internet. In this paper, performance of RED and Adaptive RED are compared from the viewpoint of nonlinear dynamics. In particular, we reveal the relationship between the performance of the network and its nonlinear dynamical behavior. We measure the maximal Lyapunov exponent and Hurst parameter of the average queue length of RED and Adaptive RED, as well as the throughput and packet loss rate of the aggregate traffic on the bottleneck link. Our simulation scenarios include FTP flows and Web flows, one-way and two-way traffic. In most situations, Adaptive RED has smaller maximal Lyapunov exponents, lower Hurst parameters, higher throughput and lower packet loss rate than that of RED. This confirms that Adaptive RED has better performance than RED.**

I. INTRODUCTION

With the rapidly increase of Internet, network congestion analysis and control has attracted great interests over the last decade. Congestion of the network can cause high packet loss rate and long delay. In the severe condition it can even cause the *congestion collapse* [1]. The goal of congestion control is to maximize throughput and minimize delay and packet loss rate. Congestion control mechanism is the basis of network stability and quality of service (QoS). The whole Internet congestion and avoidance mechanism is a combination of the end-to-end TCP (transmission control protocol) congestion control mechanism [2] and queue management mechanism at the router. TCP congestion control algorithm adapts source's transmission rate by adjusting congestion window according to the packet loss rate.

The simplest queue management algorithm used in routers is drop-tail that discards the arriving packets if the buffer of the router overflows. The drawback of drop-tail buffer management is high queuing delays and flow synchronization [3]. In recent years, active queue management (AQM) mechanisms have been proposed for better managing the congestion at the bottleneck routers. A widely studied AQM is Random Early Detection (RED) [4], which is recommended by the Internet Engineering Task Force (IETF) and implemented in some commercial routers. The basic idea of RED is to drop packets earlier that can notify the sources about the incipient congestion. In the "gentle" version of RED [5] the average queue length is computed using an exponentially weighted moving average of the instantaneous queue length and the weight parameter is w_q. When the average queue length is smaller than the RED parameter minthresh (denoted by $q_{\min}$), the packet drop rate p is equal to zero. The packet drop rate p increases linearly from zero to the maxthresh (denoted by $p_{\max}$), when the average queue length is between the $q_{\min}$ and the maxthresh (denoted by $q_{\max}$). The p increases linearly from $p_{\max}$ to 1 when the average queue length is larger than the $q_{\max}$ and smaller than the $2q_{\max}$. The average queue length is updated every time a new packet arrives at the queue. The main goal of RED algorithm is to achieve the low average queuing delay and at the same time reach the high throughput.

However, there are some drawbacks with RED algorithm as has been confirmed by simulations and real network experiments. It has been argued that the performance of RED is not better than that of drop-tail, and that the performance of RED is rather sensitive to system parameters (such as the traffic load and the round-trip times (RTTs) of connections) and control parameters ($q_{\min}$, $q_{\max}$, $p_{\max}$, w_q) [6][7]. In deed, if the control parameters of RED are not tuned properly, RED algorithm may result in heavy oscillation of queue at the router and give rise to severe delay variation. When the TCP traffic is two-way, it has been found that the RED queue has larger oscillation than the situation of one-way traffic [8]. RED also suffers from low throughput in the mixture of burst and greedy sources [9]. To overcome main drawbacks of RED, Floyd et al. have recently proposed a robust RED named Adaptive RED (ARED) [10]. The new algorithm uses AIMD (Additive Increase Multiplicative Decrease) mechanism to adaptively adjust $p_{\max}$ and makes the average queue length stay around a target queue length.

Recently, it has been found that the Internet congestion control system can display complex nonlinear dynamical behaviors. It has been demonstrated that TCP-Drop tail congestion control system can present complex chaotic behavior in certain circumstances [11]. Such chaotic behavior has also been investigated by symbolic time series analysis [12]. Recently, we have performed chaotic time series analysis of the average queue length in TCP-RED [13][14]. The estimated maximal Lyapunov exponent, correlation dimension and Kolmogorov entropy indicate that under certain parameters the dynamic behavior of aggregate traffic of bottleneck link in TCP-RED is chaotic. Although these works have confirmed the existence of chaos phenomena in the congestion control system, they don't analyze the qualitative changes of the system performance and dynamical behavior as the system/control parameters varies. If the oscillation amplitude of FTP flows increases under

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some circumstances, some network performance indexes will deteriorate. Moreover, if the chaotic level of FTP flows increases, the FTP flows will be more and more unpredictable. Therefore, it is of practical importance to analyze and control the complex nonlinear dynamics of Internet congestion control system. Recently, Ranjan et al. proposed a discrete time model of TCP-RED and found that border collision bifurcation could lead to chaos in this model [15]. By the discrete time model, Richard J. La et al. analytically demonstrated that ARED has some desirable properties compared to RED [16].

The aim of this work is to achieve a deeper understanding of the relationship between the performance and nonlinear dynamical behavior of RED and Adaptive RED. To simulating the real Internet traffic our simulation scenarios include one-way and tight coupling two-way FTP flows and Web flows. We calculate both the ordinary network performance indexes which include throughput and packet loss rate and nonlinear dynamics indexes which include the maximal Lyapunov exponent and Hurst parameter of the average queue length. We find that in most scenarios Adaptive RED exhibits significantly better performance than RED, such as smaller maximal Lyapunov exponents, lower Hurst parameters, higher throughput and lower packet dropping rate.

The rest of the paper is organized as follows. In Section 2, we describe the simulation environment that includes the definition of performance metrics, simulation parameter setting and two kinds of network traffic. In Section 3 we compare the performance of RED and Adaptive RED under FTP traffic. In Section 4 we compare the performance of RED and Adaptive RED under the combination FTP and Web traffic. Finally, we summarize this paper and discuss future works in Section 5.

II. PERFORMANCE METRICS AND SIMULATION ENVIRONMENT

A. Performance Metrics

In order to investigate the relationship between the network performance and nonlinear dynamics of TCP-RED with different RED parameters, we choose the following metrics:

Throughput: It is the observed rate at which data is sent through a link.

Packet loss rate: It is defined as the rate of dropped packets on a link.

Maximal Lyapunov exponent: We use phase space delay reconstruction method [17] for the investigation of the nonlinear dynamics of average queue series in TCP-RED. The maximal Lyapunov exponent is a measure of divergent rate of initial close trajectories on the reconstructed attractor. A positive but finite Lyapunov exponent indicates the existence of deterministic chaos. The method of estimating maximal Lyapunov exponent used in our work is a robust algorithm introduced in [18]. This algorithm searches the nearest neighbor of each point on the trajectory of reconstructed attractor. Then it measures the average separation of these neighbors and plots the logarithm of the divergence versus time. If the curve exhibits a linear increase for some range, its slope is an estimation of maximal Lyapunov exponent.

Hurst parameter: The Hurst parameter is a key measure of self-similarity. We compute the Hurst parameter of the time series of average queue length sampled from the bottleneck link. Given a weakly stationary stochastic process X, with constant mean and finite variance, we define the m-aggregated series $X^{(m)}$ as

$$X_k^{(m)} = \frac{1}{m}\sum_{i=(k-1)m+1}^{km} X(i), k = 1, 2, 3\cdots \quad (1)$$

The process X is said to be *asymptotically second-order self-similar*, if for all k large enough,

$$\text{Variance: } Var\left(X^{(m)}\right) = Var\left(X\right)/m^{\beta} \quad (2)$$

$$\text{Autocorrelation: } r_{X(m)}\left(k\right) = r_X\left(k\right) \text{ as } m \to \infty \quad (3)$$

The Hurst parameter can be defined as $H = 1 - \beta/2$. We use wavelet analysis method [19] to compute Hurst parameter. A large Hurst parameter ($H > 0.8$) implies obvious long range dependence of the traffic.

B. Simulation Setting

The simulations have been performed using NS-2 network simulator [20]. The form of network topology is like a dumbbell as shown in Fig. 1. There are a number of TCP flows sharing a bottleneck link implementing RED/Adaptive RED queue management algorithm at the router. All of non-bottleneck links implement drop-tail queue management algorithm. All flows use TCP-Reno as transport control protocol. Two classes of simulation setting are shown in Table 1 and Table 2. The capacity of bottleneck link is C Mbps and the propagation delay is d ms. Other FTP links' capacity is C2 Mbps and the other Web links' capacity is C3 Mbps. The C2 and C3 can ensure that the congestion and packet dropping only happen at the bottleneck router. The propagation delays of the links that connect the sources to node n1 are random distribution in [10ms, 50ms] and those of the links that connect node n2 to the destinations are uniform random distribution in [5ms, 15ms]. We set the buffer size as B packets and the packet size of TCP as PS bytes. The number of FTP flows is num1 and the pair of Web Server-Client is num2. The control parameters of RED are shown in Table 2. To compare the performance of RED and Adaptive RED algorithm, we set the same control parameters (q_{min}, q_{max}) for both algorithms. Simulations last 2160 seconds and for the scenario of two-way Web flows the simulation time is 400 seconds. We sample the queue length and average queue length every 0.01 seconds.

C. Network Traffic

There are two kinds of traffic used in our simulations. The first kind of traffic is infinite FTP flows. The forward FTP flows start one by one from zero second and the reverse

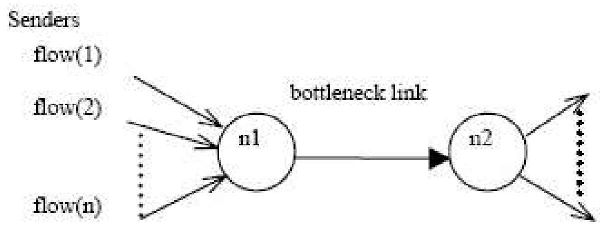

Fig. 1. Network Topology

TABLE I
SIMULATION SETTING

	C	C2	C3	d	B	PS	num1	num2
Simulation1	30	30	30	5	1500	500	100	0
Simulation2	10	1000	2000	5	230	500	10	50

FTP flows can be active one by one from 100 seconds. The second kind of traffic is the mixture the FTP flows and Web flows and Web flows make up of the major part of the whole traffic. The Web traffic employs a model of HTTP 1.0 traffic [21]. The model describes a situation where a user requests a Web page from a Web server. There are a number of inlined objects that is defined as Objects per page within a Web page. The time between requests to the inlined objects within a Web page is defined as Inter-object time. The Inter-page time is the time between the end of previous page download and the start of the following page by the same user. After the user has requested some Web pages he ends the session and after an Inter-session time the next user requests a new Web page that means the new session begins. The Web traffic model has six parameters: Inter-session time, pages per session, Inter-page time, Objects per page, Inter-object time, and Object size, which can be adjusted probability distributions. Four parameters and their distributions are summarized in Table 3. There are 2,000 Web sessions per server and 1,000 Web pages per session.

Our two-way scenario is a case of tight coupling between forward and backward TCP traffic [8]. The number of reverse FTP/Web flows is a half of forward FTP/Web flows. From 100 seconds the reverse flows starts in succession.

According to the type of the traffic our simulations have two groups: FTP flows, a mixture of Web flows and FTP flows. Each group has four kinds of scenarios: one-way RED, one-way Adaptive RED, two-way RED and two-way Adaptive RED. To investigate the effect of nonlinear dynamics with varying control parameter w_q, we select six different w_q values (0.00006, 0.0001, 0.00019, 0.0006, 0.0014, 0.002) for every simulation scenario.

TABLE II
RED SETTING

	$q_{\min}$(packets)	$q_{\max}$ (packets)	$p_{\max}$
Simulation1	250	1000	1/3
Simulation2	35	150	1/3

TABLE III
THE PARAMETERS' PROBABILITY DISTRIBUTION OF WEB TRAFFIC MODEL

	Inter-page time	Objects per page	Inter-object time	Object Size
Distribution	Pareto	Pareto	Pareto	Pareto
mean	50ms	4	0.5ms	12Kb
shape	2	1.2	1.5	1.2

III. SIMULATION WITH FTP FLOWS

A. Throughput and Packet Loss Rate

We use Simulation 2 setting in Table 2 in the following simulations, unless we point out explicitly. As shown in Fig. 2 and Fig. 3, with the increasing value of control parameter w_q in RED, the throughput reduces and the packet loss rate increases. Under the different values of w_q, Adaptive RED has ratherish higher throughput and lower packet loss rates than that of RED. Floyd *et al.* have suggested that the w_q of Adaptive RED should better be set according to the bottleneck link capacity as following [10]:

$$w_q = 1 - e^{-\frac{1}{c}} \tag{4}$$

This corresponding to $w_q = 0.004$ in Simulation 2 setting of Table 2. Using the Adaptive RED the throughput is almost unchanged in the scenario of one-way traffic. However, the performance (throughput and packet loss rate) of Adaptive RED with $w_q = 0.004$ may not be the optimal in the scenario of two-way traffic. This indicates that it would be better if we could adjust the parameter w_q of RED according to traffic variation [22]. Similar to the observation of heavy oscillation from the time series of instantaneous queue length of two-way FTP traffic [8], the two-way scenario has lower throughput and higher packet loss rate, especially for the situation of RED.

B. Maximal Lyapunov Exponents

We perform nonlinear time series analysis to sampled average queue length of TCP-GRED/ARED. To compare with the results of numeric simulations of [15], we use Simulation 1 setting in Table 2 when we compute the maximal Lyapunov exponents of one-way scenarios. Figure 4 is a bifurcation plot of the one-order discrete time model of TCP-RED proposed in [15]. The parameters of discrete time model keep the same as Simulation 1 setting in Table 2. The map exhibits period-doubling bifurcation around the w_q value of 0.0012, as shown in Fig. 4. However, when the w_q of one-way RED is equal to 0.00019 that is much smaller than 0.0012, the estimated maximal Lyapunov exponent of average queue length is 0.41 (the third point of real line on Fig. 5) that means the congestion control system is chaotic. These results reveal that, the exponential averaging weight w_q for producing chaotic average queue length should be

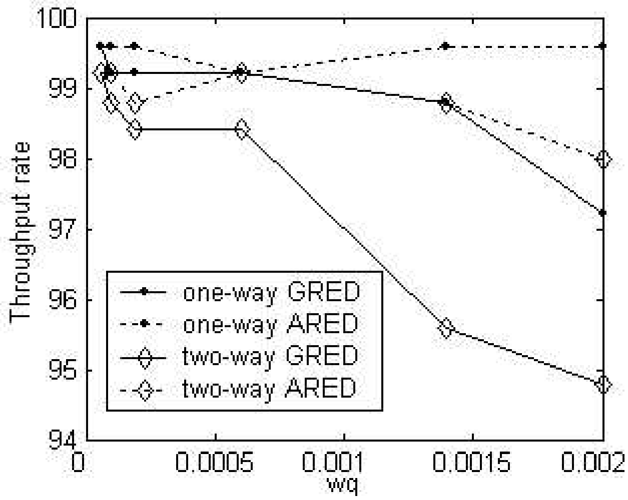

Fig. 2. Throughput rate as a function of w_q for FTP flows, where GRED represents Gentle RED, and ARED represents Adaptive RED

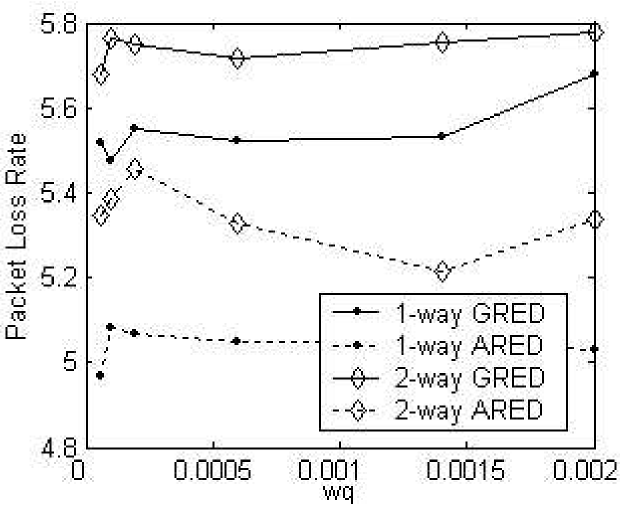

Fig. 3. Packet loss rate as a function of w_q for FTP flows

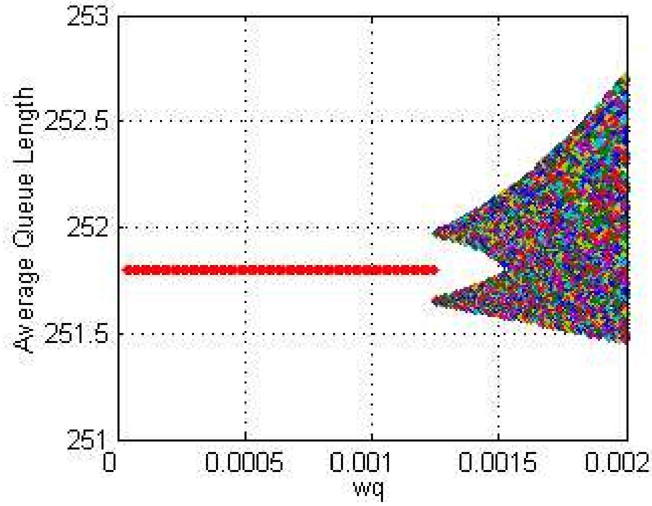

Fig. 4. Bifurcation plot of w_q for FTP flows

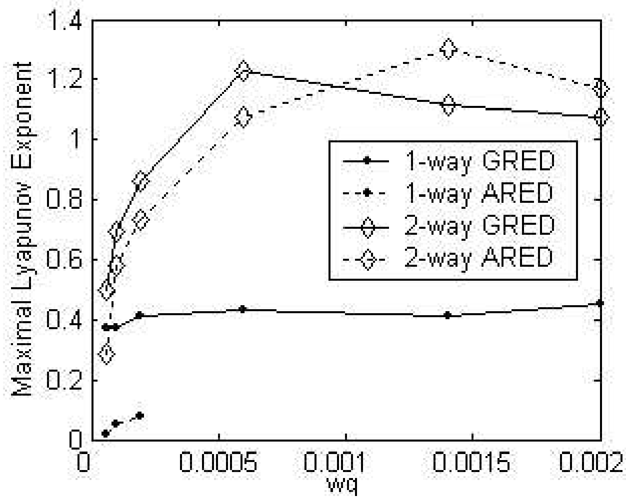

Fig. 5. Maximal Lyapunov exponent as a function of w_q for FTP flows

much smaller than the value w_q predicted by the discrete time model of TCP-RED.

In general, as shown in Fig. 5, the maximal Lyapunov exponent increases with the value of exponential averaging weight w_q no matter what kind of queue management algorithm we use. Under one-way FTP traffic the maximal Lyapunov exponents of Adaptive RED are close to zero and much lower than those of RED, but under two-way FTP traffic the maximal Lyapunov exponents of Adaptive RED are higher than those of RED when the value w_q is bigger than 0.001. According to [8] the two-way FTP traffic has heavier oscillation than one-way traffic. Our computing results of maximal Lyapunov exponents indicate that the chaotic level of two-way FTP traffic is higher than one-way traffic.

C. Hurst Parameters

In general, the monotonic increase relationship between the Hurst parameters and exponential averaging weight w_q is not obvious in all kinds of FTP's scenarios. From Fig. 6 we can find that most Hurst parameters are in the area of [0.4, 0.6], which means the traffic of most w_q is short range dependence. Under Adaptive RED algorithm most Hurst parameters is very similar to those under RED algorithm, but when the w_q of Adaptive RED is equal to 0.00019 the Hurst parameter of one-way traffic is large than 1. The mechanism for such a strange phenomena is still not clear.

IV. SIMULATION WITH WEB FLOWS

The following simulations use Simulation 2 setting in Table 2. The traffic is a mixture of FTP flows and predominating Web flows.

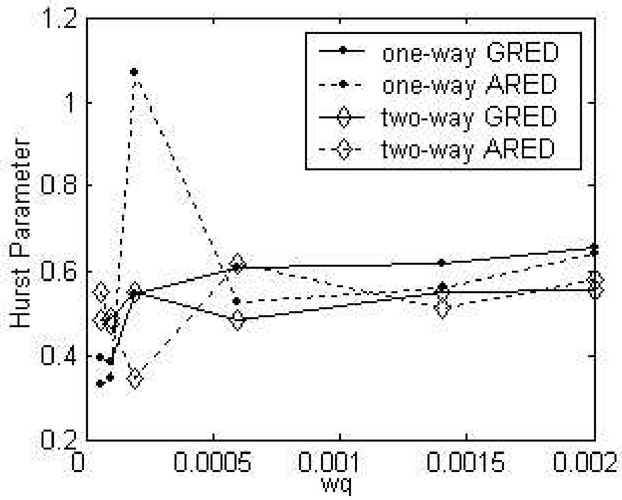

Fig. 6. Hurst parameter as a function of w_q for FTP flows

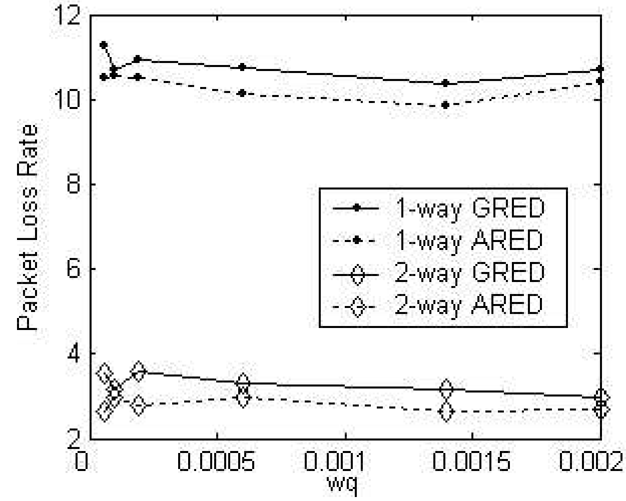

Fig. 8. Packet loss rate as a function of w_q for Web flows

A. Throughput and Packet Loss Rate

As shown in Fig. 7 and Fig. 8, for two-way Web traffic the throughput monotonic decreases and packet loss rate keeps the same value as the exponential averaging weight w_q increases no matter what kind of queue management algorithm we use. For one-way Web traffic the throughput and packet loss rate keep almost unchanged as the value of w_q increases. For two-way Web traffic the throughput of RED algorithm is the lowest in all scenarios, but the packet loss rate of this situation is lower than most other scenarios. For one-way Web traffic the packet loss rate of RED/Adaptive RED is the highest in all scenarios, but the throughput rate of this situation is higher than many other scenarios. Under all kinds of circumstances Adaptive RED has higher throughput and lower packet loss than that of RED.

B. Maximal Lyapunov Exponents

As shown in Fig. 9, in all kinds of scenarios, the maximal Lyapunov exponents increase as the exponential averaging weight w_q increases. However, Adaptive RED has much smaller maximal Lyapunov exponents than that of RED for the same value of w_q. We don't find sharp difference in the amplitude of queue length oscillation between tight coupling two-way Web traffic scenario and one-way Web traffic scenario for both RED and Adaptive RED. Also, The maximal Lyapunov exponents haven't obvious difference between one-way and two-way traffic scenarios with RED algorithm. However, there is distinct diversity of the throughput and packet loss rate between one-way and two-way Web traffic. It has been suggested that the oscillations persist for FTP flows as well as Web flows in the case of tight coupling two-way traffic [8].

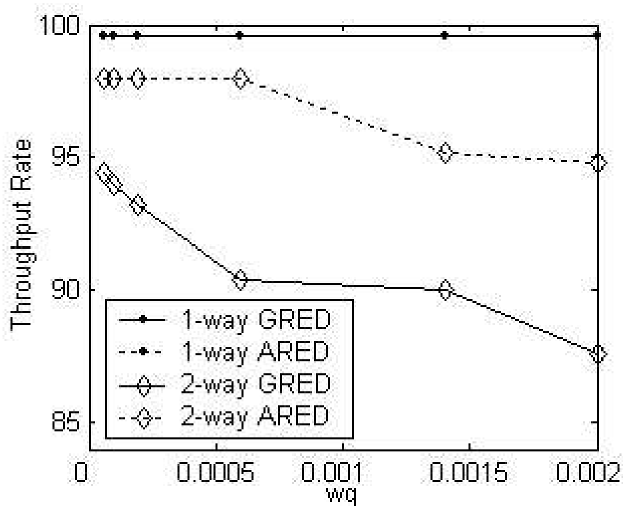

Fig. 7. Throughput rate as a function of w_q for Web flows

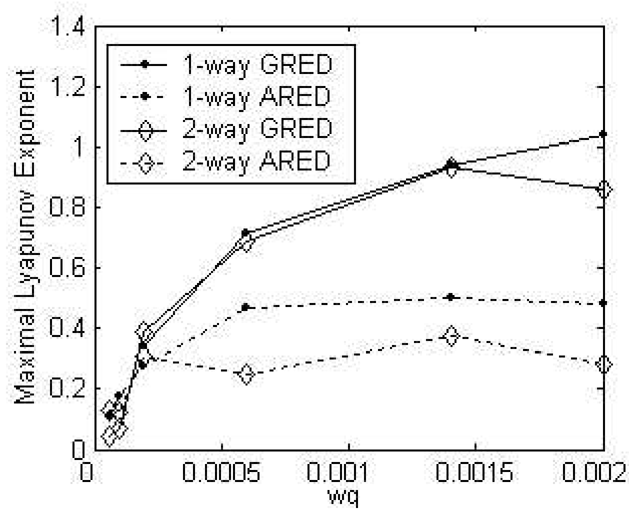

Fig. 9. Maximal Lyapunov exponent as a function of w_q for Web flows

C. Hurst Parameters

Due to the terrible long simulation time required for two-way Web traffic when estimating Hurst parameter, we have to compute only the Hurst parameters for the scenarios of one-way Web traffic. Hurst parameter of one-way Web traffic does not increase monotonically as the value of the exponential averaging weight w_q increases. All the Hurst parameters corresponding to RED are greater than 0.8 which means the long range dependence for one-way Web traffic. On the contrary, most Hurst parameters corresponding to Adaptive RED are smaller than 0.5 which means Adaptive RED succeeds in regulating long range dependent traffic into short range dependent traffic.

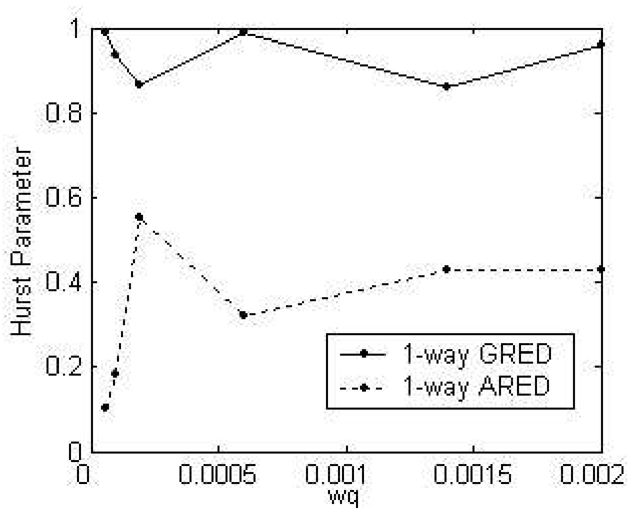

Fig. 10. Hurst parameter as a function of w_q for Web flows

V. CONCLUSIONS

We have compared the performance of RED and Adaptive RED via ordinary network performance indexes which include throughput and packet loss rate and nonlinear dynamics indexes which include maximal Lyapunov exponent and Hurst parameter. The simulations are performed with FTP traffic, as well as a mixture FTP and Web traffic for various exponential averaging weight w_q. We have also performed the simulations of tight coupling two-way traffic, as well as one-way traffic.

The major findings of this work can be summarized as following. (1) We demonstrate the advantage of Adaptive RED against RED. In most situations Adaptive RED has smaller maximal Lyapunov exponent, lower Hurst parameter, higher throughput and lower packet loss rate than that of RED. (2) The maximal Lyapunov exponents increase as the value of weight parameter w_q of RED and Adaptive RED increases, but the Hurst parameters keep almost unchanged. (3) Compare to the scenario of one-way FTP flows, we can observe larger oscillation of queues and higher maximal Lyapunov exponents for two-way FTP flows. However we have not observed the similar phenomena in the scenario of two-way Web flows. We will further analysis the nonlinear dynamic behavior of RED and Adaptive RED for different values of other system and control parameters.

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