Parallel Compensator for Control Systems with Nonminimum Phase Plants

Ryszard Gessing

***Abstract*— Following the Smith compensator the parallel compensator designed for nonminimum phase plants is introduced in the paper. The compensator connected in parallel to the plant changes its properties so that the replacement plant model becomes minimum phase and may be shaped dependently upon the goal of the control. In the case of regulation on a constant level a first order lag may be chosen for the replacement plant model. In the case of tracking or disturbance rejection of signals with frequencies belonging to some working frequency band, the replacement plant model should have its frequency response close to that of the plant, in the working frequency band. The proposed approach simplifies the design and improves accuracy of the control.**

I. INTRODUCTION

It is known that there are so called difficult plants for which it is difficult to design a regulator assuring appropriate accuracy of control. One could mention here plants with pure time delay and nonminimum phase plants. For the mentioned plants an insignificant increase of the proportional regulator gain causes instability and for small gain the system has unsatisfactory accuracy in steady state. Also the demand of stability creates a limitation for the gain of integral part of regulator. This part removes the steady state error, but related with small gain transients are very slow.

For the plants with pure time delay Smith [5] proposed a compensator with effectively takes the delay outside the loop and allows a feedback design based on the plant dynamics without delay. The result is that the regulator designed in this manner is faster and assures higher accuracy. Now this compensator is commonly called Smith compensator [1] (or predictor [3]).

In the present paper, following the idea of the Smith compensator a parallel compensator is proposed, which may be applied to nonminimum phase plants. Using this approach, the regulator may be designed for the replacement plant with appropriately chosen minimum phase model. Similarly as for the Smith compensator the assumption is that the plant is stable (in the case of Smith compensator this is not exactly formulated in literature). This approach may be also applied for the system with relay implementation of the control and a preliminary idea was presented in [2].

The contribution of the paper is in proposing to nonminimum phase plants the parallel compensator which improves the accuracy of control and in showing that the compensator may be applied both, for the systems with continuous–time and relay control implementation.

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Poltechnika Śląska, Instytut Automatyki, ul. Akademicka 16, 44-101 Gliwice, Poland, gessing@ia.gliwice.edu.pl

II. PARALLEL COMPENSATOR

Consider the linear plant described by the transfer function (TF)

$$G(s) = \frac{Y(s)}{U(s)} = \frac{L(s)}{M(s)} \tag{1}$$

where $Y(s)$ and $U(s)$ are the Laplace transforms of the plant output and input, respectively, while $L(s)$ and $M(s)$ are polynomials of m-th and n-th order, respectively and $m < n$. Assume that the plant is stable, that is its poles $p_i,\ i = 1, 2, ..., n$ have negative real parts i.e. $Rep_i < 0$.

In the case of difficult plant (e.g nonminimum phase, or with higher order dynamics), when it is difficult to design the regulator assuring an appropriate accuracy, the parallel compensator shown in Fig. 1 may be applied. The idea of parallel compensator described by the TF

$$G_c(s) = \frac{Y_c(s)}{U(s)} = G_1(s) - G(s) \tag{2}$$

is similar to that of the Smith predictor. Here $Y_c(s)$ is the Laplace transform of the output y_c of the compensator, while $G_1(s)$ is the TF which should be appropriately chosen.

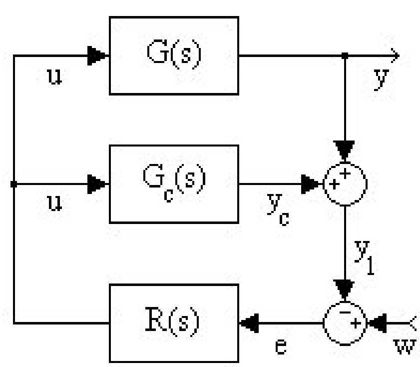

Fig. 1. The connection of parallel compensator.

Note that in the proposed structure shown in Fig. 1 the TF $G_r(s)$ of the replacement plant is described by

$$G_r(s) = \frac{Y_1(s)}{U(s)} = G(s) + G_c(s) = $$
$$= G(s) + G_1(s) - G(s) = G_1(s) \tag{3}$$

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Thus the replacement plant is described by the TF $G_1(s)$ and the regulator $R(s)$ should be designed for the replacement plant described by the TF $G_1(s)$. Therefore the crucial point in the proposed method is the choice of the TF $G_1(s)$.

We will distinguish two cases dependently upon the goal of the control and the way of its implementation .

As concerns the goal of the control we will distinguish:

I. Regulation with some accuracy under stepwise excitations;
II. Tracking and disturbance rejection with some accuracy for frequencies belonging to a working frequency band $[0, \omega_{mx}]$.

As concerns control implementation we will distinguish:

1) Continuous–time implementation;
2) Relay implementation – modified sliding mode.

III. REGULATION WITH PRESCRIBED ACCURACY

In this case we are mainly interested in the accuracy of the constant steady state, appearing after some time from occurrence of stepwise excitation (set point or disturbance). Since in the case of nonminimum phase plants we have a limited possibility of shaping transient response, which is dependent upon the placement of zeros and poles of the plant we do not formulate some special demands concerning transient response, though it should be acceptable.

In this case the model $G_1(s)$ may be chosen in the form of a first order lag i.e

$$G_1(s) = \frac{k_0}{Ts+1}, \qquad k_0 = G(0), \tag{4}$$

assuring that

$$G_1(0) = G(0) \tag{5}$$

Since the gain of constant signals ($\omega = 0$) is the same for both the models $G_1(s)$ and $G(s)$, then in steady state for constant signals, as results from (2) and (5), it is

$$y_1 = 0 \quad \text{and} \quad e = w - y$$

For the replacement plant (4) the proportional (P) regulator with high gain may be applied, or

$$R(s) = k, \qquad k \text{ is high} \tag{6}$$

and the researched CL system with (4) and (6) has appropriate stability degree for high at will gain (frequency response $kG_1(j\omega)$ lies in the first negative quadrant of the Nyquist plane).

Additionally, it may be noticed that the researched system is very robust. Really the change of some parameter of the plant causes the change of the frequency response of the plant to the form denoted by $G^*(j\omega)$. For any frequency ω the vector $G^*(j\omega) - G(j\omega)$ changes the replacement plant characteristic $G_1(j\omega)$. However, since the frequency response $G_1(j\omega)$ lies in the first negative quadrant of the Nyquist plane, its distance from the critical point $(-1, j0)$ is so big that only sufficiently large vector $G^*(j\omega) - G(j\omega)$ and sufficiently large change of parameter may cause instability. This resonning justifies robustness of the system with proposed parallel compensator. The simulations performed in the following Example confirms this observation.

A. Example 1

Consider the nonminimum phase plant described by the following TF

$$G(s) = k_p \frac{-2s+3}{s^3+4s^2+4s+3}, \quad k_p = 1 \tag{7}$$

To design the parallel compensator for regulation with prescribed accuracy we choose $G_1(s)$ in the form (4) with $T = 0.5$ and $k_0 = G(0) = 1$. Assume $k = 50$ which gives 2% accuracy in steady state.

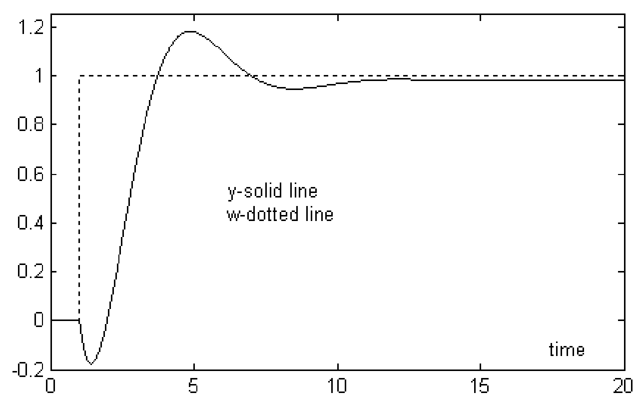

Fig. 2. Plots of y and w for Example 1.

Step response $y(t)$ of the CL system, for $w = \mathbf{1}(t)$ ($\mathbf{1}(t) = 1$ for $t \geq 0$ and $\mathbf{1}(t) = 0$ for $t < 0$), is shown in Fig. 2. The CL system is stable even for $k_p = 2$ (without change of the parallel compensator), but then the undershot is ~ -0.4, overshot is ~ 1.9 and there are some oscillations in the response. When we take $T = 0.25$ then the undershot and overshot is somewhat smaller and the response is somewhat slower.

IV. TRACKING AND DISTURBANCE REJECTION

In this case we are mainly interested in the accuracy during tracking or disturbance rejection of varying signals with frequencies belonging to some working frequency band $[0, \omega_{mx}]$. Similarly as in the case of regulation systems, in the case of nonminimum phase systems we have limited possibility of shaping transient response, which however should be acceptable.

Choosing the model $G_1(s)$ we should take into account the fact that in the proposed system with the parallel compensator, the replacement plant has the model $G_1(s)$ and to this plant the regulator should be designed. Therefore the model $G_1(s)$ should be minimum phase and it is recommended that the relative degree of the rational TF $G_1(s)$ is equal to one, since for this kind of the replacement plant even the proportional (P) regulator with high gain k gives usually a satisfactory solution. For the stable and minimum phase model $G_1(s)$, this is caused by the fact that the corresponding frequency response $kG_1(j\omega)$ lies usually in the first and second positive and/or negative quadrants of the Nyquist plane, sufficiently far from the critical point $(-1, j0)$, which assures stability.

Additional demand is that the frequency responses of $G(j\omega)$ and $G_1(j\omega)$ in the working frequency band $[0, \omega_{mx}]$ should be close one to other or

$$G_1(j\omega) \approx G(j\omega) \quad \text{for} \quad \omega \in [0, \omega_{\text{mx}}] \tag{8}$$

Thus the process of design contains the following steps

1) choose the rational TF $G_1(s)$ with relative degree equal to one;
2) find the values of the coefficients of polynomials in the numerator and denominator of $G_1(s)$ so that the approximation (8) is fulfilled in some interval $[0, \omega_{mx}]$;
3) design the regulator for which the CL system has a satisfactory stability degree.

Different algorithms may be proposed for solving the problem of the coefficients finding in step 2. Some formal algorithm will be described in the next section. In the following Example a simple method based on Nyquist plots and fundamental knowledge is used.

A. Example 2

Consider the plant described by the TF (7) which has the poles $p_1 = -3$, $p_{23} = -0.5 \pm j0.866$ and one nonminimum phase zero $z_1 = 0.6667$. We would like to design CL system with parallel compensator, which for some frequency band $[0, \omega_{mx}]$ tracks varying set point and rejects varying disturbance.

First, we decide to use the second order (simpler) model $G_1(s)$ in the form

$$G_1(s) = \frac{b_1 s + 1}{a_0 s^2 + a_1 s + 1} \tag{9}$$

for which $G_1(0) = G(0) = 1$.

Second, the first approach for coefficients a_0, a_1 is obtained from assumption that the faster mode of TF (7), related with pole p_1 is neglected and the remaining slower modes related with poles p_{23} are retained. In this manner we obtain denominator of (9) in the form $s^2 + s + 1$, which has the roots p_{23} i.e. $a_0 = 1,\ a_1 = 1$ and we choose a rather small b_1, say $b_1 = 0.25$.

Third, using MATLAB command $\texttt{nyquist}(G, G_1)$, the plots of the frequency responses $G(j\omega)$ and $G_1(j\omega)$ are obtained on Nyquist plane. By means of successive trials the coefficients a_0, a_1, b_1 are changed in this manner that visually initial segments of $G(j\omega)$ and $G_1(j\omega)$ are almost cover each other. Here ω_{mx} is the maximal frequency on $G(j\omega)$ for which this cover occurs.

Forth, to suite the frequencies in the initial segment for both the plots, we take the readings of the highest frequencies ω_{mx} and ω'_{mx} on both the plots of $G(j\omega)$ and $G_1(j\omega)$, respectively , for which the corresponding points $G_1(j\omega_{mx})$ and $G_1(j\omega'_{mx})$ are almost cover. We obtain $\omega_{mx} = 0.304$ and $\omega'_{mx} = 0.231$.

Fifth, to change the scale of frequency on the plot of $G_1(j\omega)$ the new coefficients a'_0, a'_1, b'_1 are calculated in accordance with the following rules

$$c = \omega_{mx}/\omega'_{mx} = 0.304/0.231$$

$$a'_0 = a_0/c^2, \quad a'_1 = a_1/c \quad b'_1 = b_1/c$$

In this manner we have obtained the TF (9) with the following coefficients

$$a'_0 = 2.5928, \quad a'_1 = 1.8977 \quad b'_1 = 0.1139 \tag{10}$$

The Nyquist plots of $G(jw)$ and $G_1(s)$ with parameters (10) are shown on Fig. 3 where $\omega_{mx} = 0.304\ rad/sec$.

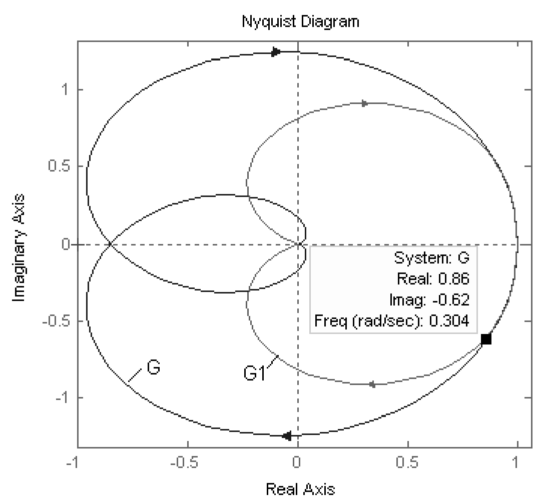

Fig. 3. Nyquist plots of $G(j\omega)$ and $G_1(j\omega)$ for Example 2.

On Fig. 4 the time response y of the CL system for sinusoidal set point $w = \sin(0.304t) \cdot \mathbf{1}(t)$ is compared with w. It is shown that beyond the initial transient period both the plots are almost the same. For sinusoidal set point w with higher frequency it appears a phase shift and a difference in the amplitudes of the signals w and y. It becomes that the system with $G_1(s)$ (9) and parameters (10) is more sensitive to plant parameters changes. For instance it is stable for k_p from 0.7 to 1.1.

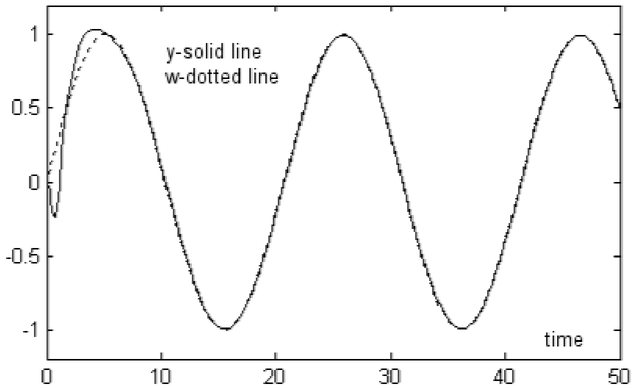

Fig. 4. Plots of y and w for Example 2.

V. DETERMINATION OF $G_1(s)$ IN GENERAL CASE

Assume that the model $G_1(s)$ has the form

$$G_1(s) = \frac{b_1 s^{p-1} + b_2 s^{p-2} + ... + b_p}{s^p + a_1 s^{p-1} + ... + a_p} \quad (11)$$

and is stable and minimum phase. We would like to find the coefficients a_i, b_i, $i = 1, 2, ..., p$, $p \le n$, for which the frequency response $G_1(j\omega)$ approximates $G(j\omega)$ in some interval $[0, \omega_{mx}]$ and $G_1(0) = G(0)$.

To formulate more precisely, let

$$G_1(j\omega) = ReG_1(j\omega) + jImG_1(j\omega)$$
$$G(j\omega) = ReG(j\omega) + jImG(j\omega)$$

where Re and Im denote real and imaginary part of appropriate frequency response. Then

$$d(\omega) = ||G_1(j\omega) - G(j\omega)|| =$$
$$\sqrt{[ReG_1(j\omega) - ReG(j\omega)]^2 + [ImG_1(j\omega) - ImG(j\omega)]^2}$$

(where $||\cdot||$ is Euclidean norm) denotes the distance between the corresponding points of both the responses $G_1(j\omega)$ and $G(j\omega)$. Of course it should be

$$d(\omega) \le \Delta \quad \text{for} \quad \omega \in [0, \omega_{mx}]$$

where Δ is an assumed small number determining the accuracy of approximation (say $\Delta = 0.01$, or 0.05).

Now, assume some value for ω_{mx} and denote by

$$\omega_i = \frac{i}{2\pi}\omega_{mx} \quad i = 1, 2, ..., 2p \quad (12)$$

the successive, equally distributed frequencies from interval $[0, \omega_{mx}]$.

Denote by Ω the set of admissible coefficients a_i, b_i, $i = 1, 2, ..., p$, for which the polynomials appearing in numerator and denominator of the TF (11) are stable and additionally $b_p = G(0)a_p$. The exact form of the set Ω may be determined for any function $G_1(s)$ using Routh-Hurwitz criterion of stability, applied for polynomials appearing in numerator and denominator of (11), together with dependence $b_p = G(0)a_p$.

Determine the measure of distance between the points of $G_1(j\omega)$ and $G(j\omega)$, corresponding to the same frequencies $\omega \in [0, \omega_{mx}]$ in the form

$$d = \max_i d(\omega_i), \quad i \in [1, 2, ..., 2p]. \quad (13)$$

To calculate the values of the coefficients we may use the following

Algorithm

1) Choose the value ω_{mx} and determine ω_i using (12).
2) Calculate the coefficients a_i, b_i, $i = 1, 2, ..., p$ minimizing
$$d_{min} = \min_{\Omega} d$$
3) If $\Delta - d_{min} \ge 0.1\Delta$ then increase ω_{mx} and repeat the points 1) and 2) of the algorithm. If $0 < \Delta - d_{min} \le 0.1\Delta$ then – end.
4) If $\Delta - d_{min} < 0$ then decrease ω_{mx} and repeat the points 1) and 2) of the algorithm.

As the result of applying the above algorithm we obtain ω_{mx} and the coefficients a_i, b_i, $i = 1, 2, ..., p$ for given $G(s)$ and assumed $G_1(s)$ and Δ.

For solving the optimization problem formulated in point 2) any non-gradient algorithm (e.g. genetic algorithm) may be used.

VI. RELAY IMPLEMENTATION

It is known, that if the plant is nonminimum phase then it is impossible to implement a relay control, such as on–off control or sliding mode control, assuring appropriate accuracy. One possibility of implementing such a control is applying the appropriate parallel compensator. The block diagram of the relay control with parallel compensator and the characteristic of the relay is shown on Fig. 5.

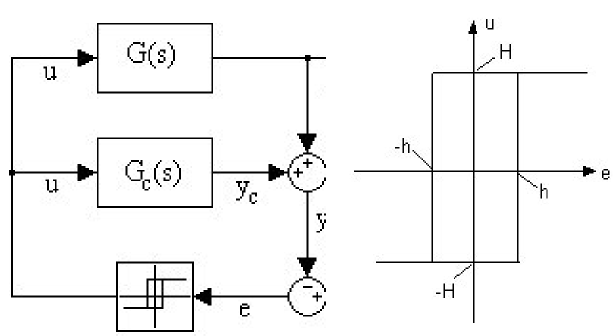

Fig. 5. CL system with parallel compensator and relay implementation; characteristic of the relay.

As in the continuous–time implementation also now, the critical point is the choice of the model $G_1(s)$ which together with $G(s)$ determines the parallel compensator (2). Dependently upon the goal of the control, as it is described in section II, the recommendations for the choice of $G_1(s)$ are the same as those for the continuous–time control, described in section III and IV.

It is worthwhile to note that the discussed relay implementation, when the TF $G_1(s)$ has relative order equal to one, may be treated as some modification of sliding mode control. Really in sliding mode control to obtain fast switching of the relay and to determine so called sliding surface the appropriate derivatives of the error signal are used [4]. In the proposed solution the fast switching of the relay is obtained owing to the fact that the relative order of the TF $G_1(s)$ is equal to one. It is worthwhile to stress that in this solution the derivatives of the error signal with usually gain noises are not utilized at all.

Below the results of simulations for two examples are given.

A. Example 3

Consider the nonminimum phase, stable plant described by the TF (7). Assume that the goal of the control is regulation of the plant output on a constant value determined by the set point w. A stepwise change of the set point (or output disturbance) may occur. The relay control with parallel compensator is used, as shown on Fig. 5.

Since the plant and the goal of control is the same as in Example 1 the same parallel compensator with

$$G_c(s) = \frac{1}{0.5s+1} - \frac{-2s+3}{s^3+4s^2+4s+3} \tag{14}$$

may be used.

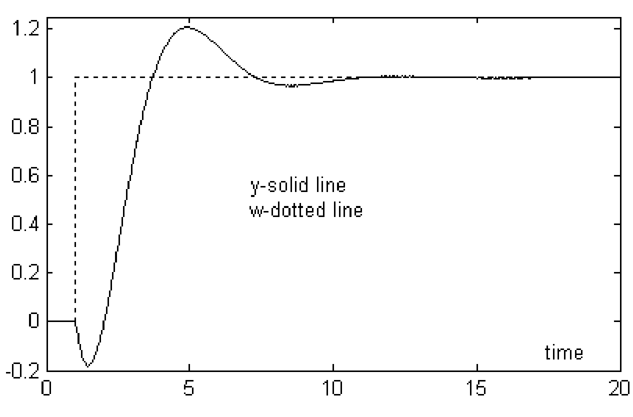

Fig. 6. Plots of y and w for Example 3.

In Fig. 6 the step response y of the CL system, with relay parameters $h = 0.2,\ H = 6$ and parallel compensator (14), to the set point $w = \mathbf{1}(t)$ is shown. It was obtained from simulations performed in SIMULINK ver 4.0. It is shown that the step response is almost the same as that for the continuous–time system, considered in Example 1 and shown on Fig. 2. Only the steady state error is smaller, now. The relay parameters h and H influence insignificantly the step response. Increase of h even to 0.4 or decrease of H even to 3 gives almost the same step response. Increase of the plant gain to $k_p = 2$ (without change of the parallel compensator) gives similar effect as in the continuous–time system considered in Example 1.

B. Example 4

For the same nonminimum phase plant (7) assume now, that the goal of the control is tracking and disturbance rejection for signal with some working frequency band $[0, \omega_m]$. The relay control with parallel compensator is used, as shown in Fig. 5. Since the plant and the goal of control is the same as in Example 2 the same parallel compensator with

$$\begin{aligned} G_c(s) = {} & \frac{0.1139s+1}{2.5928s^2+1.8977s+1} - \\ & - \frac{-2s+3}{s^3+4s^2+4s+3} \end{aligned} \tag{15}$$

may be used.

In fig. 7 the time response y of the CL system with relay parameters $h = 0.01,\ H = 6$ and parallel compensator (10) to set point $w = \sin(0.304t) \cdot \mathbf{1}(t)$, obtained from simulations, is shown. The hysteresis parameter h is now 20–times smaller than that in Example 3 to avoid switching oscillations in the output y. This is caused by the fact that the slope of the step response of $G_1(s)$ determined by $0.1139/2.5928 = 0.0439$, which for given h decides about frequency of switching is now $2/0.0439 = 45.5581$ – times smaller than that of $G_1(s)$ from Example 3. Decrease of h and increase of H influence the time response, insignificantly. Similarly as in the continuous–time control from Example 2 the system now is more sensitive to plant parameter changes than in Example 3. For instance the system is stable (neglecting switching oscillations not shown in the output y) for k_p from 0.7 to 1.17 without change of compensator (14) parameters.

From the performed simulations it results that there is great similarity in the properties of the system with continuous–time and relay implementation.

VII. CONCLUSIONS

Design of regulators assuring appropriate accuracy for nonminimum phase plants meets great difficulties. This is caused by the fact that usually insignificant increase of the proportional regulator gain causes instability and small gain causes law accuracy even in a constant steady state. If the integral part is introduced in regulator, to reduce the steady state error, then its gain is also very limited giving very slow transients.

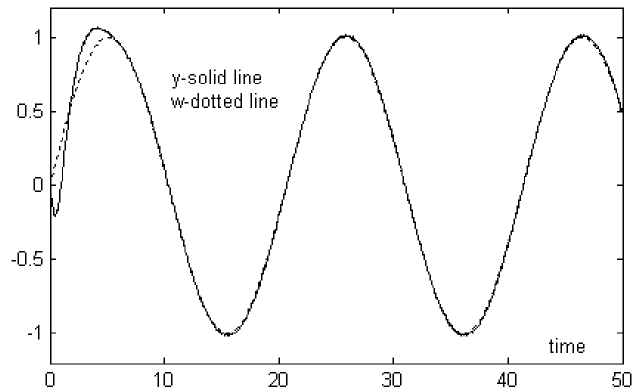

Fig. 7. Plots of y and w for Example 4.

In the present paper, following the Smith compensator [5] we propose for nonminimum phase plants the compensator which connected in parallel to the plant changes its model which becomes minimum phase. For the changed replacement plant model it is easy to design regulator with high gain which assures appropriate accuracy. The kind of the changed model depends upon our choice and the goal of the control.

If the main goal of the control is the accuracy of regulation in steady state under stepwise excitations, then the changed model may take the form of a first order lag with the gain equal to that of the plant. The time constant of this model has also a limited influence on under– and over–shot of the step response.

If the main goal of the control is tracking or disturbance rejection of signals with frequencies belonging to some working frequency band, then the changed model in the form of rational transfer function with relative order equal to one, should be chosen in this manner that it is minimum phase and in the working frequency band its frequency response is approximately the same as that of the plant.

Especially in the case of regulation the proposed system structure is very robust since the frequency response of the replacement plant model lies in the first negative quadrant of the Nyquist plane (first order lag). In the case of tracking or disturbance rejection the demand of closing the frequency response of the replacement plant to that of the plant causes some decrease of robustness, since the frequency response of the replacement plant may lay now in the first and second negative quadrants of Nyquist plane (closer to the critical point $(-1, j0)$).

To the replacement plant the relay implementation of the control may be applied; it has similar properties as a continuous–time one, which results from performed simulations.

It seems that the described idea of parallel compensator may be also used for other difficult plant improving accuracy at least in steady state and also robustness of the control.

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