# Fault detection in a belt-drive system using a proportional reduced order observer.

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Abstract— In this paper a fault detection method is proposed to detect the belt breakdown in a belt drive system where it is assumed that a DC motor drives an inertial load through a belt. The proposed approach is based on a proportional reduced order observer designed using differential algebraic techniques. Experimental results are given to evaluate the proposed approach.

<u>Keywords:</u> Fault detection, algebraic observability, proportional reduced order observer.

### I. INTRODUCTION

The high reliability required in industrial processes has created the need for detecting abnormal conditions while the processes are operating. These conditions are called faults and it is important to detect them in the early stages. Belt drive systems are ubiquitous in industry and they are used to drive fans, machine tools and many other mechanical devices. It is worth remarking that in most cases the belt works at constant speed. The most likely fault in belt drive systems with possible catastrophic consequences is belt breakdown.

Literature about fault detection in belt drive systems is rather scarce, in particular, the detection of belt breakdown. The fault detection problem in a drive belt system has been studied in [1] using parameter estimation techniques combined with heuristic knowledge from a human

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As an alternative to parameter estimation, in this paper we propose a new approach to detect the breakdown of a belt in a belt drive system. Here, a proportional reduced order observer, designed via algebraic differential techniques, is employed for detecting the belt breakdown. An advantage of this approach is that the persistence of excitation condition needed in parameter identification is not longer necessary. Moreover, the resulting observer has linear dynamics and then it can be easily implemented using analog electronics or digital processors. The paper is organized as follows.

## II. STATEMENT OF THE PROBLEM

The model of a drive belt system consisting of a DC motor connected to a load through a belt is given by the following equations

$$J_1 \dot{\theta}_1 + f_1 \dot{\theta}_1 + 2\rho (r_1 \theta_1 - r_2 \theta_2) r_1 = \tau$$
  

$$J_2 \dot{\theta}_2 + f_2 \dot{\theta}_2 + 2\rho (r_2 \theta_2 - r_1 \theta_1) r_2 = 0$$
(1)

where

- $J_1$ : Motor inertia
- $J_2$ : Load inertia
- $f_1$ : Motor friction
- $f_2$ : Load friction

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 $\rho$ : Belt elasticity coefficient

 $\tau$ : Motor torque

- $\theta_l$ : Motor angle
- $\theta_2$ : Load angle
- $r_1$ : Radius of the pulley motor
- $r_2$ : Radius of the pulley's load

The fault detection consists of determining the belt breakdown. In terms of model (1) the belt breakdown happens when the belt elasticity coefficient  $\rho$  is equal to zero. Under the above condition system (1) becomes

$$J_{1}\ddot{\theta}_{1} + f_{1}\dot{\theta}_{1} = \tau$$

$$J_{2}\ddot{\theta}_{2} + f_{2}\dot{\theta}_{2} = 0$$
(2)

Using the following changes of variables:  $x_1 = \dot{\theta}_1$ ,  $x_2 = \dot{\theta}_2$ ,  $x_3 = r_1 \theta_1 - r_2 \theta_2$ , system (1) may be written as

$$\dot{x}_1 = -\frac{f_1}{J_1} x_1 - 2\frac{\rho r_1}{J_1} x_3 + \frac{\tau}{J_1}$$
(3)

$$\dot{x}_2 = -\frac{f_2}{J_2}x_2 + 2\frac{\rho r_2}{J_2}x_3 \tag{4}$$

$$\dot{x}_3 = r_1 x_1 - r_2 x_2 \tag{5}$$

$$y = x_1 \tag{6}$$

The state  $x_1$  is the motor angular velocity;  $x_2$  is the load angular velocity and  $x_3$  represents the difference between the angular position of the pulley motor and pulley load. Rather than estimating directly the belt elasticity coefficient  $\rho$ , the proposed approach consists in detecting belt breakdown indirectly through estimation of the state  $x_3$ . Assuming that only velocity measurements  $x_1$  are available from the DC motor, then,  $x_3$  is not available and it must be estimated. The next section deals with the reduced order observer design for estimating  $x_3$  and a methodology for applying the observer is also given.

# III. OBSERVER DESIGN.

Algebraic observability of state  $x_3$  is concluded as follows. Using (3) and (6) it can be shown that

$$\dot{y} = -\frac{f_1}{J_1} x_1 - 2\frac{\rho r_1}{J_1} x_3 + \frac{\tau}{J_1} = -\frac{f_1}{J_1} y - 2\frac{\rho r_1}{J_1} x_3 + \frac{\tau}{J_1}$$
(7)

then:

$$x_{3} = \frac{-f_{1} y - J_{1} \dot{y} + \tau}{2\rho r_{1}}$$
(8)

Hence, from (8) it is clear that  $x_3$  satisfies the algebraic observability condition [2], [4], i.e.,  $x_3$  depends on input and output measurements and their time derivatives. Note that  $x_3$  loose the algebraic observability property when the belt elasticity coefficient  $\rho$  is equal to zero, in other words, when the belt breaks down.

Now, let us consider the following proportional reduced order observer [3]

$$\dot{\hat{x}}_3 = \overline{K} \left( x_3 - \hat{x}_3 \right) \tag{9}$$

where  $\hat{x}_3$  denotes the estimate of  $x_3$  and  $\overline{K} \in R^+$  determines the desired convergence rate of the observer. Substituting (8) in (9) leads to

$$\dot{\hat{x}}_{3} = \frac{\overline{K}}{2\rho r_{1}} \left( -J_{1} \dot{y} + \tau - f_{1} y \right) - \overline{K} \hat{x}_{3}$$
(10)

Since the time derivative  $\dot{y}$  is not available, observer (10) cannot be implemented. In order to overcome this problem let us consider the following auxiliary variable  $\sigma$ 

$$\sigma \triangleq \hat{x}_3 + \bar{K} \frac{J_1 y}{2\rho r_1} \tag{11}$$

Then

$$\hat{x}_3 = \sigma - \overline{K} \frac{J_1 y}{2\rho r_1} \tag{12}$$

The time derivative of (12) is

$$\dot{\hat{x}}_3 = \vec{\sigma} - \overline{K} \frac{J_1 \dot{y}}{2\rho r_1} \tag{13}$$

Then, from (10), (12) and (13) it can be easily shown that the time derivative  $\dot{\sigma}$  is given by

$$\dot{\sigma} = \frac{\bar{K}}{2\rho r_1} \left[ \tau - \sigma + (\bar{K}J_1 - f_1)y \right]$$
(14)

Then, the reduced order observer is given by equations (12) and (14). It is worth remarking that the observer depends only on the belt and mechanical motor parameters. In practice, since these parameters may be unknown, estimates of them are used in the observer, then, the reduced order observer with parameter estimates is given by

$$\hat{x}_{3} = \sigma - \overline{K} \frac{\hat{J}_{1}y}{2\hat{\rho}r_{1}}$$

$$\dot{\sigma} = \frac{\overline{K}}{2\hat{\rho}r_{1}} \left[\tau - \sigma + (\overline{K}\hat{J}_{1} - \hat{f}_{1})y\right]$$
(15)

The behaviour of the reduced order observer is different before and after the belt breakdown. When the belt is unbroken and assuming a constant torque  $\tau_s$  applied to the motor, it can be shown that the steady state output of the observer is

$$\hat{x}_{3s} = \frac{1}{2\hat{\rho}r_1} \left[ \frac{f_2 r_1^2 + (f_1 - \hat{f}_1)r_2^2}{f_2 r_1^2 + f_1 r_2^2} \right] \tau_s$$
(16)

Moreover, if the belts breaks down the steady state value of  $\hat{x}_3$  is

$$\hat{x}_{3s} = \frac{1}{2\hat{\rho}r_1} \left[ \frac{f_1 - \hat{f}_1}{f_1} \right] \tau_s$$
(17)

The above observation is the basis for detecting the belt breakdown. The first step to apply the fault detection scheme is to obtain values of  $\hat{x}_3$  with the belt disconnected from the load and let  $\hat{x}_{3swl}$  the steady state value of  $\hat{x}_3$ without load. The next step is to connect the belt to the load maintaining the same value of  $\tau_s$ . When the belt breaks down, estimate  $\hat{x}_3$  takes values near from  $\hat{x}_{3swl}$  and the above condition indicates a fault. In practice, the voltage applied to the power electronics is available rather than the mechanical torque, then, assuming a linear relationship between the torque and the input voltage, then  $\tau = K_a u$ where u is the input voltage and  $K_a$  is the amplifier gain. Note that the above equality is also true for steady state values, i.e.  $\tau_s = K_a u_s$ 

## IV. EXPERIMENTAL RESULTS.

In order to test the method outlined in Section 3, a laboratory prototype was employed and it is shown in Figure 1. A DC brushed motor that transfers the torque to an inertial load through a belt. Belt breakdown was simulated through an electrical clutch. Engaging the clutch enables the motor to drive the load. Disengaging the clutch mechanically disconnects the load from the motor. Angular velocity is measured using an optical encoder with 2500 pulses per turn. The encoder is directly attached to the motor and the pulse train produced by the encoder is fed to a frecuency to voltage converter. The motor is driven by a Copley Controls, model 413, power amplifier, configured in voltage mode. Data acquisition is performed using the MultiQ 3 card from Quanser Consulting. The card also has 12 bits digital to analog converters with an output voltage range of  $\pm 5$  Volts. The proportional reduced order observer was implemented using the MatLab-Simulink software running under the WINCON program from Quanser Consulting. The WINCON environment was used in the client and the server running at 200 MHz. The client is allocated in other Pentium based computer running at 350 MHz. Sampling rate was set to 1 KHz



Fig. 1 Laboratory prototype employed in the experiments

The observer was implemented using the following values.  $\overline{K} = 12$ ,  $\hat{f} = 0.147 \ Nm/rad$ ,  $\hat{J} = 0.0001 \ Nm^2/rad$ ,  $\hat{\rho} = 0.1$ ,  $K_a = 10$ ,  $r_1 = 0.013 \ m$ . The initial condition for the observer was set to  $\hat{x}_3(0) = -10 \ rad$ . The first experiment was performed using  $u_s = 2 \ volts$ . Figure 2 shows the motor angular velocity and Figure 3 the estimate  $\hat{x}_3$ . The second experiment was performed using  $u_s = 1.7 \ volts$ . Figures

4 and 5 show the motor angular velocity and the estimate  $\hat{x}_3$  respectively. From the above results it is clear that the observer detects the fault.

# V. CONCLUSIONS.

In this paper a fault detection method was proposed to detect the belt breakdown in a belt drive system. the approach was applied to a DC motor driving an inertial load through a belt. The proposed approach is based on a proportional reduced order observer designed using differential algebraic techniques. Experimental results shows that the reduced order observer detects effectively the fault even if the motor and belt parameters are not exactly know, a key feature for practical application of the proposed approach. Further work includes applying the scheme when the motor is in closed loop control and to other types of electrical machines.



Fig. 2 Motor velocity with fault at 20 s and  $u_s = 2$  volts.



Fig. 3 Observer output with fault at 20 s and  $u_s = 2 \ volts$  .



Fig. 4 Motor velocity with fault at 30 s and  $u_s = 1.7 \ volts$  .



Fig. 5 Observer output with fault at 30 s and  $u_s = 1.7 \ volts$  .

# VI. REFERENCES

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