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State and Input Estimation for Descriptor Systems with Unknown Inputs

Sun Liying and Cheng Zhaolin

Abstract— **The problem of inputs decoupled observer design for linear descriptor systems with unknown inputs was considered.When the system was impulsively controllable, an equivalent standard state space system was derived by regarding the partial state as unknown input.The sufficient conditions for the existence of the observer and simple method to design the observer are given.**

Index Terms — **unknown input,descriptor systems, state and input estimation, input-decoupled.**

I. INTRODUCTION

Recently, a great deal of work has been devoted to the observer design for standard systems and descriptor systems with unknown inputs[1], however few results have been presented to estimate unknown inputs. In [2] the observer contains output derivative of system, so it is essentially sensitive to noise. Although the observer don't contain output derivative of system, its output can not gradually estimate unknown inputs of system in [3]. In [4] observer for descriptor systems has been developed, however when the descriptor systems are not square, the work is less [5]. In this paper, we design an input-decoupled observer for descriptor systems that are not square with unknown inputs, and asymptotically estimate the state and unknown inputs of the descriptor systems using the state of observer and output of systems.

II. OBSERVER DESIGN AND ESTIMATION OF UNKNOWN INPUTS

Consider the descriptor system

$$\left\{\begin{array}{rcl} E\dot{x} & = & Ax + Bu + Dd \\ y & = & Cx + Fd \end{array}\right. \tag{1}$$

where $E, A \in R^{m\times n}, B \in R^{m\times k}, C \in R^{q\times n}, D \in R^{m\times p}, F \in R^{q\times p}$ are known constant matrices; x, y, u, d represent the state, output, control input and unknown input respectively; E is singular matrix with $0 < \text{rank}E = r < min\{m, n\}$. Without loss of generality,we assume $m < n$.

To design observer we assume that

(A1) rank $\begin{bmatrix} 0 & E & 0 \\ E & A & D \end{bmatrix} = m + r$

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L. Sun is with the School of Science, Jinan University, Jinan, 250022, P.R. China `sunliying66@beelink.com`

Z. Cheng is with the School of Mathematics and System Sciences, Shandong University, Jinan, 250100, P.R. China `Chengzha@jn-public.sd.cninfo.net`

(A2) rank $\begin{bmatrix} 0 & E & 0 \\ E & A & D \\ 0 & C & F \end{bmatrix} = n + p + r$

(A3) rank $\begin{bmatrix} A - sE & D \\ C & F \end{bmatrix} = n + p, \quad \forall s \in C, \text{Re}s \geq 0$

where C denotes complex domain.

These conditions are not restrictive. The assumption (A1) is equivalent to impulse controllable of the system (E, A, D). The assumption (A2) implies the unknown input dimension of the descriptor systems $[E, A, D, C]$ is less than its output dimension when $m < n$, if $m = n$, then the systems(1) becomes descriptor square systems,the assumption (A2) implies the output dimension of the descriptor systems is not less than its unknown input dimension[4]. The assumption (A3) is equivalent to that the invariant zeros of the system (E, A, D, C) are stable.

Without loss of generality,we assume (1) as

$$\left\{\begin{array}{rcl} \dot{x}_1 & = & A_1x_1 + A_2x_2 + B_1u + D_1d \\ 0 & = & A_3x_1 + A_4x_2 + B_2u + D_2d \\ y & = & C_1x_1 + C_2x_2 + Fd. \end{array}\right. \tag{2}$$

where $x_1 \in R^r$, $x_2 \in R^{n-r}$.

It is well known (A1) is equivalent to that $[A_4, D_2]$ has full row rank, so there exists nonsingular matrix P, such that $[A_4 \quad D_2]P = [I_{(m-r)\times(m-r)} \quad 0_{(m-r)\times(n-m)} \quad 0_{(m-r)\times p}]$. Denote $P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \end{bmatrix}$. Take the nonsingular transformation

$$\begin{bmatrix} x_1 \\ x_2 \\ d \\ u \end{bmatrix} = \begin{bmatrix} I_r & 0 & 0 & 0 & 0 \\ 0 & P_{11} & P_{12} & P_{13} & 0 \\ 0 & P_{21} & P_{22} & P_{23} & 0 \\ 0 & 0 & 0 & 0 & I_k \end{bmatrix} \begin{bmatrix} x_1 \\ \bar{x}_{21} \\ \bar{x}_{22} \\ \bar{d} \\ u \end{bmatrix} \tag{3}$$

then the system (2) is transformed into the system

$$\left\{\begin{array}{rcl} \dot{x}_1 & = & \bar{A}_1x_1 + \bar{B}_1u + W_1w \\ \bar{x}_{21} & = & -A_3x_1 - B_2u \\ y & = & \bar{C}_1x_1 + \bar{B}_2u + W_2w \end{array}\right. \tag{4}$$

where $\bar{x}_{21} \in R^{m-r}$, $\bar{x}_{22} \in R^{n-m}$, $\bar{A}_1 = A_1 - \bar{A}_{21}A_3$, $\bar{B}_1 = B_1 - \bar{A}_{21}B_2$, $\bar{C}_1 = C_1 - \bar{C}_{21}A_3$, $\bar{B}_2 = -\bar{C}_{21}B_2$, $W_1 = [\bar{A}_{22}, \ \bar{D}_1]$, $W_2 = [\bar{C}_{22}, \ \bar{F}]$, $w = \begin{bmatrix} \bar{x}_{22} \\ \bar{d} \end{bmatrix}$.

Remark 1: The system (4) is a standard state space system. So, the problem of observer design for descriptor system (1) with unknown inputs d is transformed into the same problem for standard state space system (4) with unknown inputs w.

Lemma 1: (A2) is equivalent to rank$W_2 = n - m + p$. So we can obtain

$$w = W_2^+(y - \bar{C}_1 x_1 - \bar{B}_2 u). \quad (5)$$

where W_2^+ is Penrose-Moore inverse of W_2.Then the system (4) is transformed into the system

$$\left\{ \begin{array}{rcl} \dot{x}_1 & = & \tilde{A}_1 x_1 + \tilde{B}_1 u + W_1 W_2^+ y \\ \bar{x}_{21} & = & -A_3 x_1 - B_2 u \\ \tilde{y} & = & \tilde{C}_1 x_1 + \tilde{B}_2 u \end{array} \right. \quad (6)$$

where $\tilde{A}_1 = \bar{A}_1 - W_1 W_2^+ \bar{C}_1,\ \tilde{B}_1 = \bar{B}_1 - W_1 W_2^+ \bar{B}_2,$ $\tilde{y} = (I_q - W_2 W_2^+)y,\ \tilde{C}_1 = (I_q - W_2 W_2^+)\bar{C}_1,\ \tilde{B}_2 = (I_q - W_2 W_2^+)\bar{B}_2.$

Remark 2: In the system (6), the state is x_1, inputs are u and y, output is $\tilde{y}$, u is control input, y and $\tilde{y}$ is measurable. So inputs and output of the system(6) are known, its observer exists if and only if the pair $[\tilde{A}_1,\ \tilde{C}_1]$ is detectable.

Lemma 2: Assume (A1)∼(A3) hold, then the pair $[\tilde{A}_1,\ \tilde{C}_1]$ is detectable.

Theorem 1: Assume (A1)∼(A3) hold, then the observer of system (6) exists. An observer of full order is given by

$$\left\{ \begin{array}{rcl} \dot{z} & = & (\tilde{A}_1 - L\tilde{C}_1)z + (\tilde{B}_1 - L\tilde{B}_2)u \\ & & +(W_1 W_2^+ + L(I_q - W_2 W_2^+))y \\ \hat{x}_1 & = & z \\ \hat{\bar{x}}_{21} & = & -A_3 z - B_2 u \end{array} \right.$$

where $\tilde{A}_1 - L\tilde{C}_1$ has eigenvalues with negative real parts.

So we have $\hat{w} = W_2^+ y - W_2^+ \bar{C}_1 \hat{x}_1 - W_2^+ \bar{B}_2 u,$ $\hat{\bar{x}}_{22} = [I_{n-m}\ \ 0]\hat{w},\ \ \hat{d} = [0\ \ I_p]\hat{w}.$

From (3), (4)and (5), we can obtain

$$\begin{bmatrix} x_2 \\ d \end{bmatrix} = R_1 x_1 + R_2 u + R_3 y$$

where

$$\left\{ \begin{array}{l} R_1 = \begin{bmatrix} -P_{11}A_3 - (P_{12}, P_{13})W_2^+ \bar{C}_1 \\ -P_{21}A_3 - (P_{22}, P_{23})W_2^+ \bar{C}_1 \end{bmatrix} \\ R_2 = \begin{bmatrix} -P_{11}B_2 - (P_{12}, P_{13})W_2^+ \bar{B}_2 \\ -P_{21}B_2 - (P_{22}, P_{23})W_2^+ \bar{B}_2 \end{bmatrix} \\ R_3 = \begin{bmatrix} (P_{12}, P_{13})W_2^+ \\ (P_{22}, P_{23})W_2^+ \end{bmatrix} \end{array} \right.$$

Theorem 2: Assume (A1)∼(A3) hold, then the input-state observer of system (1) is given by

$$\left\{ \begin{array}{rcl} \dot{z} & = & Hz + J_1 u + J_2 y, \qquad \mathrm{Re}\lambda(H) < 0 \\ \hat{x} & = & Kz + T_1 u + T_2 y \\ \hat{d} & = & Gz + Q_1 u + Q_2 y \end{array} \right.$$

where

$H = \tilde{A}_1 - L\tilde{C}_1,\ J_1 = \tilde{B}_1 - L\tilde{B}_2,$
$J_2 = W_1 W_2^+ + L(I_q - W_2 W_2^+),$
$K = N\begin{bmatrix} I_r \\ (I_{n-r},\ 0)R_1 \end{bmatrix},\ T_1 = N\begin{bmatrix} 0 \\ (I_{n-r},\ 0)R_2 \end{bmatrix},$
$T_2 = N\begin{bmatrix} 0 \\ (I_{n-r},\ 0)R_3 \end{bmatrix},\ G = [0,\ I_p]R_1,$
$Q_1 = [0,\ I_p]R_2,\quad Q_2 = [0,\ I_p]R_3.$

III. EXAMPLE

Consider the descriptor system

$$\left\{ \begin{array}{l} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ + \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} d \\ y = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 2 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} d \end{array} \right.$$

where $x_1 \in R^2,\quad x_2 \in R^2.$

It is simply known that (A1)∼(A3) are satisfied, by calculating, we have $W_2^+ = \begin{bmatrix} -\frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ 1 & 1 & 0 \end{bmatrix}$, so the system is transformed into

$$\left\{ \begin{array}{l} \dot{x}_1 = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{4}{3} & \frac{2}{3} \end{bmatrix} x_1 + \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} u \\ \qquad + \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & \frac{2}{3} \end{bmatrix} y \\ \bar{x}_{21} = -[0\ \ 2]x_1 - [0\ \ 1]u \\ \tilde{y} = \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & -\frac{2}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} x_1 \end{array} \right.$$

Note that, in this example, the pair $[\tilde{A}_1,\ \tilde{C}_1]$ is detectable, we can choose matrix $L = \begin{bmatrix} 0 & 6 & 1 \\ 6 & 2 & 0 \end{bmatrix}$ such that $\tilde{A}_1 - L\tilde{C}_1$ has eigenvalues -1 and -2. So the input-state observer of the system is:

$$\dot{z} = \begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix} z + \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} u + \begin{bmatrix} -1 & 2 & -2 \\ 1 & -1 & 2 \end{bmatrix} y$$

$$\hat{x} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -2 & 1 \\ -\frac{4}{3} & -\frac{1}{3} \end{bmatrix} z + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} u + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \\ -\frac{1}{3} & \frac{1}{3} & \frac{2}{3} \end{bmatrix} y$$

$$\hat{d} = \begin{bmatrix} \frac{5}{3} & \frac{4}{3} \end{bmatrix} z + \begin{bmatrix} 0 & -1 \end{bmatrix} u + \begin{bmatrix} -\frac{1}{3} & -\frac{2}{3} & -\frac{1}{3} \end{bmatrix} y$$

IV. CONCLUSIONS

In this paper, we present a simple design method of an input-decoupled observer for descriptor systems with unknown inputs. The existent conditions of the observer are given, and we give the gradually estimation of state and unknown inputs.

REFERENCES

[1] M. Hou and P. C. Muller, “Disturbance Decoupled Observer Design: A Anified Viewpoint,” *IEEE Trans. Automat. Contr.*, vol. 39, no. 6, 1994, pp. 1338-1341.

[2] M. Hou and R.J. Patton, “Input Observability and Input Reconstruction,” *Automat.*, Vol. 34, no.6, 1998, pp. 789-794.

[3] M. Corless and J. Tu, “State and Input Estimation for A Class of Uncertain System,” *Automat.*, Vol. 34, no. 6, 1998, pp. 757-764.

[4] S. Zhu, L. Sun and Z. Cheng, “Input Estimation for Uncertain Linear Singular Systems and Robust Stabilization,” *in Proc. 40th IEEE CDC, Orlando,USA* , 2001, pp. 2856-2857.

[5] M. Darouach, M.Zasadzinski and M. Hayar, “Reduced-Order Observer Design for Descriptor Systems with Unknown Inputs,” *IEEE Trans. Automat. Contr.*, Vol. 41, No. 7, 1996, pp. 1068-1072.