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Washout Filter-Aided RED Control

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***Abstract*— We apply a dynamic feedback control algorithm, namely washout filter-aided control, to stabilize TCP-RED system that can suffer from instabilities caused by both smooth and non-smooth bifurcations. We demonstrate that a linear feedback control scheme can be used to considerably increase the stable operating regime of the system, and that a nonlinear feedback control can be adopted to further enhance the stability of bifurcations.**

I. INTRODUCTION

It has been shown that the interaction of nonlinearity and delay in a network can lead to undesirable behavior that can degrade network performance [13]. For robust operation of the network, it is important to understand its dynamical behavior beyond the linear stability regime. In our earlier work [13], [14] we have shown that for a class of seemingly diverse looking models like that of Firoiu and Borden [4] and for Kelly's rate control framework [9] the natural mode of transition from fixed point operation to oscillations is through a period doubling bifurcation in naturally arising discrete-time maps.

Nonlinear instability and its control is a well studied area in the control system literature [1], [3]. In this paper, we utilize the theory of bifurcation control using washout filters and other feedback based chaos control techniques to extend the stable parameter range of TCP-RED. A washout filter is a simple high-pass filter and has been shown to be very effective for robust control of nonlinear instabilities [1]. In this control scheme, the stability of the original equilibrium point is enhanced by feedback-based, small parametric modulations introduced in RED control parameters. The structure of washout filter aided control ensures that the control does not result in any movement of the nominal (open-loop) fixed points of the system, even in the presence of model unertainty. The control cancels the instability effect introduced by variation in other system parameters such as the number of connections and round trip propagation delay that are beyond the control of the network manager.

In this paper, using our analytical model we characterize the linear stability conditions given a washout filter parameter and linear control gain pair. Moreover, we analytically show that there exists a washout control parameter and linear control gain pair that can stabilize the system for all RED exponential averaging weights. This is in contrast with a previous result [8] that Proportional-Integral (PI) control is only stabilizing for a limited range of averaging weight values. We provide numerical examples to demonstrate that washout filter based control can significantly improve the stability of TCP-RED.

The paper is organized as follows. In Section II, we summarize the first-order model for TCP-RED in a congested network from [13] and review its linear analysis. In Section III, we outline the theory behind washout filter-based control. In Section IV, we describe the application of washout filters to TCP-RED and give a characterization of the stability region in parameter space. Section V contains the simulation results showing the bifurcation delay in parameter space along with an NS-2 implementation of the washout filter based control scheme. Finally, in Section VI we collect and discuss the results.

II. MODEL AND PREVIOUS WORK

In [13], we have proposed a first-order discrete-time dynamic model for studying the interaction of TCP connections with a RED gateway. Let $\mathcal{I}, \mathcal{I} = \{1, \cdots, N\}$, denote the set of connections. Each connection is assumed to be a TCP connection. Throughout this paper we assume that all connections are long-lived TCP Reno connections that are Explicit Congestion Notification (ECN) capable. The capacity of the shared link and the packet size of connections are denoted by C and M, respectively. We assume that the connections are homogeneous, *i.e.*, they have the same round-trip propagation delay (denoted by R_0) and have the same rate or throughput. Rather than interpreting this assumption as a requirement that the connections must have the same propagation delay, one should consider the delay R_0 as the effective delay that represents the overall propagation delay of the connections. This allows us to reduce the problem with N connections to a single connection system that represents the set of connections, and then study the behavior of this simpler system. We assume that the Random Early Detection (RED) queue management mechanism with ECN capability is implemented at each node in order to control the average queue size at the router. A RED gateway marks a packet with a probability p, which is a function of the average queue size q^{ave} as follows [6]:[1]

$$p(q^{ave}) = \begin{cases} 0 & \text{if } q^{ave} < q_{min} \\ 1 & \text{if } q^{ave} > q_{max} \\ \frac{q^{ave} - q_{min}}{q_{max} - q_{min}} p_{max} & \text{otherwise} \end{cases}, \qquad (1)$$

[1] In practice a RED gateway marks a packet with a modified probability in order to lead to a more uniform marking pattern [6].

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where q_{min} and q_{max} are the lower and higher threshold values, and p_{max} is the selected marking probability when $q^{ave} = q_{max}$. The average queue size is updated at the time of packet arrival through exponential averaging:

$$q^{ave}_{new} = (1-w)q^{ave}_{old} + w \cdot q_{cur} \ , \qquad (2)$$

where q_{cur} is the queue size at the time of arrival, and w is the exponential averaging weight, which determines the time constant of the averaging mechanism. Therefore, the control parameters of the RED mechanism are w, q_{min}, q_{max}, and p_{max}.

We use a first-order discrete-time nonlinear dynamic model to analyze the interaction of the RED gateway with TCP connections, which was first proposed by Firoiu and Borden [4]. We define the control system as follows. The packet marking probability p_k at period $k, k \geq 1$, determines the throughput of the connections and the queue size q_{k+1} at period $k+1$, based on the system constraints. The queue size at period $k+1$ is used to compute the average queue size q^{ave}_{k+1} at period $k+1$ according to the exponential averaging rule in (2). Then, the average queue size q^{ave}_{k+1} is used to calculate the packet marking probability p_{k+1} at period $k+1$, which is the control variable of the AQM mechanism. This can be written mathematically as follows:

$$\begin{aligned} q_{k+1} &= G(p_k) && (3)\\ q^{ave}_{k+1} &= A(q^{ave}_k, q_{k+1}) && (4)\\ p_{k+1} &= H(q^{ave}_{k+1}) \ , && (5) \end{aligned}$$

Here, $A(q^{ave}_k, q_{k+1}) = (1-w)q^{ave}_k + w \cdot q_{k+1}$ as given in (2), and the control function $H(q^{ave}_{k+1}) = p(q^{ave}_{k+1})$ from (1).

The exact form of the plant function $G(\cdot)$ depends on system parameters such as the number of connections, nature of connections, round-trip delays, etc. The plant function is derived in [13] and is given by

$$G(p_k) = \begin{cases} 0 \ , & \text{if } p_k > p_u \\ B \ , & \text{if } p_k < p_l \\ \frac{NK}{\sqrt{p_k}} - \frac{R_0 C}{M} \ , & \text{otherwise} \end{cases} \qquad (6)$$

where B is the buffer size, p_u is the smallest probability such that the queue size at the next period is zero, and p_l is the largest probability such that the next period queue size equals B. ¿From (3) - (6), the average queue size at period $k+1$ is given by

$$\begin{aligned} \overline{q}_{e,k+1} &= \begin{cases} (1-w)\overline{q}_{e,k} & \text{if } \overline{q}_{e,k} > q^{ave}_u \\ (1-w)\overline{q}_{e,k} + wB & \text{if } \overline{q}_{e,k} < q^{ave}_l \\ (1-w)\overline{q}_{e,k} + \\ +w\left(\frac{NK}{\sqrt{\frac{(\overline{q}_{e,k}-q_{min})p_{max}}{(q_{max}-q_{min})}}} - \frac{R_0 c}{M}\right) & \text{otherwise} \end{cases} \\ &=: f(\overline{q}_{e,k}, \rho) \end{aligned} \qquad (7)$$

where $q^{ave}_l = \frac{p_l(q_{max}-q_{min})}{p_{max}} + q_{min}$, and

$$q^{ave}_u = \begin{cases} \frac{p_u(q_{max}-q_{min})}{p_{max}} + q_{min} & \text{if } p_{max} \geq p_u \\ q_{max} & \text{otherwise} \end{cases} .$$

III. Feedback Control of Instabilities

In this section, we summarize a simple delayed feedback control algorithm to control instabilities [1].

A. Washout filter-based control

The washout filter mechanism has been successfully utilized to control a number of bifurcations in nonlinear models with uncertainty [1]. This approach for TCP-RED systems differs considerably from other schemes where the control scheme tries to keep the operating point invariant under significant parametric variations [2], [5]. For example, the adaptive RED (ARED) scheme also modulates a control parameter, namely p_{max}, to adapt to dynamically changing operating conditions, using an additive increase and multiplicative decrease algorithm [5]. However, the adaptation is done based on the difference between the current average queue size and *fixed* target queue size, and hence keeps system operation independent of other parameter variations. An inherent problem with such an approach is that the range over which it is effective may be severely limited in the parameter space [10].

A simple discrete time high-pass filter can be used as an analogue of washout filter in continuous time. Consider the following high-pass filter discussed in [1].

$$G(z) = \frac{1-z^{-1}}{1-dz^{-1}} \qquad (8)$$

This can have the following time domain implementation:

$$z_{k+1} = x_k + (1-d)z_k, \quad y_k = x_k - dz_k \qquad (9)$$

where $\{x_k\}$ is the input sequence to the washout filter, $\{y_k\}$ is the output sequence, and the washout filter constant d should satisfy $0 < d < 2$. At steady state, $z_{k+1} = z_k$ and $x_{eq} - d \cdot z_{eq} = 0$. Hence, from (9), we have $y_k \equiv 0$ and the output of the washout filter vanishes at the steady state.

Now, we can consider a scalar nonlinear dynamical system with washout filter control:

$$x_{k+1} = f(x_k, u_k) \qquad (10)$$

where u_k is a scalar control input. If washout filter is put in the feedback loop with feedback function $h(\cdot)$, we have following modified system:

$$\begin{aligned} x_{k+1} &= f(x_k, u_k), \quad z_{k+1} = x_k + (1-d)z_k && (11)\\ y_k &= x_k - dz_k, \quad u_k = h(y_k) && (12) \end{aligned}$$

where $h : R \rightarrow R$ is any smooth function such that $h(0) = 0$. It can be shown that this type of feedback control does not modify the equilibrium point of the original system under no control, *i.e.*, $u_k = 0$ [1]. However, with a proper choice of feedback function $h(\cdot)$ and washout filter constant, it can enhance the stability of the original equilibrium point without the need for accurate knowledge of the system model or equilibrium value.

IV. Application to TCP-RED

In this section we look at the stabilization of map in (7) with linear control terms in the neighborhood of fixed point q^*, *i.e.*, $q^* = f(q^*, \rho)$. For this we need to compute the linearization of the map ($x_{n+1} = Ax_n + bu_n$) around the intended fixed point of the system. We have

$$\left.\frac{\partial f(\overline{q}_{e,k+1},\rho)}{\partial \overline{q}_{e,k+1}}\right|_{\overline{q}_{e,k+1}=q^*} = 1 - w - \frac{0.5wNK}{(q^* - q_{min})^{\frac{3}{2}}} := \lambda_0(\rho) \quad (13)$$

Also, depending on the RED parameter to be modulated, $b(p_{max}) = \frac{\partial f}{\partial p_{max}}$ or $b(q_{max}) = \frac{\partial f}{\partial q_{max}}$ can be computed:

$$b(p_{max}) = -\frac{0.5wNK}{\sqrt{\frac{(\overline{q}_{e,k}-q_{min})}{(q_{max}-q_{min})}}p_{max}^{1.5}} \quad (14)$$

$$b(q_{max}) = \frac{0.5wNK}{\sqrt{(\overline{q}_{e,k}-q_{min})(q_{max}-q_{min})p_{max}}} \quad (15)$$

It is clear from the above that $b(\cdot) \neq 0$ for a nominal range of parameters. For one dimensional system with nonzero eigenvalue, both left (l) and right (r) eigenvectors are 1.

From the two observations above we conclude that $l \cdot b(\cdot) \neq 0$. This has consequences for linear stabilizability due to the Popov-Belevitch-Hautus (PBH) eigenvector test for controllability of modes of linear time invariant systems, and tells us that linear stabilizing feedback exists in this case. This also implies that cubic stabilizing feedback exists, which we study in Section IV-E.

In the view of PBH test for controllability and the washout filter described above, we can view the averaged queue size of RED as an input to the state estimation filter that provides the estimate y_k. This estimate can be used to construct the control depending on the functional form of h. In this section we consider only the linear control law, *i.e.*, $u_k = k_l \cdot y_k$, because in linear analysis all the nonlinear terms vanish when the system is linearized at the fixed point. Throughout this section we assume that we modulate q_{max} unless stated otherwise. In this framework, the TCP-RED system given by (7), when augmented by a washout filter, can be rewritten as follows:

$$z_{k+1} = \overline{q}_{e,k} + (1-d)z_k \quad (16)$$

$$u_k = h(\overline{q}_{e,k} - dz_k) \quad (17)$$

$$\overline{q}_{e,k+1} = \begin{cases} (1-w)\overline{q}_{e,k} & \text{if } \overline{q}_{e,k} > q_u^{ave} \\ (1-w)\overline{q}_{e,k} + wB & \text{if } \overline{q}_{e,k} < q_l^{ave} \\ (1-w)\overline{q}_{e,k} + w\Big(\frac{NK}{\sqrt{\frac{(\overline{q}_{e,k}-q_{min})p_{max}}{(q_{max}^{wo}-q_{min})}}} - \frac{R_0 c}{M}\Big) & \text{otherwise} \end{cases} \quad (18)$$

Here, $q_{max}^{wo} = \min\{\frac{B}{2}, \max\{\alpha \cdot q_{min}, q_{max} + u_k\}\}$. We upper limit q_{max} to 0.5 B due to the consideration of GENTLE_ mode of RED and lower limit it to $\alpha \cdot q_{min}$, where $1 < \alpha < 2$.

A. Stability analysis with washout filter

In this section we analyze the stability of washout enabled TCP-RED given by (18). Clearly, $[q^*/d, q^*]$ is the fixed point of the new system given by (16) - (18) for $d \neq 0$. The Jacobian matrix evaluated at the fixed point $[q^*/d, q^*]$ is given by

$$A = \begin{pmatrix} 1-d & 1 \\ b\frac{\partial h(\overline{q}_{e,k}-dz_k)}{\partial z_k} & \frac{\partial f(\overline{q}_{e,k},\rho)}{\partial \overline{q}_{e,k}} + b\frac{\partial h(\overline{q}_{e,k}-dz_k)}{\partial \overline{q}_{e,k}} \end{pmatrix} \quad (19)$$

where $b = b(q_{max})$ given in (15).

If we evaluate (19) at the fixed point $[q^*/d, q^*]$ with linear control, *i.e.*, $u_k = k_l(\overline{q}_{e,k} - d \cdot z_k)$, (19) simplifies to

$$A = \begin{pmatrix} 1-d & 1 \\ -dbk_l & \lambda_0 + bk_l \end{pmatrix} \quad (20)$$

where $\lambda_0 = \left.\frac{\partial f(\overline{q}_{e,k},\rho)}{\partial \overline{q}_{e,k}}\right|_{\overline{q}_{e,k}=q^*}$ from (13).

Next we recall Jury's stability test for second order discrete-time systems:

Lemma 1: (Jury's stability test for second order systems [11]) A necessary and sufficient condition for the zeros of the polynomial

$$p(\lambda) = a_2\lambda^2 + a_1\lambda + a_0$$

($a_2 > 0$) to lie within unit circle is

$$p(1) > 0,\ p(-1) > 0 \text{ and } |a_0| < a_2$$

The characteristic equation for matrix in (20) is given by

$$\lambda^2 - \lambda((1-d)) + \lambda_0 + bk_l) + (1-d)\lambda_0 + bk_l = 0$$

Using Jury's test for stability, the conditions for linear asymptotic stability are given as follows.

$$d(1-\lambda_0) > 0 \quad (21)$$

$$2 + 2bk_l + 2\lambda_0 - d(1+\lambda_0) > 0$$
$$\Rightarrow k_l > \frac{(d-2)(1+\lambda_0)}{2b} \text{ for } b > 0 \quad (22)$$

$$|\lambda_0(1-d) + bk_l| < 1$$
$$\Rightarrow \frac{-1-\lambda_0(1-d)}{b} < k_l < \frac{1-\lambda_0(1-d)}{b} \text{ for } b > 0 \quad (23)$$

Similar inequalities can be formulated for linear stability in the case of $b < 0$, *e.g.*, p_{max} is modulated. As we see here the stability region for pair (d, k_l) is made up of three straight lines in (d, k) plane, which are described below:

$$(l_1): \quad k = \frac{(1+\lambda_0)d}{2b} - \frac{1+\lambda_0}{b} \quad (24)$$

$$(l_2): \quad k = \frac{\lambda_0 d}{b} - \frac{(1+\lambda_0)}{b} \quad (25)$$

$$(l_3): \quad k = \frac{\lambda_0 d}{b} + \frac{(1-\lambda_0)}{b} \quad (26)$$

Under the generic assumption of $\lambda_0 < -1$ and $b > 0$, we can see that lines (l_2) and (l_3) are parallel as they have the same slope. Lines (l_1) and (l_3) intersect each other at $(d_0, k_0) = (\frac{4}{1-\lambda_0}, \frac{(1+\lambda_0)^2}{(1-\lambda_0)b})$. Similarly, lines ($l_1$) and ($l_2$) intersect each other at $(d_1, k_1) = (0, \frac{(1+\lambda_0)}{b})$.

Proposition 1: For a (d, k) pair to be stabilizing, it must lie within the triangle with the vertices $\left(0, \frac{(1+\lambda_0)}{b}\right)$, $\left(0, \frac{(1-\lambda_0)}{b}\right)$, and $\left(\frac{4}{1-\lambda_0}, \frac{(1+\lambda_0)^2}{(1-\lambda_0)b}\right)$

Proof of this proposition and other results in this paper can found in [15].

The parameter that will be modulated for control will determine the value of b, *e.g.,* $b < 0$ for p_{max} and $b > 0$ for q_{max}. Gain k_l needs to be chosen accordingly. Parameter d is chosen such that $0 < d < 2$, and $\lambda_0 < -1$ in the regime after period doubling bifurcation. This shows that, theoretically, it is possible to control the average queue size of RED locally near critical parameter value, although allowable range for parameters such as p_{max} or q_{max} is limited by physical constraints in the real system. Also, these control gains need to be limited so as to not cross the basin of attraction for the fixed point. Hence, though local stabilization near the critical value of a parameter is possible, it may not be possible to stabilize in an arbitrarily large parameter range. Next, using Jury's test we compute the parameter range where stabilization is possible for a fixed value of k_l as different parameters such as exponential averaging weight w, round-trip propagation delay R_0, and the number of active connections N, are varied.

B. Stabilization with respect to exponential averaging weight

Stabilization with respect to exponential averaging weight w is simpler to analyze since the fixed point is independent of w and the eigenvalue λ_0 decreases linearly with w from (13). Hence, due to linear stabilizability of the original system, *i.e.,* $l \cdot b \neq 0$, it is possible to stabilize the RED averaged queue by picking appropriate k_l and d that obey the conditions given by (21) - (23). Here we are interested in investigating the possibility of local linear stabilizing over all values of $0 < w < 1$. It turns out that due to some interesting properties of λ_0 and $b = b(q_{max})$ as given by (15) it is possible to pick a (d, k_l) pair to stabilize the system for all possible values of $w > w_{crit}$, where w_{crit} is the value of w at which first period doubling bifurcation happens in the uncontrolled system and is given by [13]

$$w_{crit} = \frac{2}{1 + \frac{NK}{2(q^* - q_{min})^{\frac{3}{2}}} \sqrt{\frac{q_{max} - q_{min}}{p_{max}}}}. \quad (27)$$

From (13) and (15) one can show that $\frac{(1-\lambda_0)}{b(q_{max})}$ is independent of w. This provides important insight into the locus of triangular stability region as given by Proposition 1. It shows that one of the vertices $(0, \frac{(1-\lambda_0)}{b})$ is invariant of w. We now need to understand the behavior of λ_0 and $b(q_{max})$ and that of $\frac{\lambda_0}{b(q_{max})}$ as w is varied in unit interval. It is clear from (13) that λ_0 decreases linearly as a function of w. Similarly, $b(q_{max})$ as given by (15) increases linearly with w, and $\frac{\lambda_0}{b(q_{max})}$ is strictly decreasing with w, which can be seen directly by differentiating the expression. This means that all three constraint lines given by (24) - (26) become steeper with increasing w. This leads to decreasing area of stability triangle shown in Fig. 1. Finally, we use the fact that w is bounded by one from above and evaluate the worst case stability region. Clearly, the eigenvalue remains finite for w=1. Evaluating the vertices for $w = 1$ will provide the smallest triangle. Hence, if the stabilizing pair (d, k_l) lies within this triangle, then it does for all other values of $w > w_{crit}$.

Theorem 1: TCP-RED system along with washout filter for a given washout control parameter and linear control gain pair (d, k_l) and all other parameters held fixed, will be stable for $w_{crit} < w < 1$ where w_{crit} is the value of w corresponding to the first period doubling bifurcation, if (d, k_l) lies within the triangle with vertices $\left(0, \frac{(1+\lambda_0(w=1))}{b}\right)$, $\left(0, \frac{(1-\lambda_0(w=1))}{b}\right)$, $\left(\frac{4}{1-\lambda_0(w=1)}, \frac{(1+\lambda_0(w=1))^2}{(1-\lambda_0(w=1))b}\right)$ with $b = b(q_{max})$.

C. Stabilization with respect to round-trip time of connections

The RTTs of connections are beyond the control of network managers, and tend to vary widely in practice. Moreover, the stability of TCP-RED is shown to be sensitive to the variation in RTT R_0 [13]. Thus, stabilizing the system for a larger range of R_0 is an important issue from the practical perspective. The networking research community has spent considerable amount of effort in understanding the effects of RTTs of connections and are trying to design more robust algorithms against the variations in R. In washout filter control scheme we achieve this goal by linear feedback modulation in a RED parameter and thereby increasing the stable operation domain. Based on the Jury's criteria we can state the following result:

Theorem 2: TCP-RED system along with washout filter for a given (d, k_l), $b(\cdot) > 0$ and all other parameters held fixed, will be linearly asymptotically stable for $R_0 < R_0^*$ where R_0^* is given as a solution of the following equation:

$$\frac{(d-2)(1+\lambda(R_0))}{2b} = k_l \quad \text{for } b > 0 \text{ and } 0 < d < 2 \quad (28)$$

where $\lambda(R_0)$ is the eigenvalue evaluated at the fixed point given in (13) as a function of R_0.
Solution to (28) exists due to the fact that the fixed point of map in (7) decreases with increasing R_0, and so does the eigenvalue $\lambda_0(R_0)$ in the parameter regime of interest.

D. Stabilization with respect to the number of connections

The number of connections N is another parameter that is beyond the control of network administrators. In general N may vary widely, and stabilizing the system over a large range of N has proven to be non-trivial. Similarly as in the previous subsection, based on the Jury's criteria we can state the following result:

Theorem 3: TCP-RED system along with washout filter for a given (d, k_l), $b(\cdot) > 0$ and all other parameters held fixed, will be linearly asymptotically stable for $N > N^*$

where N^* is given as a solution to the following equation:

$$\frac{(d-2)(1+\lambda(N))}{2b} = k_l \quad \text{for } b > 0 \text{ and } 0 < d < 2 \quad (29)$$

where $\lambda(N)$ is the eigenvalue evaluated at the fixed point given in (13) as a function of N.
Solution to (29) exists because the eigenvalue $\lambda_0(N)$ increases monotonically with N in the parameter regime of interest.

E. Nonlinear control

It is possible to use small nonlinear control terms to further enhance the stability of a system going through a period doubling bifurcation. We first introduce the following hypothesis:

Hypothesis 1: Eq. (7) has a period-1 orbit at $x^*(\rho^*)$ where $x^*(\rho^*)$ is the fixed point at the critical parameter value ρ^*. Furthermore, the linearization of (7) at $x^*(\rho^*)$ possesses a simple eigenvalue $\lambda_1(\rho)$ with $\lambda_1(\rho^*) = -1$ and $\lambda_1^{'}(\rho^*) \neq 0$, where $\lambda_1^{'}(\cdot)$ is the derivative of $\lambda_1(\cdot)$ with respect to ρ.
This hypothesis can be easily verified for TCP-RED map given by (7). Now we recall the nonlinear control theorem given in [1] for local control of period doubling bifurcation.

Theorem 4: Under hypothesis 1 and for $l \cdot b \neq 0$, *i.e.*, when the critical eigenvalue is controllable for linearized system, there is a feedback $u(x_k)$ with $u(x_k - x^*(\rho^*) = 0) = 0$, *i.e.*, feedback control vanishes at the fixed point, which solves the local period doubling bifurcation control problem. Moreover, this can be accomplished with third order terms in $u(x_k)$, leaving the critical eigenvalue unaffected.

Above theorem suggests a cubic control by itself can stabilize the system or a mixed control with linear terms can be used to enhance the stability of bifurcation in an extended parameter domain. This allows us to consider different functional forms for the control in (18). All these forms have been shown to enhance the stability of the fixed point, thus delaying the system bifurcations [1]. It is also important that only the error terms $x_k - x^*(\rho^*)$ from the nominal operating point is used to preserve the original operating point.

$$u_k = k_c {y_k}^3 \qquad \text{Cubic Control Law}$$
$$u_k = k_l y_k + k_c {y_k}^3 \quad \text{Mixed Control Law}$$

The stability analysis done in [1] also suggests that k_l and k_c be based on the computation of l and b. Clearly, we do not need a quadratic control due to critical eigendirection being linearly controllable. Cubic control can be used to change the nature of emerging period doubling orbit in the presence of uncertainty. According to the theoretical results in [1] it is possible to enhance the nonlinear stability terms by using just the cubic control terms. It is shown that stability coefficient β_2, which decides the nature of bifurcation in the absence of any control, equals

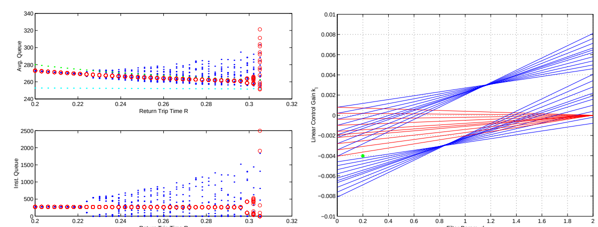
$$\beta_2 = -2\left(\frac{1}{2}\left(\frac{\partial^2 f}{\partial {q_k^n}^2}\right)^2 + \frac{1}{3}\left(\frac{\partial^3 f}{\partial {q_k^n}^3}\right)\right) . \qquad (30)$$

This coefficient β_2, when evaluated for the linearized system, decides if the bifurcation will be super ($\beta_2 < 0$) or subcritical ($\beta_2 > 0$) [7]. With the cubic control terms β_2 is changed by the following value:

$$\Delta = -4C_u(r, r, r)lb \qquad (31)$$

where $C_u(r, r, r)$ can be assigned any real value by an appropriate choice of cubic feedback to stabilize the ensuing bifurcation.

Along with the linear feedback term there is a sound reason to use small cubic terms in order to stabilize. The theorem from [1] supports this idea due to the fact that by using cubic term it is possible to stabilize the bifurcations with changes in parameters.



(a) (b)

Fig. 1. (a) Bifurcation diagram with and without control with respect to R (with p_{max} modulation). Bifurcation diagrams in blue and red are plotted without and with control, respectively. (b) Allowed (d, k_l) region lies below the red line for stability.

The effect of linear feedback control designed to stabilize the linearized version of the original system is difficult to determine. More precisely, when the bifurcation reappear at a different value of the bifurcation parameter, for instance, when the feedback control gain is small, the stability of this new bifurcation is not easily determined. Hence, using only a linear stabilizing feedback may be unacceptable if the goal is to stabilize a bifurcation and not merely to stabilize an equilibrium point for a fixed parameter value. In addition, in some cases a linear feedback that locally stabilizes an equilibrium point may result in globally unbounded behavior, whereas nonlinear feedback exists which stabilizes the equilibrium both locally and globally [12].

V. Numerical and NS-2 Simulation

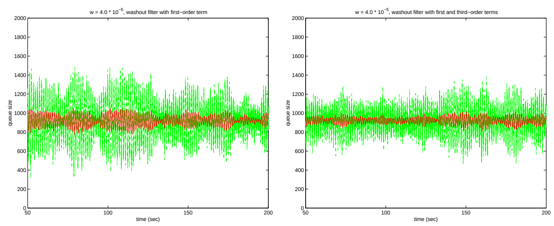
In this section we study the effect of washout filter-aided control on RED by numerical and NS-2 simulations.

A. Numerical examples

Fig. 1 plots the bifurcation diagram with respect to R_0 and the stability region of (d, k_l). Here we modulate p_{max} for feedback control, and only linear feedback control is used. The values of parameters used in the numerical example are as follows:

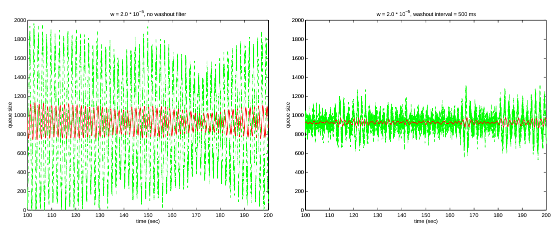
$$q_{max} = 747,\ q_{min} = 249,\ c{=}40 \text{ Mbps},\ K = \sqrt{3/2},$$
$$B = 3,735,\ w = 2^{-5},\ M = 4 \text{ kbits},\ N = 129,$$
$$k_l = -15/b,\ d = 0.2,\ R_0 = \text{bifurcation parameter}$$

As shown in the figure the washout filter-aided control delays the bifurcation. However, once the bifurcation takes place with feedback control, the system becomes even more unstable than the system without feedback control. This demonstrates the need for nonlinear feedback control as explained in the previous section.



(a) (b)

Fig. 2. Linear controller vs. nonlinear controller. (a) RED with linear feedback controller ($w = 4.0 \times 10^{-5}$), (b) RED with feedback controller with both first and third order terms ($w = 4.0 \times 10^{-5}$).



(a) (b)

Fig. 3. Long-lived connections. (a) RED without feedback controller ($w = 2.0 \times 10^{-5}$), (b) RED with feedback controller with both first and third order terms ($w = 2.0 \times 10^{-5}$).

B. NS-2 simulation

In this subsection we run the simulation with only long-lived TCP connections and compare the performance of RED with and without the feedback controller. The parameter p_{max} is updated once every 500 ms. The gains for the first and third order terms, *i.e.,* k_l and k_c, of the washout filter are set to 10^{-3} and 2.0×10^{-8}, respectively, and d of the washout filter is set to 0.1. These parameters are not optimized, and the selection of robust parameters is left for future studies. We compare the performance of the controller with only linear term and both linear and third order terms as well.

Fig. 3 shows the evolution of the instantaneous and average queue sizes. As one can see the RED without any controller shows unstable behavior, while the RED with a feedback controller shows very stable behavior. Here we only show the performance of the controller with both first and third order terms. However, the controller with only the linear term shows similar improvement in the stability. This is because the third order term does not play a significant role in this example since the system is still stable with the controller.

Fig. 2 shows the queue evolution with $w = 4.0 \times 10^{-5}$. Unlike in the previous scenario, with a larger exponential averaging weight the difference in the performance is more visible. As one can see the linear controller is not able to control the average queue size as well as the controller with both terms, which still exhibits only small oscillations. This is consistent with the claim that the third order term (nonlinear term) reduces the amplitude of the oscillations in the presence of instability.

VI. Discussion

We propose a dynamically adaptive version of RED that modulates a control parameter utilizing a washout filter based bifurcation control algorithm. This scheme is studied both analytically and numerically using a network simulator *ns-2*. Preliminary results suggest that it is possible to extend the stable operation region in parameter space significantly by using this technique. Furthermore, the stabilized domain in averaging weight w is larger than with the PI controller [8].

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