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Design and properties of feedback–feedforward sampled-data model-reference LQG control

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***Abstract*— Design rules for a 2DOF sampled-data controller which simultaneously tracks a pre-programmed set-point change and minimizes the influence of stochastic disturbance, based on the model-reference concept and LQG problem formulation, are given and its properties with regard to possible plant-model mismatch are discussed.**

I. INTRODUCTION

In this paper, which is a continuation of [4], we will employ sampled-data LQG approach, [2], [10], with continuous-time integral performance index to design a model following control system working in presence of stochastic disturbances, whose influence on the output should be attenuated.

The main point consists in splitting the control law into two parts: the deterministic feedforward component and the feedback one, whose aim is to deal with stochastic disturbances and possible model-system mismatch.

This approach allows the above components to be designed separately, the main point being a different value of the quadratic performance index control weighting factor λ for different components.

As far as reference tracking is concerned, $\lambda = 0$, the proper choice of the model's relative degree and its 'relative speed' will be shown to be main design specifications to keep control signal character under designer's control.

The main specification for feedback part is the value of $\lambda > 0$ chosen so that the variance of control is within prescribed limits and the overall system is robust enough to handle model-system mismatch.

The paper is organized as follows. A sampled-data LQG control problem with a continuous-time performance index defining the model reference control task is formulated in section II. In section III this problem is converted into an equivalent discrete-time one and solved in section IV. Design rules are presented in section V. The properties of the system under model–system mismatch are discussed in section VI, and the paper is concluded in section VII.

II. PROBLEM STATEMENT

Assume that a linear SISO plant is defined by the following set of stochastic equations

$$\frac{d\boldsymbol{x}^p(t)}{dt} = \boldsymbol{A}^p\boldsymbol{x}^p(t) + \boldsymbol{b}^p u(t) + \boldsymbol{c}^p\dot{\xi}(t), \tag{1}$$

$$y(t) = \boldsymbol{d}^{p\prime}\boldsymbol{x}^p(t), \tag{2}$$

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where $\boldsymbol{x}^p(t)$ is a p – dimensional state vector, $\boldsymbol{A}^p$ is a $p \times p$ – dimensional matrix, $\boldsymbol{b}^p$, $\boldsymbol{c}^p$ and $\boldsymbol{d}^p$ are p – dimensional vectors. The initial condition $\boldsymbol{x}_0^p$ is assumed to be a normal random vector, $\boldsymbol{x}_0^p \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{Q}_0^p)$. The colored disturbance on the output is modelled by continuous white noise $\dot{\xi}(t)$ with unit variance, var $\xi(t) = \delta(t)$ entering the disturbance channel of the system. The the control signal $u(t)$ is produced by a ZOH device with period h,

$$u(t) = u_i, \text{ for } t \in (ih, ih+h],\ i = 0, 1, \ldots, \tag{3}$$

driven by the digital controller output u_i, which changes its value at discrete time instants $t_i = ih, k = 0, 1, \ldots$. The output of the system is also sampled at t_i. Available measurements z_i are expressed by the following measurement equation

$$z_i = \boldsymbol{d}^{p\prime}\boldsymbol{x}_i^p + n_i, \tag{4}$$

where n_i that represents measurement errors is a Gaussian white noise with zero mean, $\mathrm{E}[n_i] = 0$, and variance $\mathrm{E}[n_i^2] = \nu^2$.

We assume that the relative order of the plant is n_r^p, which means that $\mu_0^p = 0,\ \mu_1^p = 0,\ \ldots, \mu_{n_r-1}^p = 0$ and $\mu_{n_r}^p \neq 0$, where

$$\mu_j^p = \boldsymbol{d}^{p\prime}(\boldsymbol{A}^p)^{j-1}\boldsymbol{b}^p \tag{5}$$

is the j–th Markov parameter of the system in (1)–(2).

Assume that an r–th order continuous-time reference model is given

$$\frac{d\boldsymbol{x}^r(t)}{dt} = \boldsymbol{A}^r\boldsymbol{x}^r(t) + \boldsymbol{\beta}^r r(t), \tag{6}$$

$$y^r(t) = \boldsymbol{d}^{r\prime}\boldsymbol{x}^r(t). \tag{7}$$

It is assumed that $r(t)$ changes step-wise at $t_r = N_r h$, for certain integer N_r, i.e. $r(t) = r1(t - t_r)$. As a result $y^r(t) \equiv 0, t \le t_r$, and $y^r(t) \neq 0$ for $t > t_r$.

The aim of the system is to follow the profile signal $y^r(t)$ produced by the reference model as closely as possible based on noisy sampled measurements defined in (4), i.e. to make the error $e(t) = y^r(t) - y(t)$ small. To this end, a LQG control problem is formulated with performance index J,

$$J = \lim_{N\to\infty} \frac{1}{Nh}\mathrm{E}\int_0^{Nh}\left\{e^2(t) + \lambda u^2(t)\right\}dt,\ \lambda \ge 0, \tag{8}$$

and plant model

$$\frac{d\boldsymbol{x}^m(t)}{dt} = \boldsymbol{A}^m\boldsymbol{x}^m(t) + \boldsymbol{b}^m u(t) + \boldsymbol{c}^m\dot{\xi}(t), \tag{9}$$

$$y(t) = \boldsymbol{d}^{m\prime}\boldsymbol{x}^m(t), \tag{10}$$

with m-dimensional state vector $\boldsymbol{x}^m(t)$, matrix $\boldsymbol{A}^m$, and vectors $\boldsymbol{b}^m, \boldsymbol{c}^m, \boldsymbol{d}^m$ of appropriate dimensions.

III. Equivalent discrete-time LQG

The above sampled data control problem will be reformulated to a more classical discrete-time LQG control problem. To this end, aggregate the problem in (9)-(10) and (6)-(8) to

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t) + \boldsymbol{\beta}r(t) + \boldsymbol{c}\dot{\xi}(t), \tag{11}$$
$$y(t) = \boldsymbol{d}'\boldsymbol{x}(t), \tag{12}$$

$$J = \lim_{N\to\infty} \frac{1}{Nh} \mathrm{E} \int_0^{Nh} \left\{\boldsymbol{x}'(t)\boldsymbol{M}\boldsymbol{x}(t) + \lambda u^2(t)\right\} dt, \tag{13}$$

where $\boldsymbol{x} = [\boldsymbol{x}^{m\prime}(t), \boldsymbol{x}^{r\prime}(t)]'$, $\boldsymbol{A} = \operatorname{diag}\{\boldsymbol{A}^m, \boldsymbol{A}^r\}$, $\boldsymbol{b} = [\boldsymbol{b}^{m\prime}, \boldsymbol{0}']'$, $\boldsymbol{\beta} = [\boldsymbol{0}', \boldsymbol{\beta}^{r\prime}]'$, $\boldsymbol{c} = [\boldsymbol{c}^{m\prime}, \boldsymbol{0}']'$, $\boldsymbol{d} = [\boldsymbol{d}^{m\prime}, \boldsymbol{0}']'$,$\boldsymbol{M} = \boldsymbol{l}\boldsymbol{l}'$, $\boldsymbol{l} = [-\boldsymbol{d}^{m\prime}, \boldsymbol{d}^{r\prime}]'$. Denote

$$\boldsymbol{F}(\tau) = e^{A\tau},\ \boldsymbol{g}(\tau) = \int_0^\tau e^{A\nu}\boldsymbol{b}d\nu, \tag{14}$$
$$\boldsymbol{h}(\tau) = \int_0^\tau e^{A\nu}\boldsymbol{\beta}d\nu. \tag{15}$$

Then the problem defined by modulation equation (3), measurement equation (4), state equation (11) and performance index (13) is equivalent with the following discrete-time problem:

$$\boldsymbol{x}_{i+1} = \boldsymbol{F}\boldsymbol{x}_i + \boldsymbol{g}u_i + \boldsymbol{h}r_i + \boldsymbol{w}_i, \tag{16}$$
$$z_i = \boldsymbol{d}'\boldsymbol{x}_i + n_i, \tag{17}$$

$$J = \lim_{N\to\infty} \frac{1}{N} \mathrm{E} \sum_{j=0}^{N-1} \varphi(\boldsymbol{x}_j, r_j, u_j), \tag{18}$$

where

$$\boldsymbol{F} = \boldsymbol{F}(h), \qquad \boldsymbol{g} = \boldsymbol{g}(h), \qquad \boldsymbol{h} = \boldsymbol{h}(h), \tag{19}$$

$$\begin{aligned}\varphi(\boldsymbol{x}_j, r_j, u_j) &= \boldsymbol{x}_j'\boldsymbol{Q}_1\boldsymbol{x}_j + 2\boldsymbol{x}_j'\boldsymbol{q}_{12}u_j + 2\boldsymbol{x}_j'\boldsymbol{q}_{13}r_j \\ &+ 2r_j'q_{32}u_j + q_3r_j^2 + q_2u_j^2 + q_w,\end{aligned} \tag{20}$$

$\boldsymbol{w}_i$ is a zero mean vector Gaussian noise with $\mathrm{E}\{\boldsymbol{w}_i\boldsymbol{w}_i'\} = \boldsymbol{W}$,

$$\boldsymbol{W} = \int_0^h e^{As}\boldsymbol{c}\boldsymbol{c}'e^{A's}ds, \tag{21}$$

vectors $\boldsymbol{x}_0$ and $[\boldsymbol{w}_i', n_i]$ are independent for all $i \geq 0$, and

$$\boldsymbol{Q}_1 = \frac{1}{h}\int_0^h \boldsymbol{F}'(\tau)\boldsymbol{M}\boldsymbol{F}(\tau)d\tau, \tag{22}$$
$$\boldsymbol{q}_{12} = \frac{1}{h}\int_0^h \boldsymbol{F}'(\tau)\boldsymbol{M}\boldsymbol{g}(\tau)d\tau, \tag{23}$$
$$\boldsymbol{q}_{13} = \frac{1}{h}\int_0^h \boldsymbol{F}'(\tau)\boldsymbol{M}\boldsymbol{h}(\tau)d\tau, \tag{24}$$
$$q_{32} = \frac{1}{h}\int_0^h \boldsymbol{h}'(\tau)\boldsymbol{M}\boldsymbol{g}(\tau)d\tau, \tag{25}$$
$$q_2 = \frac{1}{h}\int_0^h \boldsymbol{g}'(\tau)\boldsymbol{M}\boldsymbol{g}(\tau)d\tau + \lambda, \tag{26}$$

$$q_w = \frac{1}{h}\boldsymbol{l}'\left\{\int_0^h\int_0^\tau \boldsymbol{F}(\tau-s)\boldsymbol{c}\boldsymbol{c}'\boldsymbol{F}'(\tau-s)dsd\tau\right\}\boldsymbol{l}. \tag{27}$$

Integrals in (21)–(27) can be computed by the methods of [7], [8], [9]

IV. Problem solution

In this section a 2 DOF feedback and feedforward structure of the control system displayed in Fig.1 will be proposed that results from the optimal control law, obtained in [4], for the problem in (16)–(18) under the following decomposition:

$$\hat{\boldsymbol{x}}_{i|i} = \bar{\boldsymbol{x}}_i + \boldsymbol{\delta}\hat{\boldsymbol{x}}_{i|i}, \quad z_i = \bar{y}_i + \delta z_i, \tag{28}$$

where $\bar{\boldsymbol{x}}_i = \mathrm{E}\{\boldsymbol{x}_i\} = \mathrm{E}\{\hat{\boldsymbol{x}}_{i|i}\}$, and $\bar{y}_i = \mathrm{E}\{y_i\}$. It reflects deterministic character of the set-point and the stochastic character of the disturbance. We have the following theorem:

Theorem 1: The optimal control law consists of the nominal deterministic feedforward part, $\bar{u}_i = \mathrm{E}\{u_i\}$, responsible for the set-point, and the feedback part, δu_i, responsible for stochastic disturbance attenuation:

$$u_i = \bar{u}_i + \delta u_i, \tag{29}$$

The feedforward component is determined by:

$$\bar{u}_i = -\boldsymbol{k}_x'\bar{\boldsymbol{x}}_i + k_r r_i + u_i^a, \tag{30}$$

where

$$\bar{\boldsymbol{x}}_{i+1} = \boldsymbol{F}\bar{\boldsymbol{x}}_i + \boldsymbol{g}\bar{u}_i + \boldsymbol{h}r_i, \tag{31}$$
$$\bar{y}_i = \boldsymbol{d}'\bar{\boldsymbol{x}}_i. \tag{32}$$

The elements of (30)

$$\boldsymbol{k}_x = \frac{\boldsymbol{q}_{12} + \boldsymbol{F}'\boldsymbol{K}\boldsymbol{g}}{q_2 + \boldsymbol{g}'\boldsymbol{K}\boldsymbol{g}}, \tag{33}$$

$$k_r = -\frac{q_{32} + \boldsymbol{g}'\boldsymbol{K}\boldsymbol{h}}{q_2 + \boldsymbol{g}'\boldsymbol{K}\boldsymbol{g}}, \tag{34}$$

$$u_i^a = -\frac{\boldsymbol{g}'\boldsymbol{p}_{i+1}}{q_2 + \boldsymbol{g}'\boldsymbol{K}\boldsymbol{g}}, \tag{35}$$

depend on the solution $\boldsymbol{K}$ of the following Riccati equation:

$$\boldsymbol{K} = \boldsymbol{Q}_1 + \boldsymbol{F}'\boldsymbol{K}\boldsymbol{F} - \frac{(\boldsymbol{q}_{12} + \boldsymbol{F}'\boldsymbol{K}\boldsymbol{g})(\boldsymbol{q}_{12} + \boldsymbol{F}'\boldsymbol{K}\boldsymbol{g})'}{q_2 + \boldsymbol{g}'\boldsymbol{K}\boldsymbol{g}}, \tag{36}$$

and $\boldsymbol{p}_i$ is calculated from

$$\boldsymbol{p}_i = \begin{cases} \boldsymbol{p}_\infty & i \geq N_r, \\ \left(\boldsymbol{F}' - \boldsymbol{k}_x\boldsymbol{g}'\right)\boldsymbol{p}_{i+1}, & i = N_r - 1, \ldots, 0 \end{cases} \tag{37}$$

with

$$\boldsymbol{p}_\infty = \left(\boldsymbol{I} - \boldsymbol{F}' + \boldsymbol{k}_x\boldsymbol{g}'\right)^{-1} \times \Big(\boldsymbol{q}_{13} + \boldsymbol{F}'\boldsymbol{K}\boldsymbol{h} - \boldsymbol{k}_x(q_{32} + \boldsymbol{h}'\boldsymbol{K}\boldsymbol{g})\Big) r. \tag{38}$$

The stochastic component is determined by

$$\delta u_i = -\boldsymbol{k}_x' \boldsymbol{\delta}\hat{\boldsymbol{x}}_{i|i}, \tag{39}$$

where

$$\boldsymbol{\delta}\hat{\boldsymbol{x}}_{i|i} = \boldsymbol{\delta}\hat{\boldsymbol{x}}_{i|i-1} + \boldsymbol{k}_i^f(\delta z_i - \boldsymbol{d}'\boldsymbol{\delta}\hat{\boldsymbol{x}}_{i|i-1}), \tag{40}$$

$$\boldsymbol{\delta}\hat{\boldsymbol{x}}_{i|i-1} = \boldsymbol{F}\boldsymbol{\delta}\hat{\boldsymbol{x}}_{i-1|i-1} + \boldsymbol{g}\delta u_{i-1}, \tag{41}$$

$$\boldsymbol{k}_i^f = \frac{\boldsymbol{\Sigma}_{i|i-1}\boldsymbol{d}}{\nu^2 + \boldsymbol{d}'\boldsymbol{\Sigma}_{i|i-1}\boldsymbol{d}}, \tag{42}$$

$$\boldsymbol{\Sigma}_{i+1|i} = \boldsymbol{W} + \boldsymbol{F}\left(\boldsymbol{\Sigma}_{i|i-1} - \frac{\boldsymbol{\Sigma}_{i|i-1}\boldsymbol{d}\boldsymbol{d}'\boldsymbol{\Sigma}_{i|i-1}'}{\nu^2 + \boldsymbol{d}'\boldsymbol{\Sigma}_{i|i-1}\boldsymbol{d}}\right)\boldsymbol{F}', \quad \boldsymbol{\Sigma}_0 = \boldsymbol{Q}_0. \tag{43}$$

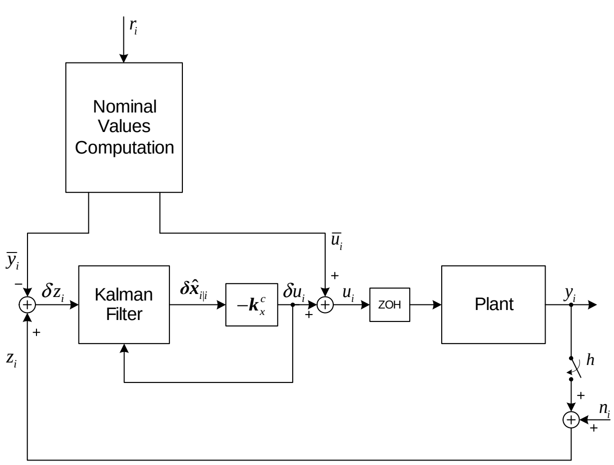

Fig. 1. Feedback-feedforward 2 DOF control structure

Remark 1: First two elements of the feedforward component (30) are equal to zero for $i < N_r$, while the third element u_i^a anticipating the change of r_i might be nonzero for $i = 0, 1, \ldots, N_r$. Control laws with negligible u_i^a are of particular practical value.

Remark 2: Following [2], the steady-state Kalman filter will be used with constant $\boldsymbol{k}^f$ calculated from the algebraic variant of Riccati equation (43). Then transfer function $D(z)$ of the controller can be found such that $\delta u_i = D(z^{-1})\delta z_i$.

V. Controller design

In this section we will concentrate on the choice of appropriate reference model (6)–(7), and the value of λ in performance index (8).

We will study the design rules separately for the feedforward part responsible for the set-point, and for feedback responsible for both disturbance attenuation and reduction of sensibility to system-model mismatch.

In our example the control and disturbance paths of the plant are defined by the transfer functions

$$G_c(s) = \frac{k_c}{(as+1)^2(Ts+1)}, \tag{44}$$

$$G_d(s) = \frac{k_d}{T_d^2 s^2 + 2\xi T_d s + 1}, \tag{45}$$

respectively. The outputs of both paths add together to form the system output, which is then sampled and corrupted by noise.

We will keep $k_c = 1$, $a = 0.2$, $T = 1$, $T_d = 1$, $\xi = 0.5$, and $h = 0.2$ in our examples. The value of k_d is chosen so that the disturbance has unity variance. We will also assume ideal fit of the model, $G_c^m(s), G_d^m(s)$, i.e. $G_c^m(s) = G_c(s)$, and $G_d^m(s) = G_d(s)$, in this chapter.

A. Design of feedforward part

The main result of [4] concerning the set-point tracking is that the reasonable reference model should has the same relative order as the plant, and $\lambda = 0$ should be chosen. It is then possible to keep the maximum value of the control signal in response to the set-point change according to the following rule of thumb

$$\frac{u(t_r)}{u(\infty)} \leq \frac{\mu_r^r}{\mu_r^p}, \tag{46}$$

where $u(t_r)$ is the magnitude of the first non-anticipative control impuls, and μ_r^r and μ_r^p are Markov parameters of the model and plant, respectively. This is readily seen in Fig. 2, displaying output and control signals for the reference model

$$G_r(s) = \frac{1}{(a^r s+1)^2(T^r s+1)}, \tag{47}$$

for $a^r = 0.2$ and $T^r = 1.0$. Acceleration of the system by reducing the main time constant requires almost 2 times greater control magnitude, which can be inferred from (46). Notice also that anticipative action of the controller is negligible in this case. Decreasing h results in approaching the continuous-time control with $u(t_r) = 2$. Imposing the

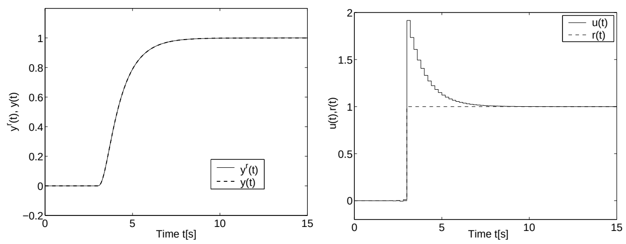

Fig. 2. Reference tracking, $a^r = 0.2,\ T^r = 1.0,\ \lambda = 0$

reference model

$$G_r(s) = \frac{1}{s+1}, \tag{48}$$

reduces the relative degree of $G_r(s)$ from $\mu_r^r = 3$ to $\mu_r^r = 1$, which results in technically unacceptable control displayed in Fig.3, with high magnitudes and prominent ringing anticipative action. It is worth noting [4] that decreasing h results in increased magnitudes of control signal and decreased anticipation time, so as to approach the continuous-time control that consist higher order Dirac impulses at $t = t_r$.

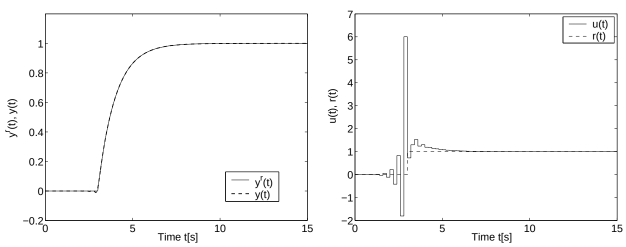

Fig. 3. Reference tracking, $a^r = 0.0,\ T^r = 1.0,\ \lambda = 0$

B. Design of feedback part

Unfortunately, the value $\lambda = 0$ leads to high magnitudes of control required for minimum-variance disturbance attenuation. A realization of control process is displayed in Fig.4

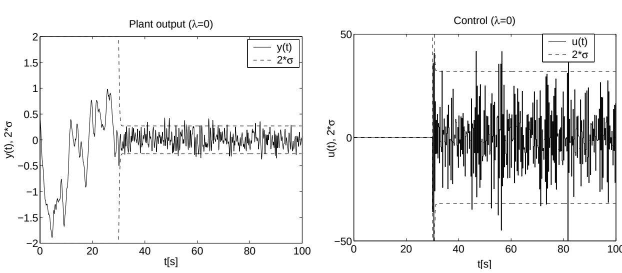

Fig. 4. MV disturbance attenuation, $\lambda = 0$, loop closed at $t = 30$

From Fig.5 it is seen[1] that small values of $\lambda > 0$ lead to dramatic reduction of the control magnitudes without loosing to much of control quality. The results for a typical value of $\lambda = 0.01$ are displayed in Fig.6. Unfortunately, as seen from Fig.7, $\lambda > 0$ causes undesirable deformation of both the feedforward component of control signal as well as loss of tracking accuracy.

A remedy is to design the feedforward component with $\lambda = 0$ and the feedback component with $\lambda > 0$.

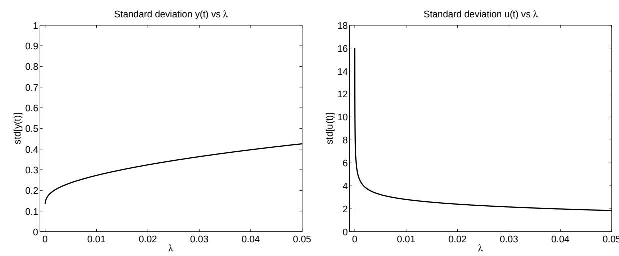

Fig. 5. Standard deviations of output and input signals vs. λ

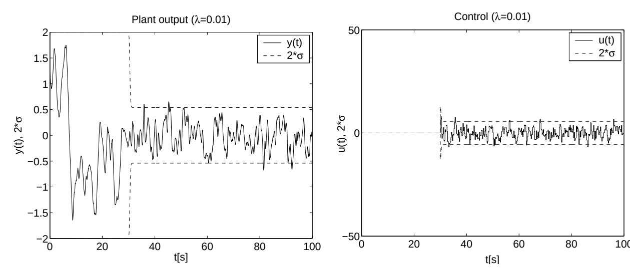

Fig. 6. Disturbance attenuation, $\lambda = 0.01$, loop closed at $t = 30$

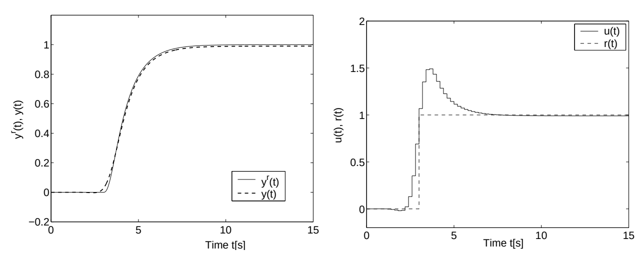

Fig. 7. Reference tracking, $a^r = 0.2,\ T^r = 1.0,\ \lambda = 0.01$

As far as sampling period h is concerned it is well known that the shorter sampling period the better the control quality. It is interesting to note that, as seen from Fig.8, increasing the sampling frequency can compensate the effect of more intensive measurement noise.

VI. Effect of the model–system mismatch

An important characteristics of the control system is its sensitivity to the model–system mismatch. To study this issue we assume at first that models have the same forms as that in (44)–(47)

$$G_c^m(s) = \frac{k_c^m}{(a^m s+1)^2(T^m s+1)}, \tag{49}$$

[1] Standard deviations used throughout the paper are square roots of mean intersample variances defineded as

$$\sigma_{yi}^2 = \frac{1}{h}\int_{ih}^{(i+1)h} \mathrm{E}\{y^2(t)\}dt$$

and computed by the methods of [7], [8].

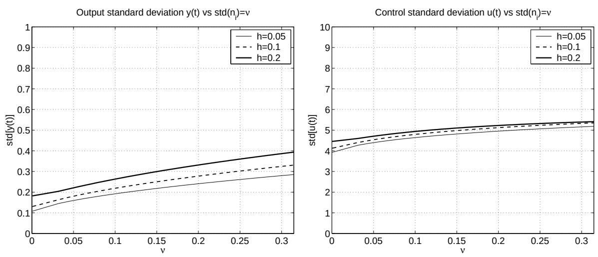

Fig. 8. Std. dev. of $y(t)$ versus ν for different h, $\lambda = 0.001$

$$G_d^m(s) = \frac{k_d^m}{T_d^{m2} s^2 + 2\xi^m T_d^m s + 1}, \qquad (50)$$

but different parameters.

A. Parameter mismatch and disturbance attenuation

The effects of the discrepancies between model parameters and the real system values on the quality of disturbance attenuation are displayed in Fig.9–11. Surprisingly, the quality of disturbance attenuation as measured by standard deviation of the output signal is almost insensitive to quite large discrepancies between the parameters of control channel including its gain, and only a little more sensitive to the parameters of the disturbance channel model, Fig.12.

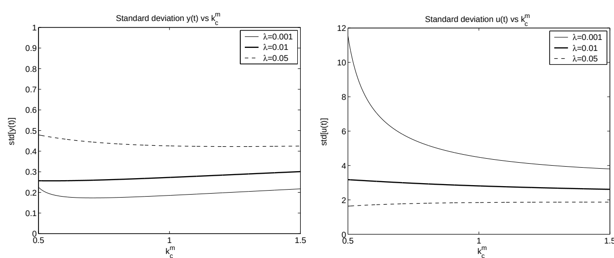

Fig. 9. Model-system mismatch, $k_c^m \neq k_c = 1$

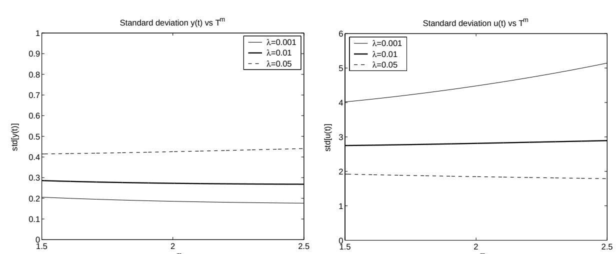

Fig. 10. Model-system mismatch, $T^m \neq T = 2$

B. Reference tracking: effect of parameter mismatch

In the case of mismatch between a model and the system, the output of the open-loop system would differ from the reference. However feedback designed primary to combat disturbances is also able to make corrections. Examples are given in Fig.13-15.

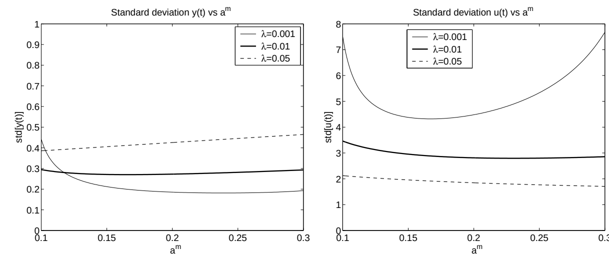

Fig. 11. Model-system mismatch, $a^m \neq a = 0.2$

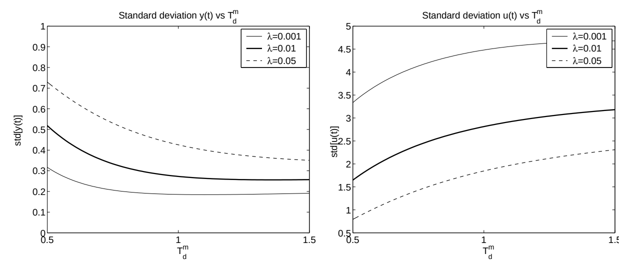

Fig. 12. Model-system mismatch, $T_d^m \neq T_d = 1$

C. Reference tracking: effect of unmodelled dynamics

The effect of unmodelled dynamics on reference tracking is shown in Fig.16, where $G_p(s)$ is multiplied by $(1 - 0.05s)$. Almost identical transients result from multiplying $G_p(s)$ by $1/(1+0.05s)$ or $(1-0.025s)/(1+0.025s)$. The plots are then very similar to those of Fig.13 and show that the system is also capable to handle unmodelled dynamics.

VII. Conclusion

In the paper several issues concerning the design and properties of sampled-data controllers were investigated for a continuous-time plant with output disturbed by a stochastic disturbance.

It has been shown that the controller can be presented as a 2DOF feedback and feedforward system, each of them to be designed separately.

For the feedforward part, which is responsible for reference tracking, it is very important that the relative degree of the control path of the continuous-time plant is determined properly. The best choice for λ is $\lambda = 0$.

Feedback part relies solely on the weighting factor $\lambda > 0$, whose value should be chosen so that the control signal has reasonable magnitudes and the entire system is robust enough against model-system mismatch.

Higher control accuracy requires better knowledge of the high frequency plant dynamics. This is a challenge for appropriate identification and system modelling methods.

It was also shown that higher sampling frequency can compensate the effect of greater measurements errors. This result is of high practical value: the use of cheaper sensors can be compensated by higher sampling frequency.

To summarize, the approach presented in the paper, competitive to those of [6], [11], leads to excellent control systems featuring important practical qualities.

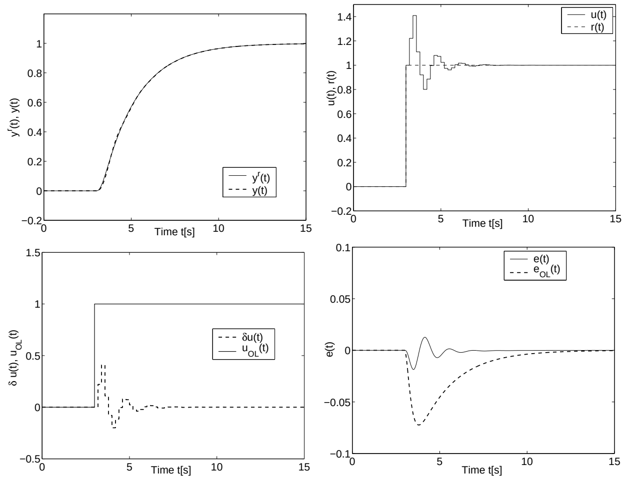

Fig. 13. Model-system mismatch, $a^m = 0.15,\ a = 0.2$

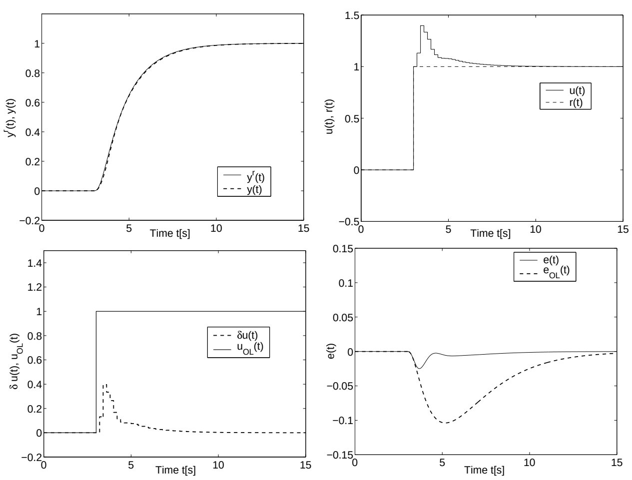

Fig. 14. Model-system mismatch, $T^m = 1.5,\ T = 1.0$

The same remarks apply to program control systems [5].

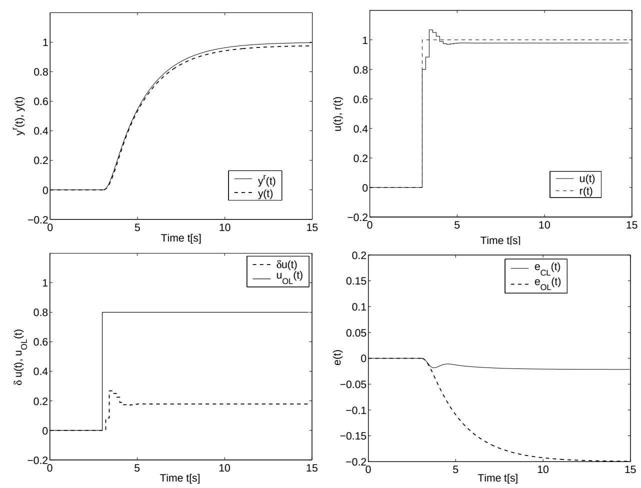

Fig. 15. Model-system mismatch, $k^m = 1.25,\ k = 1.0$

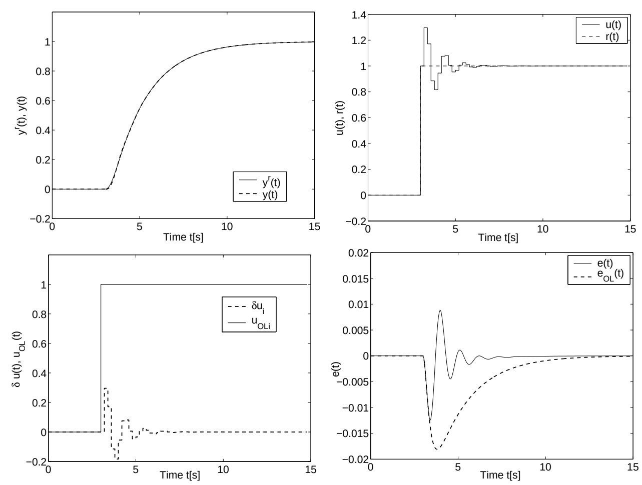

Fig. 16. $G_p(s)$ multiplied by $(1 - 0.05s)$

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