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Regulator Constrained Control and Rate Problem for Linear Systems with Additive Disturbances

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Abstract— **This communication is devoted to the control of linear systems with constrained control and rate with additive disturbances. Necessary and sufficient conditions such that the system evolution respects rate constraints are used to derive stabilizing feedback control. The control law respects both constraints on control and its rate and is robust against additive bounded disturbances. An application to a surface mount robot, where the Y-axis of the machine uses a typical ball screw transmission driven by a DC motor to position circuits boards is achieved.**

Keywords: Invariance, Pole assignment, DC Motor control.

I. INTRODUCTION

Usually, real or physical plants are subject to constrained variables. The most frequent constraints are of saturation type: limitations on the magnitude of certain variables. Hence, this topic is of continuing interest and one could find many approaches to study this problem. Not exhaustively, the positive invariance concept [3], [4]; [5]; [6]; and the references Therein), the l_1 optimization concept [8], and so on ... can be cited. Other type of rate or incremental constraints were introduced while considering predictive control and practical applications [7], [13]. In fact, for some processes, the rate of variables change is limited within given bounds. These limits can arise from physical limits or from linearization approximations that, if exceeded, could damage the process or destroy limits of linear model validity. Most synthesis methods are based on symmetric constraints. However, in most real-life applications the constraints are not symmetric, from economical and safety reasons. For example, in process control applications, the nominal working point of the valves is usually near the upper or lower limits; in mechanical systems the maximum acceleration is usually smaller than the maximum deceleration (for safety reasons). In a previous work the problem of non-symmetrical constrained control and increment or rate is successfully addressed [11]. Henceforth, this work extends these results to the case of non-symmetrical constrained perturbed systems and shows its applicability on a real process, namely, a surface mount robot. Necessary and sufficient conditions of positive invariance for incremental domains with respect to (w.r.t.) autonomous perturbed systems are then derived. Further, a link is done between a pole assignment procedure and these conditions to find stabilizing linear state feedback controllers respecting both non-symmetrical constraints on control and rate with additive disturbances. In the other hand, for the application, surface mount robots are of great interest in the modern industry. This interest is justified by the presence of positioning systems in practically all industrial applications. In the prolific literature many works about these systems classically controlled by motors can be found, [9] and references there in,. Hence, stabilizing state feedback to an assembly machine for mounting electronic components is derived. The Y-axis of the machine uses a ball screw transmission driven by a current controlled DC motor that have constrained control and rate with additive disturbances.

A. Notations:

If $x \in \Re^n$ is a vector, $\dot{x}(.)$ denotes its derivative w.r.t. time. Further, for a scalar $a \in \Re$ we define $a^+ = \sup(a, 0)$ and $a^- = \sup(-a, 0)$, we note then $x^+ = (x_j^+)$ and $x^- = (x_j^-)$ for $j = 1, ..., n$. Furthermore, for a matrix $A = (a)_{ij}$ $i, j = 1, ..., n$, the Tilde transforms are defined by

$$\widetilde{A} = \begin{bmatrix} A^+ & A^- \\ A^- & A^+ \end{bmatrix},$$

where $A^+ = (a^+)_{ij}$ and $A^- = (a^-)_{ij}, i, j = 1, ..., n$ and

$$\widetilde{A_c} \;=\; \begin{bmatrix} A_1 & A_2 \\ A_2 & A_1 \end{bmatrix}$$

where $A_1 = \begin{cases} a_{ii} \\ a_{ij}^+ \text{ for } i \neq j \end{cases}$ and $A_2 = \begin{cases} 0 \\ a_{ij}^- \text{ for } i \neq j \end{cases}$

Also, $\sigma(A)$ denotes the spectrum of matrix A; $I\!D_s$ denotes the stability domain for eigenvalues (that is, the left complex half plane).

II. PROBLEM STATEMENT

Consider the linear continuous-time invariant system :

$$\dot{x}(t) = A\ x(t)\ +\ B\ u(t)\ +\ E\ p(t) \tag{1}$$

where $x(t) \in \Re^n$ is the state of the system, $u(t) \in \Re^m$ is the input constrained to evolve in the following domain

$$D_u = \{u(t) \in \Re^m, -u_{\min} \leq u(t) \leq u_{\max} \\ u_{\min}, u_{\max} \in Int\Re_+^m\}. \tag{2}$$

The control rate is constrained as follows:

$$-\Delta_{\min} \quad \leq \dot{u}(t) \leq \quad \Delta_{\max} \tag{3}$$

$p(t)$ is an additive bounded disturbance as

$$-p_{min} \leq p(t) \leq p_{max} \tag{4}$$

Further, We denote

$$U = \begin{bmatrix} u_{\max} \\ u_{\min} \end{bmatrix}, \quad \Delta = \begin{bmatrix} \Delta_{\max} \\ \Delta_{\min} \end{bmatrix}, \quad P = \begin{bmatrix} p_{\max} \\ p_{\min} \end{bmatrix}$$

the problem is to find linear stabilizing state feedback as

$$u(t) = F\ x(t), \quad F \in \Re^{mxn} \tag{5}$$

ensuring closed-loop asymptotic stability of the system despite perturbations with non violation of rate and control non-symmetrical constraints.

III. PRELIMINARY RESULTS

The solution of the previously stated problem, without additive disturbances, is achieved in [11]. This paper presents the the extension of this problem to perturbed systems. Consider the linear time invariant perturbed autonomous system

$$\dot{z}(t) \;=\; H\ z(t)\ +\ E\ p(t),\ \ z(t_o) = z_o \tag{6}$$

where $z \in \Re^m$ is the state constrained to evolve in

$$D_z = \{z \in \Re^m, -z_{\min} \leq z(t) \leq z_{\max} \\ z_{\min}, z_{\max} \in Int\Re_+^m\} \tag{7}$$

and $p(t)$ is the perturbation bounded in the domain

$$D_P = \{p(t) \in \Re^p, -p_{\min} \leq p(t) \leq p_{\max} \\ p_{\min}, p_{\max} \in Int\Re_+^p\} \tag{8}$$

Consider also that the rate is constrained as follows:

$$-\Delta_{\min} \quad \leq \dot{z}(t) \leq \quad \Delta_{\max} \tag{9}$$

First, recall the definition of D_P-positive invariance of domain D_z which is very useful for the sequel.

Definition 1: Domain D_z given by (7) is D_P-positively invariant w.r.t. motion of system (6) if for all initial condition $z_o \in D_z$, the trajectory of the system $z(t, t_o, z_o) \in D_z$ for all $p(t) \in D_P,\ \ t > t_o$

Let us now extend to the case of perturbed systems a technical lemma that is established in [11]. Relating that result to a pole assignment procedure enables to find stabilizing controllers for systems with non-symmetrical constrained control and rate.

Lemma 2: The evolution of the autonomous system (6) respects rate constraints iff matrix H satisfies:

$$\widetilde{H}\ \ Z\ + \widetilde{E}\ P\ \ \leq \Delta \tag{10}$$

where

$$Z = \begin{bmatrix} z_{\max} \\ z_{\min} \end{bmatrix}, \quad \Delta = \begin{bmatrix} \Delta_{\max} \\ \Delta_{\min} \end{bmatrix}, \quad P = \begin{bmatrix} p_{\max} \\ p_{\min} \end{bmatrix}.$$

Proof: <u>If part</u> Assume that condition (10) is satisfied. Then,

$$-z_{\min} \leq z(k) \leq z_{\max} \tag{11}$$

next, decompose matrix $H = H^+ - H^-$, pre-multiplying (11) by H^+ and $-H^-$, gives

$$-H^+ z_{\min} \leq H^+ z(t) \leq H^+ z_{\max} \tag{12}$$

$$-H^- z_{\max} \leq -H^- z(t) \leq H^- z_{\min} \tag{13}$$

Further, the perturbation is bounded :

$$-p_{\min} \leq p(t) \leq p_{\max} \tag{14}$$

the same reasoning with matrix E and the perturbation $p(t)$ leads to the following inequalities

$$-E^+ p_{\min} \leq E^+ p(t) \leq E^+ p_{\max} \tag{15}$$

$$-E^- p_{\max} \leq -E^- p(t) \leq E^- p_{\min} \tag{16}$$

addition of the obtained inequalities enables to write the following:

$$-E^+ p_{\min} - E^- p_{\max} - H^+ z_{\min} - H^- z_{\max} \leq Hz(t) + Ep(t)$$

and

$$Hz(t) + Ep(t) \leq H^+ z_{\max} + H^- z_{\min} + E^+ p_{\max}\ +\ E^- p_{\min}$$

according to condition (10) this is is equivalent to

$$-\Delta_{\min} \leq \dot{z}(t) \leq \Delta_{\max}.$$

<u>*Only if part*</u>, Now, assume that the derivative of $z(t)$ respects the constraints, and condition (10) is not satisfied for an index $1 \leq i \leq n$ such that

$$[\widetilde{H}\ Z]_i + [\widetilde{E}\ P]_i > \Delta_i \tag{17}$$

expanding (17) :

$$[H^+ z_{\max}\ +\ H^- z_{\min}]_i\ +\ [E^+ p_{\max}\ +\ E^- p_{\min}]_i > \Delta_{\max}^i$$

Then, the following state vector for the system can be selected

$$\phi(t) = \begin{cases} z^j_{\max} \text{ if } h_{ij} > 0 \\ 0 \text{ if } h_{ij} = 0 \\ -z^j_{\min} \text{ if } h_{ij} < 0 \end{cases}, j = 1, ..., n$$

It is easy to check that $\phi(t)$ is an admissible state for the system. Further the following admissible perturbation may occur :

$$\kappa(t) = \begin{cases} p^j_{\max} \text{ if } e_{ij} > 0 \\ 0 \text{ if } e_{ij} = 0 \\ -p^j_{\min} \text{ if } e_{ij} < 0 \end{cases}, j = 1, ..., p$$

Calculation of the i^{th} component of the derivative of this state gives

$$\begin{aligned} [\tfrac{d}{dt}\phi(t)]_i &= [H\,\phi(t) \ + E\,p(t)]_i \\ &= \sum_{j=1}^{n} h_{ij}\phi_j(t) + \sum_{j=1}^{p} e_{ij}\kappa_j(t) \\ &= [H^+ z_{\max} + H^- z_{\min} + E^+ p_{max} + E^- p_{min}]_i \end{aligned}$$

taking into account inequality (17), it is possible to write

$$[\frac{d}{dt}\phi(t)]_i > \Delta^i_{\max}$$

which contradicts the assumption. ■

Remark 3: : As matrix $\widetilde{H}$ and vector Z have positive components, inequality (10) can never be satisfied if the difference $(\Delta - \widetilde{E}P)$ is negative. Hence, the interesting conclusion is that inequality (10) permits to compute the maximum perturbation set that can be admissible with these rate constraints that is the set given by :

$$D^{max}_P = \{p(t) \in \Re^p, -p_{\min} \leq p(t) \leq p_{\max} \,/ \widetilde{E}\ P = \Delta\} \quad (18)$$

Since, matrix E and vector Δ are known from the statement of the problem, it can be concluded if such rate constraints requirement can be fulfilled or not and if it is admissible or not.

Evolution of the autonomous system (6) will respect both constraints on the state $z(t)$ and constraints on its rate if domain D_z given by (7) is D_P-positively invariant and conditions given in the previous lemma are satisfied. D_P-positive invariance condition has already been proposed for the continuous time case in [10]. Let us recall it hereafter :

Theorem 4: [10] Domain D_z is D_P-posistvely invariant w.r.t. motion of the perturbed system (6) iff matrix H satisfies :

$$\widetilde{H_c}\ Z \ + \ \widetilde{E}\ P \ \leq \ 0 \quad (19)$$

Now, it is possible to derive the following :

Lemma 5: Domain (7) is D_P-positively invariant w.r.t. motion of system (6) and rate constraints (9) are respected if and only if

$$\begin{cases} \widetilde{H}\ Z \ + \ \widetilde{E}\ P \leq \ \Delta \\ \widetilde{H}_c Z \ + \ \widetilde{E}\ P \leq \ 0 \end{cases}$$

Proof: For the rate constraints, conditions can be derived from the previous lemma 2, and the D_P-positive invariance conditions are recalled in theorem above [10]. ■

Relating the previous lemma 5 to the so called inverse procedure a pole assignment procedure [1] makes possible to solve the problem stated above. Consider the time invariant system given by (1) and without loss of generality (see Remark below), assume that matrix A has $(n - m)$ stable eigenvalues. Resolution of equation

$$X\ A \ + \ X\ B\ X \ = \ H\ X \quad (20)$$

gives a state feedback assigning spectrum of matrix H $(\sigma(H) \subset I\!\!D_s)$ together with the stable part of spectrum of matrix A in closed loop. For this equation to have a valid solution, matrix H must satisfy :

$$\begin{cases} \sigma(H) \cap \sigma(A) = \emptyset \\ B\zeta_i \neq 0, \ i = 1, ..., m \\ \zeta_i, i = 1, ..., m \text{ are linearly independent} \end{cases} \quad (21)$$

for ζ_i eigenvectors of matrix H. There exits a unique solution to equation (20) if and only if

$$\{\ \chi_1 \ \ \dots \ \ \chi_n \ \} \text{ are linearly independent}$$

where $\chi_i, i = m + 1, ..., n$ are eigenvectors associated to stable eigenvalues of matrix A, and $\chi_i, i = 1, ..., m$ are computed by

$$\chi_i = (\lambda_i I_n - A)^{-1} B\zeta_i, \ i = 1, ..., m$$

Hence, the solution is given by:

$$F = [\ \zeta_1 \ ... \ \zeta_m \ 0 \ ... \ 0 \ ][\ \chi_1 \ ... \ \chi_m \ \chi_{m+1} \ ... \ \chi_n \ ]^{-1} \quad (22)$$

Remark 6: Without loss of generality, it was assumed that the system possesses $(n - m)$ stable eigenvalues. Else, it is always possible to augment the representation as : let $v \in \Re$ be a vector of fictitious inputs such that

$$-v_{\min} \leq v \leq v_{\max} \quad -\Delta^v_{\min} \leq \delta v \leq \Delta^v_{\max}$$

where $v_{\min}$, $\Delta^v_{\min}$ and $v_{\max}$, $\Delta^v_{\max}$ are freely chosen constraints. In this case, vectors U and Δ become:

$$U = \begin{bmatrix} u_{max} \\ v_{max} \\ u_{min} \\ v_{min} \end{bmatrix}, \Delta = \begin{bmatrix} \Delta_{max} \\ \Delta^v_{max} \\ \Delta_{min} \\ \Delta^v_{min} \end{bmatrix}$$

The augmented system is then given by

$$\delta x(\cdot) = Ax(\cdot) + [\ B \quad 0\ ] \begin{bmatrix} u(\cdot) \\ v(\cdot) \end{bmatrix} \quad (23)$$

It is easy to see that for the obtained square system the problem of $(n - m)$ stable eigenvalues is eliminated and controllability is not changed.

IV. MAIN RESULTS

With this background, we are now able to solve the problem stated in Section 2. Consider the linear time invariant stabilizable perturbed system with constraints on both control and rate of the control (1)-(4). Using the state feedback

$$u(t) = F\,x(t),\; F \in \Re^{mxn},\; \sigma(A + B\,F) \in \mathbb{D}_s \qquad (24)$$

induces the following domain of linear behaviour in the state space

$$D_F = \{x \in \Re^n, -u_{\min} \leq Fx \leq u_{\max} \\ u_{\min}, u_{\max} \in Int\Re^m_+\} \qquad (25)$$

If the state does not leave the domain (25), the control signal does not violate the constraints. That is, the set D_F is D_P-positively invariant w.r.t. motion of system (1). This gives the following result:

Proposition 7: Perturbed System (1) with state feedback (24) is asymptotically stable at the origin from all initial state $x_o \in D_F$ with respected constraints on both the control and its rate if there exists a matrix $H \in \Re^{mxm}$ satisfying conditions (21) such that:

$$\text{i)}\;\; F\,A + F\,B\,F = H\,F$$

$$\text{ii)}\;\; \begin{cases} \widetilde{H}\,U + \widetilde{F\,E}\,P \leq \Delta \\ \widetilde{H}_c U + (\widetilde{F\,E})\;P \leq 0 \end{cases} . \qquad (26)$$

Proof: Introduce the following change of coordinates $z = F\,x$, it is possible to write

$$\begin{aligned} \dot{z}(t) &= F\dot{x}(t) \\ &= F(A + B\,F)\,x(t) + F\,E\,p(t) \\ &= H\,F\,x(t) + F\,Ep(t) \\ &= H\,z(t) + F\,E\,p(t) \end{aligned} \qquad (27)$$

With this transformation, domain D_F is transformed to domain D_z given by (7). Further, with conditions (26), it is easy to note that domain D_z is D_P-positively invariant w.r.t. the system (27) while the constraints on the increment of the control are respected. Bearing in mind that $\sigma(A + BF) \in \mathbb{D}_s$ and that the linear behaviour is guaranteed, it is possible to conclude to the asymptotic stability of the closed-loop system. ■

Steps to follow for design of such controllers are proposed in the algorithm below :

Algorithm:

Step 1. Check if matrix A has $(n-m)$ stable eigenvalues, else augment system (see Remark 6).

Step 2. Choose matrix $H \in \Re^{mxm}$ or, $H \in \Re^{nxn}$ if the system is augmented, according to (21) and such that

$$\widetilde{H}_c U \leq -\epsilon\,U. \quad \epsilon > 0 \qquad (28)$$

Step 3. Compute the gain matrix F or F_a by using (22)

Step 4. if condition (26) is satisfied continue, else return to Step 1 and change matrix H.

Step 5. Use F or extracted F from the first m rows of F_a for the control.

Comments 8: It is true that the set of positive invariance in this case is not the absolute maximal but it is maximal with respect to the chosen feedback F. In fact for a given matrix feedback F, the maximal set where no violation of control constraints may occur is the set D_F as proposed above. Nevertheless, piece-wise techniques [2] or maximization procedure for the set of positive invariance [12] can be used to overcome this difficulty.

Remark 9: The introduction of the variable ϵ can be seen as an optimization parameter that can be used to choose matrix H. Further, its introduction in this case makes more possibilities as a margin to find a matrix H fulfilling all the required conditions.

A. the maximal disturbance set

As stated in remark 3, the maximal disturbance set such that asymptotic stability of the closed loop system with no violation of constraints on both rate and control can be estimated. Consider the perturbed system

$$\dot{x}(t) = A\,x(t) + B\,u(t) + E\,p(t) \qquad (29)$$

stabilized by state feedback (24). The maximal disturbance allowed set can be estimated :

Corollary 10: The maximal disturbance set such that closed loop asymptotic stability, rate and control constraints are not violated is given by :

$$D_P^{max} = \{p(t) \in \Re^p / -p_{min}^{max} \leq p(t) \leq p_{max}^{max}\} \qquad (30)$$

where vector P^{max} satisfies :

$$\left\{ \widetilde{F\,E}\,P^{max} = min(\Delta,\; -\widetilde{H}_c U) \right. \qquad (31)$$

the minimum here is taken component-wise.

Proof: Assume that we look to stabilize the system by state feedback with $p(t)$ a perturbation vector with unknown limits. From condition (26), one can write :

$$\begin{cases} \widetilde{F\,E}\,P \leq \Delta - \widetilde{H}\,U \\ \widetilde{F\,E}\,P \leq -\widetilde{H}_c U \end{cases}$$

since matrices $\widetilde{H}$ and $\widetilde{H}_c$ have special constructions and their respective spectra are involved in closed loop dynamics, one must have :

$$\widetilde{H}\,U \geq 0, \quad \widetilde{H}_c U \leq 0 \qquad (32)$$

which is equivalent to write

$$\widetilde{F\,E}\,P \leq \Delta \qquad (33)$$

$$\widetilde{F\,E}\,P \leq -\widetilde{H}_c U \qquad (34)$$

hence, the maximal disturbance set that can be seen as the limit of fulfillment of the previous two conditions. Further, for any vector $T \in \Re^p$ such that $T \leq P^{max}$, it is easy to check that condition (26) is satisfied. ■

V. APPLICATION TO A SURFACE MOUNT ROBOT

Application of the previous results to an assembly machine or a surface mount robot is considered. The Y-axis of the machine uses a ball screw transmission driven by a current controlled DC motor. The rotation of the DC motor is converted into a translation motion by ball screw. A positioning table attached to the ball nut carries different loads. The process is simplified as a two mass system :

$$J_m\ddot{\theta}_m + b_m\dot{\theta}_m = K_m i_m - T_f - T_l \quad (35)$$

$$T_l = k_t(\theta_m - x_l/p) \quad (36)$$

$$m_l\ddot{x}_l + b_l\dot{x}_l = T_l/p \quad (37)$$

where θ_m and x_l are the motor angle and table displacement respectively, i_m is the motor current, T_l is the load torque due to the torsion of ball screw, T_f is the friction torque, p is the screw pitch, J_m is the motor inertia plus ball screw inertia, b_m, b_l and K_m are respectively the damping coefficients and the constant torque, k_t is the stiffness, m_l is the equivalent mass of load, table and nut [9]. Converting the motor angle position to linear position and rewriting the model in the state space gives :

$$\dot{x(t)} = Ax(t) + Bu(t) - Bp(t) \quad (38)$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -9.67 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 7.35 \end{bmatrix} \quad (39)$$

and $x = [\ p\theta \ \ p\dot{\theta}\ ]^t$, $u(t) = i_m(t)$ and $p(t)$ is the lumped disturbance of the load torque, friction and other external disturbances. The current input of the DC motor, its rate and the perturbation are respectively constrained as ;

$$-4 \leq u(t) \leq 4, \ \ -80 \leq \dot{u}(t) \leq 80, \ \ -1 \leq p(t) \leq 1.$$

Comments 11: The origin of the rate constraints is that the limit variation of $i(t)$, the control current, is taken as a peak to peak value at a sampling time. As the system possesses 1 stable eigenvalue, it is not necessary to augment the system. Further, for this system, the constraints are symmetric from the original example, this fact simplify a number of conditions but we insist to present the theoretical non-symmetrical case for the seek of generality. Furthermore, the resulting closed loop can not be unstable as it is our goal to stabilize the system. Hence, assuming the perturbation is limited within the given set is always true although it is a function of the state of the system as presented.
Matrix H reduces to a scalar in this case, let us select $h = -\alpha$, where $\alpha \neq 0$, is any positive number which satisfies all the required conditions (21).

$$\widetilde{H}_c U \ = \ [-4\alpha \ \ -4\alpha\ ]^t \ \leq \ 0$$

It is clear that α here may be any positive number. Let us choose $\alpha = 12$. Solution of equation (20) leads to the following gain matrix F:

$$F = [-15.7878 \quad -1.6327]$$

At this step, one has to check that all required conditions (26) are fulfilled. In fact :

$$\widetilde{H}U \ + \ (\widetilde{-F B})P \leq \ \Delta$$

$$\begin{bmatrix} 4\alpha \\ 4\alpha \end{bmatrix} + \begin{bmatrix} 12p_{min} \\ 12p_{max} \end{bmatrix} \leq \begin{bmatrix} \Delta_{max} \\ \Delta_{min} \end{bmatrix}$$

that is, $4\alpha + 12p_{max} = 60 \leq 80$ which is satisfied. Then,

$$\widetilde{H}_c U \ + \ (\widetilde{-F B})P \leq \ 0$$

$$\begin{bmatrix} -4\alpha \\ -4\alpha \end{bmatrix} + \begin{bmatrix} 12p_{max} \\ 12p_{min} \end{bmatrix} = \begin{bmatrix} -36 \\ -36 \end{bmatrix} \leq 0.$$

Finally, all required conditions are fulfilled, asymptotic stability is obtained with the obtained state feedback. Simulation results are summarized in figures below.

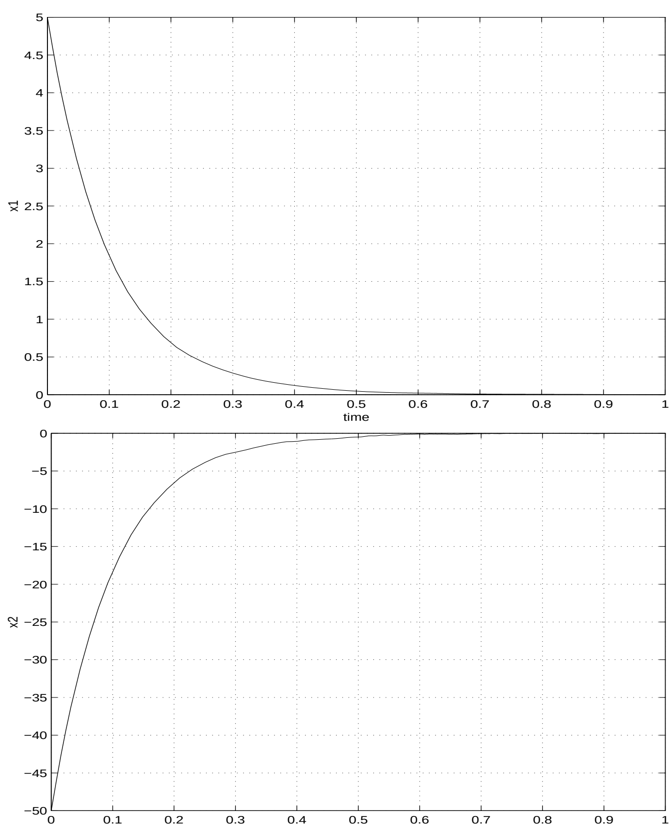

Figure 1 : States $x_1(t)$ and $x_2(t)$ motion from the initial condition $xo(t) = [5 \ \ -50]^t$.

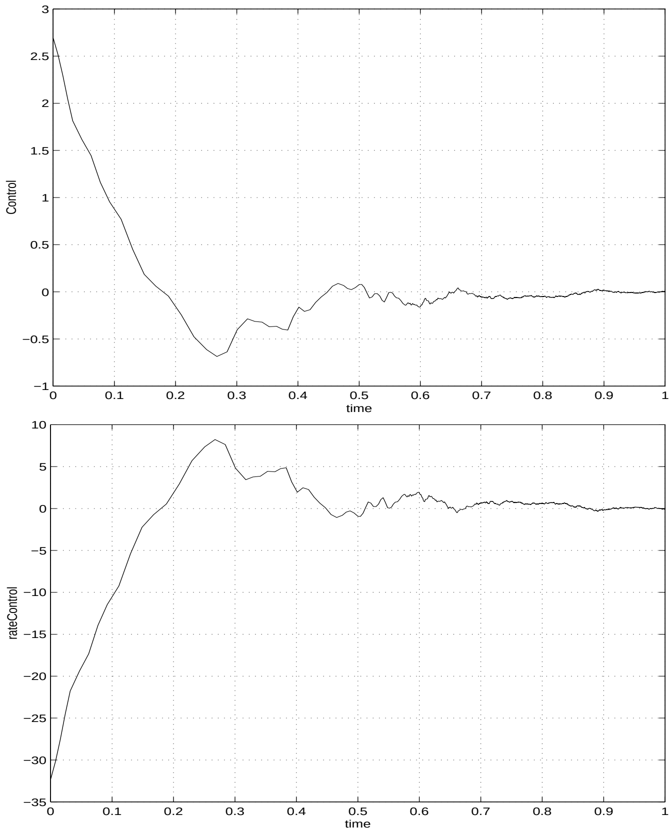

Figure 2 : input and rate's input evolution in time.

The maximal allowed disturbance set, which does not change stability and does not violate imposed constraints, in this case, can be easily deduced as follows :

$$(-\widetilde{F\ B})\ P^{max} \ = \ min(\Delta,\ -\widetilde{H_c}U) \qquad (40)$$

$$\text{simple calculation leads to,} P^{max} = \begin{bmatrix} 4 \\ 4 \end{bmatrix} \qquad (41)$$

VI. CONCLUSION

In this paper, the regulator problem for linear perturbed systems with non-symmetrical constrained control and rate is studied. Necessary and sufficient conditions, established for linear autonomous systems such that their motion respects rate constraints together with D_P-positive invariance, are the key to solve this problem. These conditions linked to the inverse procedure, a pole assignment method for constrained control, are the cornerstone of this work. In fact, this link enables to give a simple algorithm to compute a robust stabilizing state feedback respecting non-symmetrical constraints on both control and rate. The maximal disturbance, such that robust asymptotic stability, rate and control constraints are not destroyed is easily deduced. An application to a positioning table is successfully achieved. It is shown that the algorithm to find robust stabilizing controller is quite simple to apply.

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