Power Control for Wireless Networks Using Multiple Controllers and Switching

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***Abstract*— Controlling transmitted power in a wireless network is critical for maintaining quality of service, maximizing channel utilization and minimizing near-far effect for suboptimal receivers. In this paper, a general PID (Proportional–Integral–Derivative) type algorithm for controlling transmitted powers in wireless networks is studied and a systematic way to adapt or tune the parameters of the controller in a distributed fashion is suggested. The proposed algorithm utilizes multiple candidate PID gains. Depending on the prevailing channel conditions, it selects an optimal PID gain from the candidate gain set at each instant and places it in the feedback loop. The algorithm is data driven and can distinguish between stabilizing and destabilizing controller gains as well as rank the stabilizing controllers depending on their performance. Simulation results indicate that the proposed scheme performs better than several candidate controllers, including the well known distributed power control algorithm.**

***Index Terms*— Power control, cellular system, switched systems, PID tuning and adaptation**

I. INTRODUCTION

Power control in wireless networks is important to maintain reliable communication links between base stations and mobile users, and to maximize the battery life. This objective can be met by using a centralized algorithm [1], [2], [3], which also minimizes the total transmitted power and interference. However, centralized algorithms are not practical as they require complete information on the link gains. This difficulty can be avoided by using the distributed power control algorithm proposed by Foschini and Miljanic [4]. Their algorithm, widely known as distributed power control (DPC) in the power control literature, converges to the optimal solution of the centralized case provided that the channel gains are fixed and the system is feasible.

Adjusting transmitted powers has also been addressed using control–theoretic methods including PID (Proportional+Integral+Derivative) or PI type controllers in order to improve the convergence rate [5], [6]. In [5], Gunnarsson *et al.* provide specific parameter values for their PI controller in log–linear scale, albeit it leads to a suboptimal system performance in general because the design ignores the cross coupling between powers of various users through the interference term. Later, they extended their work by computing a range of stabilizing PID controller parameters [6]; however it uses linearization of the non–linear interference term and the small gain theorem, which is known to produce conservative results.

Although the algorithms in [5], [6] with appropriately chosen parameters lead to better convergence rates than DPC, there is not a systematic procedure that would update the controller parameters based on the dynamic wireless environment which could improve performance. Furthermore, a single set of controller parameters might not be suitable for all users in the network, *i.e.*, the optimal controller for an individual user might be a function of its location within the cell, the current channel conditions, the total number of active users in the cell, shadow and fast fading, and the interference it is experiencing. For instance, a mobile user close to the base station may find it more suitable to use a low gain controller whereas other users far from the base station may have to use some other controller to achieve faster convergence.

In this paper, we propose an adaptive distributed scheme which can tune the controller parameters of individual users in order to improve the overall system performance. The proposed system utilizes a set of candidate PID controllers. The idea of using multiple controllers has become popular over the last decade [7], [8], [9], [10], [11], [12], [13]. The basic problem here is to control a complex, unknown, possibly time–varying system for which there is no way to construct a traditional controller that would give satisfactory results; the limitation in constructing such a controller might be due to lack of sufficient knowledge about the system or the complexity of the system. Hence, instead of using a single controller to control the system, a set of candidate controllers is used. Based on measurement data collected from the system, a supervisory unit switches among these candidate controllers and tries to place the best available controller in the feedback loop.

Most switching based methods belong to two broad categories: Indirect method based on system identification [7], [8], [9] and methods that directly identify the controller [10], [11], [12], [13]. In this paper, we use the direct method to choose the most suitable PID gains for each user from a candidate set.

In direct methods, there is a set of candidate controllers, and at each instant, the potential performance of every candidate controller is evaluated from the measured data using some suitably defined performance index. This index is a measure of how closely the output of the closed loop system would have followed some reference input, had the candidate controller been in the feedback loop. Note that the performance index of all the candidate controllers can be evaluated without actually inserting them in the feedback loop. Depending on this index, a suitable controller is selected and switched in the feedback loop to control the system. The underlying theory is quite general and can be applied to many other power control algorithms that satisfy some mild assumptions on invertibility of the controller, stated in Section III.

The rest of the paper is organized as follows. In Section II, we present the power control problem and a simple PID–type distributed power control algorithm. In Section III, we describe the proposed multiple controller based switching scheme. In Section IV, we apply this technique to tune the parameters of the PID power control algorithm discussed in Section II. Finally, we present the simulation results and the conclusions in Sections V and VI, respectively.

II. Distributed power control

We consider N mobile users sharing the same channel. We assume that the i^{th}, $i = 1, \ldots, N$, mobile user is connected to the i^{th} base station. If two mobile users, say i and j are assigned to the same base station, then the indices i and j refer to the same physical base station. All values are in linear scale unless otherwise mentioned. We study only the uplink, although the results are also applicable to the downlink.

Let g_{ij} represent the channel gain between the j^{th} transmitter and the i^{th} receiver and the transmitted power vector be given by $P(t) = [p_1(t),\ p_2(t), \ldots, p_N(t)]^T$, with $p_i(t) \geq 0$ denoting the power from the i^{th} transmitter. The achieved SINR for the i^{th} user can be expressed as

$$\gamma_i(t) = \frac{g_{ii} p_i(t)}{\sum_{j=1,\ j\neq i}^{N} g_{ij} p_j(t) + \nu_i}\ ,\ i = 1, \ldots, N, \qquad (1)$$

where ν_i is the thermal noise at the i^{th} receiver. The objective of the power control algorithm is to update the power levels $p_i(t)$ such that the achieved SINR satisfies

$$\gamma_i(t) \geq \Gamma, \ \ i = 1, \ldots, N, \qquad (2)$$

where Γ is the target SINR that is determined from the Quality of Service constraints. This goal can be achieved using several distributed power control algorithms [4], [5], [6]. In this paper, we employ a general linear PID–type controller structure and propose a method to tune the PID gains. However, the theory behind the proposed tuning scheme is quite general in nature and can be easily applied to the afore–mentioned power control algorithms of [4], [14], [5], [6].

Our starting point is the DPC algorithm, which is an Integral (I)–type algorithm proposed in [4]; we extend it by adding the proportional and derivative terms as well. Since practical power control algorithms are usually implemented in discrete–time, we directly present the discrete–time version of the proposed PID algorithm. The power update equation for the i^{th} user is given by:

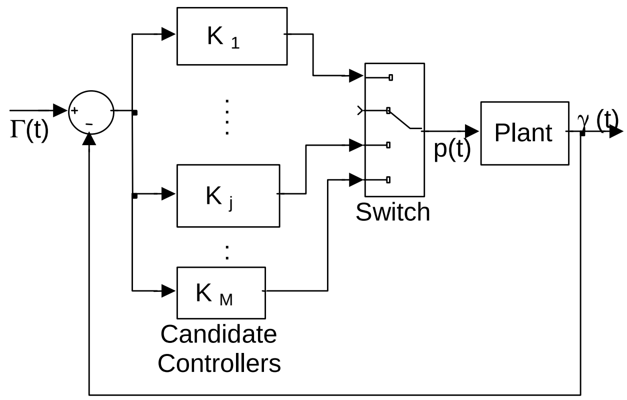

Fig. 1. General plant controller configuration

$$p_i(k) = \alpha e_i(k) + \beta x_i(k) + \theta[e_i(k) - e_i(k-1)], \qquad (3)$$

where

$$e_i(k) = \left[1 - \frac{\Gamma}{\gamma_i(k-1)}\right] p_i(k-1), \qquad (4)$$

$$x_i(k) = x_i(k-1) + e_i(k), \qquad (5)$$

with α, β, θ forming the controller parameters. This is a typical PID–controller, with the proportional, integral and derivative gains represented by α, β and θ, respectively.

In this paper, we apply the proposed multiple–controller based approach to tune the PID–parameters α, β and θ. The general idea is as follows: There are multiple candidate controllers, having different sets of gains for the controller parameters α, β and θ. The proposed scheme monitors the data and associates a performance index with each candidate controller. Based on this index, the best available controller is selected from the candidate set and is used in the feedback loop.

III. Multiple Controller System

In this section, we present a general overview of a direct multi controller based switching system which we will use in the next section to tune the gains of the PID controller parameters. This concept was introduced in [10]; it was later applied to several systems [11], [12], [13]; and its stability analysis was done in [15].

Consider an unknown plant $\mathcal{P}$, with $(p(t), \gamma(t))$ being the plant input/output (I/O) data that can be measured (see Fig. 1). The plant output $\gamma(t)$ is required to track a reference input, $\Gamma(t)$. The proposed scheme utilizes a set of M candidate controllers, denoted by $C_j, j \in \mathcal{M} \triangleq \{1, ..., M\}$, where each candidate controller is characterized by the triplet $(\alpha_j, \beta_j, \theta_j)$. The objective is to select at each instant the best controller among the set of available controllers using some performance index and place it in the feedback loop. Construction of this set of candidate controllers is crucial for the success of the algorithm. The assumption is that the set $\mathcal{M}$ contains at least one controller which can stabilize the plant. For the power control problem, this assumption is easily satisfied by including the controller with the parameters, $\alpha = \theta = 0,\ \beta = 1$, in the candidate controller set, as this controller corresponds to the DPC algorithm in [4] and is known to produce a stable system under static channel conditions. The candidate controller set should be constructed such that there are several stabilizing controllers in the set, including some potential good ones. When the ranges of stabilizing candidate controller parameters are partially or completely unknown, this set has to be made large by including several candidate controllers.

Let us define the control error

$$e_c(t) \triangleq \Gamma(t) - \gamma(t)\ . \tag{6}$$

Given a set of past plant I/O data $(p(t), \gamma(t))$, we now define the fictitious reference input $\widetilde{\Gamma}(C_j, p, \gamma)$ for the j^{th} candidate controller $C_j,\ j \in \mathcal{M}$. This is the hypothetical reference signal that would have produced exactly the measured data $(p(t), \gamma(t))$ had the candidate controller C_j been in the feedback loop with the unknown plant during the entire time period over which the measured data $(p(t), \gamma(t))$ were collected. As this signal is not the actual reference signal, hence the name fictitious. For example, for the system in Fig. 1, let K_j be the j^{th} candidate controller. Note that the controller used can be in the form of a gain or a transfer function; in that case, we will have to use the inverse of the transfer function to obtain the fictitious reference signal. Let $(p(t), \gamma(t))$ be the plant I/O data collected using some other controller in the loop. Then, the fictitious reference signal for the j^{th} candidate controller K_j is given as

$$\widetilde{\Gamma}(C_j, p, \gamma) = K_j^{-1} p + \gamma\ , \tag{7}$$

where K_j^{-1} is the inverse of the j^{th} candidate controller transfer function. If the j^{th} controller is actually in the loop during the time for which plant I/O data (p, γ) are collected, then the j^{th} fictitious reference signal would be same as the actual reference signal; otherwise it would be different from the actual reference signal.

Remark: To uniquely determine the fictitious reference signal, the controller has to be causally-left-invertible [11], that is, from the past and present output of the controller, one should be able to uniquely determine its present input and hence the inverse of the controller should also be a proper transfer function. Controllers with bi–proper transfer function, including the controller studied in this paper have this property.

Let us now define the fictitious error as

$$\widetilde{e}(C_j, p, \gamma) \triangleq \widetilde{\Gamma}(C_j, p, \gamma) - \gamma. \tag{8}$$

For convenience, in the sequel, we will use the following notations for the fictitious reference signal and fictitious error signal of the j^{th} candidate controller :

$$\widetilde{\Gamma_j}(t) \triangleq \widetilde{\Gamma}(C_j, p, \gamma, t), \quad \widetilde{e_j}(t) \triangleq \widetilde{e}(C_j, p, \gamma, t).$$

From the definition of the fictitious reference signal given above, it is clear that the error $\widetilde{e_j}$ would have been the control error of (6) with (p, γ) as the plant I/O data, had the j^{th} candidate controller C_j been in the feedback loop. So, $\widetilde{e_j}$ is a measure of the effectiveness of the j^{th} candidate controller, if it were used to control the plant. Hence, switching among candidate controllers must be based on this error. We can construct some suitably defined performance indices, $\widetilde{J}(j, t), j \in \mathcal{M}$, which will be discussed explicitly in the next section. Given a set of candidate controllers $C_j, j \in \mathcal{M}$, the problem then is to identify and switch to the best available controller C^* that would minimize the performance index at each instant, that is:

$$C^*(t) = C_{j^*(t)} \ \text{ where } \ j^*(t) = \arg\min_{j \in \mathcal{M}} \widetilde{J}(j, t). \tag{9}$$

IV. Switching Based Approach to Power Control Problem

In this section, we apply the switching based approach to the power control algorithm described earlier. We first construct a bank of PID controllers, $C_j,\ j \in \mathcal{M}$, each with different values of α, β and θ, i.e., the j^{th} controller has α_j, β_j and θ_j as its parameters. Given this set, we now proceed to construct the fictitious reference signal, as described in the previous section.

As we consider the local loops, that is, the signals corresponding to individual users are dealt with separately, we henceforth discard the index i. Thus, in the sequel, $p(k)$ will indicate the power $p_i(k)$ of the i^{th} user; similarly $\gamma(k)$ will denote the achieved SINR $\gamma_i(k)$ for the i^{th} user. Using the delay operator q, the PID algorithm in (3) can be written as

$$\begin{aligned} p(k) = &\frac{(\alpha+\beta)q^2 - \alpha q + \theta(q^2 - 2q + 1)}{q(q-1)} \\ &. \left[p(k-1) - \frac{\Gamma}{\gamma(k-1)} p(k-1) \right] \end{aligned} \tag{10}$$

or

$$\begin{aligned} \Gamma = &\ \gamma(k-1) \\ &- \frac{q^2 - q}{(\alpha+\beta+\theta)q^2 - (\alpha + 2\theta)q + \theta} \frac{p(k)\gamma(k-1)}{p(k-1)}\ . \end{aligned} \tag{11}$$

Hence, the fictitious reference signal $\widetilde{\Gamma_j}(t)$ for the j^{th} candidate controller C_j, with parameters as α_j, β_j and θ_j can be expressed as

$$\begin{aligned} \widetilde{\Gamma}_j(k) = &\ \gamma(k-1) - \\ &\frac{q^2 - q}{(\alpha_j+\beta_j+\theta_j)q^2 - (\alpha_j + 2\theta_j)q + \theta_j} \frac{p(k)\gamma(k-1)}{p(k-1)} \end{aligned} \tag{12}$$

Note that here j is the index of the candidate controller and all signals correspond to the i^{th} user. This would have been the reference SINR, had the j^{th} controller C_j been in the loop, with $p(k)$ and $\gamma(k)$ being the actual transmitted power and the actual achieved SINR. The fictitious error signal $\widetilde{e_j}$ for the j^{th} controller is then given by (8), which is an indication of how closely the achieved SINR would have followed the target SINR, had C_j been used to control the plant.

A. Choice of Performance Index

An appropriate choice of the index is crucial for the performance of the switched system. The performance index proposed here is motivated by the loop shaping techniques, a well known tool used in robust control for H_∞ design [16], [11], [6].

The objective in any power control algorithm is that the achieved SINR satisfies the requirement in (2) using minimum transmitted power. The control error for the power control algorithm is, $e_c(t) = \Gamma - \gamma(t)$, and the reference input is the target SINR (Γ). Hence, the sensitivity function for the power control problem, which is defined as the transfer function from the reference signal to the control error of the system, is given by [16]

$$S = \frac{e_c}{\Gamma}.$$

The sensitivity function S assumes an important role in robust performance of any system and is widely used in H_∞ control system design [16]. It can be shown that it is also the transfer function from external disturbances to the output of the system [16]. Thus to minimize the effect of disturbance on the output of the system and minimize the error, it is desirable that the magnitude of the sensitivity function be low at all frequencies. However, for a stable plant, the sensitivity function has to fulfill the requirement

$$\int_0^\infty \ln(|S(j\omega)|)d\omega = 0 \qquad (13)$$

and hence, if the sensitivity is low at certain frequencies, it has to be high at some other frequency (known as the "waterbed effect" in robust control theory). In the low frequency region (inside the system bandwidth), it is desirable that the magnitude of the sensitivity be small, and is allowed to be large in the high frequency region to compensate for small values in the low frequency region.

In robust control theory and $\mathcal{H}_\infty$ design, to ensure that the sensitivity function of a system meets the above specification, a controller is designed such that the sensitivity function lies below the frequency response of the reciprocal of some known filter W_1 at all frequencies, that is [16],

$$|S(j\omega)| \le 1/|W_1(j\omega)|, \ \ \forall\omega \ , \qquad (14)$$

which can be written as

$$\|W_1(j\omega)S(j\omega)\|_\infty \le 1 \qquad (15)$$

where $\|.\|_\infty$ denotes the $\mathcal{H}_\infty$ norm. W_1 is a filter to be specified by the designer[1]. The filter $1/W_1$ has low gain at low frequency and is allowed to have high gain at high frequency.

Let the system be required to satisfy the integral performance specification

$$J(t) \triangleq \zeta\, T_{spec}(\Gamma(t), p(t), \gamma(t)) + \delta \int_0^t \exp^{-\phi(t-\tau)} T_{spec}(\Gamma(\tau), p(\tau), \gamma(\tau))d\tau \le \mu, \qquad (16)$$

where the objective is to minimize the parameter μ by optimally choosing the controller; $T_{spec}(\cdot)$ is the performance specification to be chosen later and ζ, δ, ϕ are non-negative design constants. The performance index for the j^{th} candidate controller can then be given by

$$\widetilde{J}(j,t) = \zeta\, T_{spec}(\widetilde{\Gamma}_j(t), p(t), \gamma(t)) + \delta \int_0^t \exp^{-\phi(t-\tau)} T_{spec}(\widetilde{\Gamma}_j(\tau), p(\tau), \gamma(\tau))d\tau. \qquad (17)$$

The parameters ζ and δ penalize the instantaneous and accumulated performance specifications, respectively, and ϕ is the forgetting factor that determines the weight of past data and ensures the boundedness of the integral term. Making ζ large results in very fast switching whereas large δ results in comparatively slow switching. The forgetting factor ϕ helps reduce the importance of old data set. A qualitative discussion on how to choose these constants and their effect on switching can be found in [7]. The optimal controller C^* can be selected and switched in the feedback loop using (9). For convenience, we use the following notation in the sequel to represent performance specification of the j^{th} candidate controller: $\widetilde{T}_{spec}(j,t) \triangleq T_{spec}(\widetilde{\Gamma}_j(t), p(t), \gamma(t))$.

Let us define a signal $f(j,t)$ corresponding to the j^{th} candidate controller as

$$f(j,t) = w_1(t) * (\widetilde{\Gamma_j}(t) - \gamma(t)), \qquad (18)$$

where $*$ is the convolution operator and $w_1(t)$ is the time response of filter W_1 described before. The performance specification is taken as

$$\widetilde{T}_{spec}(j,t) = \frac{|f(j,t)|^2}{|\widetilde{\Gamma_j}(t)|^2} \ . \qquad (19)$$

Note that

$$\max_{\Gamma(t)} \frac{||w_1(t) * (\Gamma(t) - \gamma(t))||^2}{||\Gamma(t)||^2} = \|W_1(j\omega)S(j\omega)\|_\infty$$

forms the left hand side of (15), where $||\cdot||$ is the 2-norm of the signal and $||\cdot||_\infty$ is the $\mathcal{H}_\infty$ norm of transfer function $W_1 S$. The continuous–time state space representation of the performance index of (17) can be obtained as

$$\begin{aligned} \dot{x}(t) &= -\phi\, x(t) + \delta\, \widetilde{T}_{spec}(j,t) \\ \widetilde{J}(j,t) &= x(t) + \zeta\, \widetilde{T}_{spec}(j,t), \end{aligned} \qquad (20)$$

where $x(t)$ is the state of the filter generating the performance specification. For implementation, a discrete–time version of the above can be easily obtained.

The following algorithm summarizes the multiple controller based switching scheme.

[1]For details, refer to [16].

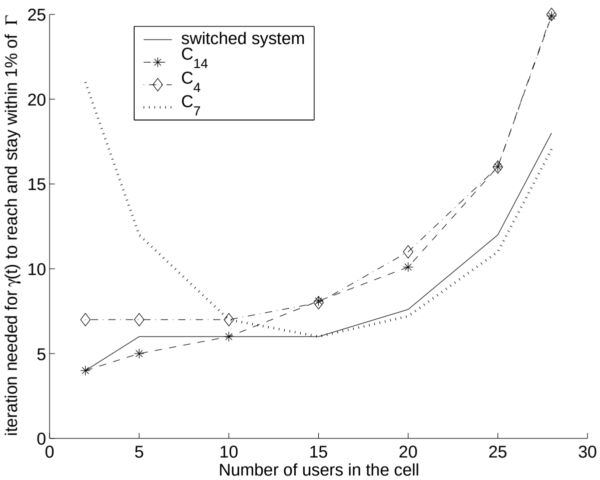

Fig. 2. Average number of iterations required for all users to reach and stay within 1% of the target SINR.

Algorithm 1:

INITIALIZATION : For each user $i = 1, ..., N$:

- Define a set of candidate controllers given by $C_j,\ j \in \mathcal{M}$. Choose an initial controller to be used in the loop at time $\tau = 0$.
- Set initial performance specification $\widetilde{J}(j, 0) = 0,\ j \in \mathcal{M}$.

PROCEDURE (at each time $\tau = k\Delta t$) : For each user $i = 1, ..., N$, repeat the following till the user is connected to the network:

(1) Measure $(p(k\Delta t), \gamma(k\Delta t))$
(2) Calculate $\widetilde{J}(j, k\Delta t),\ \ j \in \mathcal{M}$, using discrete-time version of (17) and (19).
(3) Determine the best available controller C^* using (9) and update the controller parameters of the i^{th} user.
(3) Update the transmitted power using (3). □

V. SIMULATION RESULTS

In this section, we numerically test our proposed scheme by considering a simple CDMA cell in which all users are assumed to share the same channel and are served by a single base station [17]. The mobiles are uniformly distributed around the base station in a square cell of length 2000 meters.

The success of any switching scheme depends on judicious choice of the candidate controller set. When a range of stabilizing controller parameters is known, the candidate controllers can be selected such that their parameters lie evenly in that range. In absence of such knowledge, if a stabilizing controller is known, it can be used as a nominal controller and several other candidate controllers can be constructed with parameters lying around those of the nominal controller. From [4], we know that the controller corresponding to the DPC algorithm ($\alpha = 0,\ \beta = -1,\ \theta = 0$) produces a stable system. Hence, DPC is set as a nominal controller and the candidate controllers are chosen from all possible combinations of the parameters (α, β, θ), where $\alpha \in \{-0.15, 0, 0.15\},\ \beta \in \{-1.5, -1, -0.5\},\ \theta \in \{-0.003, 0, 0.003\}$, which yields a total of 27 candidate controllers.

The target SINR is assumed to be 1/30. The performance index used is as in (17) with the performance specification of (19), with $\zeta = 1,\ \delta = 1,\ \phi = 0.01$. The filter W_1 is given by $W_1(s) = \frac{s+10}{2(s+0.1)}$. Constant channel gains, without any fading and a path loss exponent of 4 are implemented. Receiver thermal noise is taken as 1e-6. Initial transmitted powers are set to 0.001 for all users. As DPC is known to produce a stable system, Algorithm 1 is executed with DPC as the initial controller for all users.

As a measure of performance, we consider the number of iterations needed for all users to reach and stay within 1% of the target SINR, which is similar to the notion of settling time in control theory. As we assume that all mobiles are uniformly distributed within the cell, the average settling time is computed based on 1000 Monte Carlo simulations. We repeat this process for various number of users in the cell and plot the average settling time versus the number of users in the cell.

Fig. 2 summarizes the simulation results for the proposed switching scheme (solid curve) and for three other candidate controllers, C_4 ($\alpha = -0.15,\ \beta = -1,\ \theta = -0.003$), C_7 ($\alpha = -0.15,\ \beta = -1.5,\ \theta = -0.003$) and C_{14} ($\alpha = 0,\ \beta = -1,\ \theta = 0$). Note that C_{14} corresponds to DPC. The curves for these three controllers are obtained by using them for all users at all times, without any switching. Responses of other controllers are not plotted as they have comparable or worse performance.

From Fig. 2, it is seen that when the number of users is low ($N \leq 10$), the DPC algorithm (C_{14}) outperforms other candidate controllers when used by all users without any switching. On the other hand, when the cell has a larger number of users, other candidate controllers perform better. The switched system utilizes DPC mainly when the number of users is low and uses other candidate controllers when the number of users is high, and thus performs better than most of the candidate controllers for all possible user configurations. For example, with 28 users distributed uniformly in the cell, it is seen that C_7 gives good performance (low settling time). In simulation, it is observed that most users converge to this controller when the proposed switching scheme is implemented. However, for certain distribution of the mobiles, *e.g.*, when one or several users are very close to the base station, and with different initial transmitted powers, C_7 may not always give the best performance. Even in such cases, we have observed that the proposed switching scheme selects different candidate controllers for different users, thereby still improving the overall performance. The achieved SINR of all users for a random distribution of 28 mobiles, with the proposed switching scheme, is shown in Fig. 3; the settling time is seen to be 20. When DPC (C_{14})

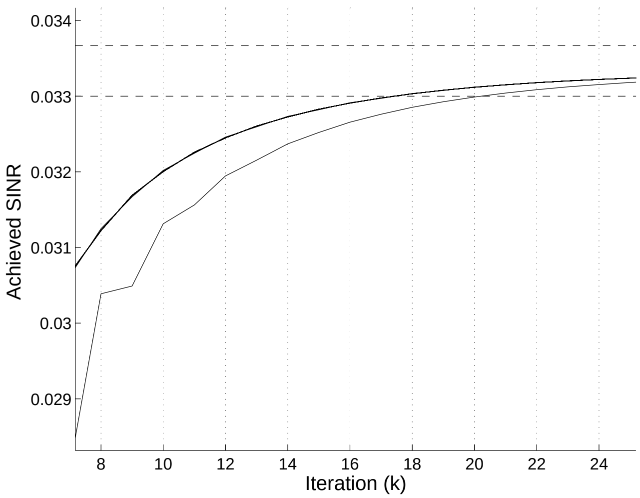

Fig. 3. Achieved SINR (solid line –) for 28 users using the proposed switched system. Dashed line (- -): 99% and 101% of the target SINR. Settling time = 20 iterations.

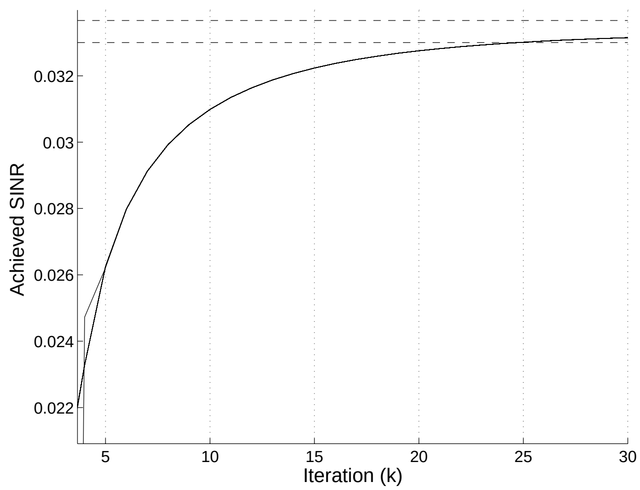

Fig. 4. Achieved SINR (solid line –) for 28 users with the same mobile distribution as in Fig. 3 and with all of them using only C_{14} (DPC). Dashed line (- -): 99% and 101% of the target SINR. Settling time = 25 iterations.

is used for all users without any switching for the same mobile distribution, the settling time is found to be 25 (Fig. 4).

VI. Conclusions

In this paper, we have described an adaptive technique to tune the gains of a PID–type controller to be used for power control in a wireless network. The proposed algorithm selects controllers in a distributed fashion using the available data only, without any prior knowledge about the existing channel conditions. Simulation results show that the algorithm performs better than most of the available candidate controllers (including the DPC for higher number of users in the cell) and can adapt to different number of users in the cell by using different candidate controllers.

VII. Acknowledgement

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