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Qualitative Results for a Hierarchical Discrete Event Control Paradigm Applied to Structures Operating Under Nominal and Fault Conditions

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***Abstract*— This paper presents a mathematical discrete event state space model for plants and controllers that is amenable to classical state space control theory. The model is based upon industry-standard N-squared diagrams which are shown to readily translate into a state space matrix form. A hierarchical structure is defined that allows the design to scale in dimension and remain tractable. The motivation for using this state space model approach is to develop reachability, observability and stability results using approaches based upon existing control theory, and well as to adapt certain control design paradigms. The state space model defined is based upon Boolean algebra, and so the desired theoretical results must be adapted accordingly. The model is described in the context of two examples, the first being a Bouc-Wen modified hysteresis model, and the second a general supervisory discrete-event servo controller.**

I. Introduction

The discrete event model defined in this paper is most readily defined in the context of examples. A four-state model of a Bouc hysteresis is used to illustrate the fundamental concepts underlying N^2 diagrams, and a larger more complicated supervisory servo controller is used to motivate the idea of hierarchically structured N^2 diagrams and to provide a basis for the state space discrete event control paradigm derived from the N^2 diagrams.

II. N^2 Diagram Definition Illustrated Using an MR Damper Model

The state space discrete event model presented herein was motivated while investigating means of stabilizing a three degree of freedom (3DOF) structure during seismic excitation. The nonlinear controller considered utilized a hysteretic magnetorheological damper (MRD) to control the structure's effective damping, a so-called semi-passive approach. In the 3DOF structure shown Fig. 1, v is the applied control signal to the MRD, f is the force applied to the MRD by the structure, m_i and x_i, $i = 1, 2, 3$, are the respective masses and the positions of the three floors in the structure, and $\ddot{x}_g$ is the applied ground-level seismic acceleration. The equations of motion for the structure are detailed by Dyke *et al.* [1], [2], [3], but only the discrete event portion of the model is presented here.

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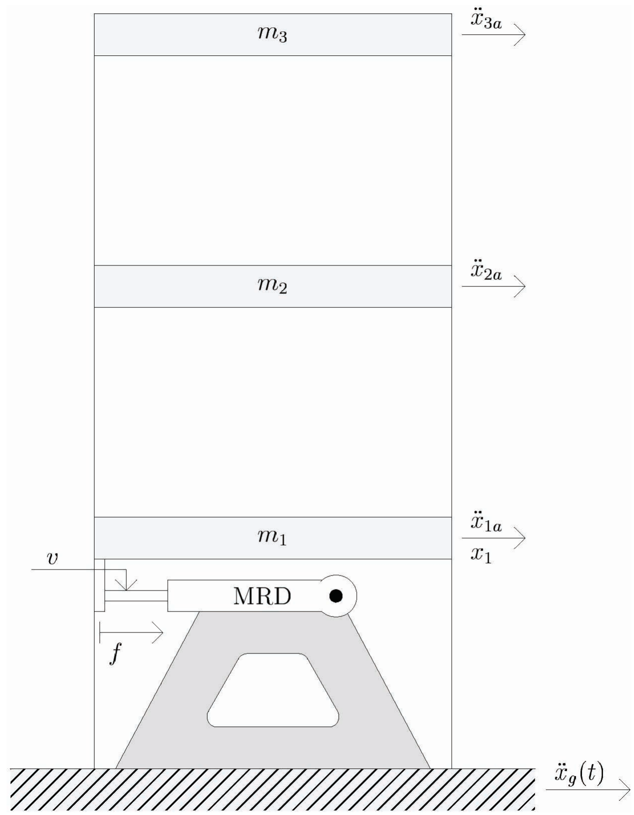

Fig. 1. Connection of an MR damper to a 3DOF building.

A. An MR Damper Bubble Chart State Diagram

The hysteretic nature of the MRD is represented using a second-order modified Bouc-Wen hysteresis model [4], [5], [6] described by a differential equation of the form

$$\dot{z} = (a - b_1 z^2)\dot{x}, \qquad \text{sgn}\, z = \text{sgn}\, \dot{x}, \qquad (1)$$
$$\dot{z} = (a - b_2 z^2)\dot{x}, \qquad \text{sgn}\, z \neq \text{sgn}\, \dot{x}, \qquad (2)$$

where a, b_1, b_2, x and z are real scalars, x represents displacement of the MRD, z is a nonphysical variable representing the hysteretic portion of the restoring force applied by the MRD, and a, b_1 and b_2 are loop-shaping parameters for the hysteresis.

Use of the Bouc model led to the discrete event bubble chart model in Fig. 2. The appealing intuitive clarity displayed by bubble charts, however, generally disappears as the system becomes more complex, limiting this form

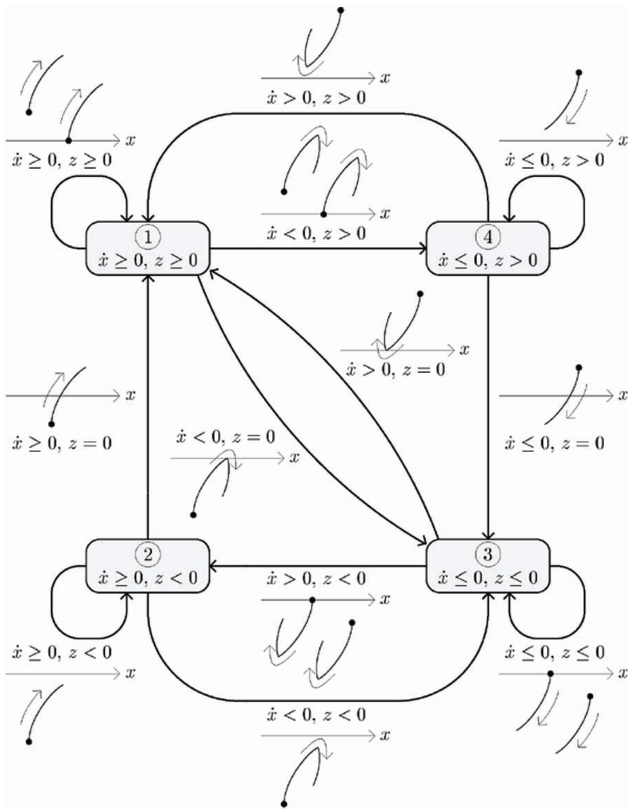

Fig. 2. Connection of an MR damper to a 3DOF building.

to relatively simple models with few states and transitions. For example, even the relatively simple chart for the MRD threatens to become an intractable tangled web of transitions if a second damper should be added to the system.

B. An MR Damper N^2 Diagram

N-squared diagrams are an industry standard for representing complex discrete event plant and controller behavior, and provide a basis for the mode and fault control designs in autonomous and semi-autonomous systems. Additionally, their matrix-like structure leads readily to the development of state space realizations, paving the way for qualitative assessment of discrete event control system properties.

For a given model having a set of n distinct operating conditions, let the corresponding discrete event model have n unique states. The corresponding N^2 diagram is an $n \times n$ grid with the n discrete states of the model placed on the squares on the main diagonal of the grid. The first or entrance state is placed in the upper left corner and the exit or final state is placed in the lower right corner. Intermediate states can be placed in any order along the main diagonal. Fig. 3 illustrates placement of the states shown in the state chart in Fig. 3 into an N^2 diagram (entries A1, B2, C3 and D4).

Transitions are represented on the off-diagonal elements in an N^2 diagram. Let s_i, $i = 1, ..., n$, denote the set of

	1	2	3	4
A	$\dot{x} \geq 0$ $z \geq 0$		$\dot{x} < 0$ $z = 0$	$\dot{x} \leq 0$ $z > 0$
B	$\dot{x} \geq 0$ $z = 0$	$\dot{x} \geq 0$ $z < 0$	$\dot{x} < 0$ $z < 0$	
C	$\dot{x} > 0$ $z = 0$	$\dot{x} > 0$ $z < 0$	$\dot{x} \leq 0$ $z \leq 0$	
D	$\dot{x} > 0$ $z > 0$		$\dot{x} \leq 0$ $z = 0$	$\dot{x} \leq 0$ $z > 0$

Fig. 3. N^2 diagram corresponding to Fig. 2.

	1	2	3	4
A	s_1		t_{13}	t_{14}
B	t_{21}	s_2	t_{23}	
C	t_{31}	t_{32}	s_3	
D	t_{41}		t_{43}	s_4

Fig. 4. Compact N^2 diagram for Bouc-Wen hysteresis model.

discrete states. Each transition is uniquely associated with a single *source* state and a single *destination* state. Given a unique pair of states (s_i, s_j), where $i \neq j$, let s_i denote the source state, and s_j denote the destination state. The transition associated with this pair, denoted t_{ij}, is placed in grid location (i, j) of the N^2 diagram. Fig. 3 illustrates placement of the transitions shown in the state chart in Fig. 2 into an N^2 diagram.

Note that the transitions in Fig. 2 having the same source and destination state are shown only for completeness' sake in illustrating the hysteretic behavior of the actuator model, and are omitted from the N^2 diagram. Additionally, for purposes of this work, all possible or relevant operating conditions of the modeled system are assumed to be represented in the N^2 diagram.

Evaluation or reading an N^2 diagram proceeds as follows. Let s_i represent the state that corresponds to the present operating condition of the system. The system is then said to be *in state* s_i, with the implication that the logical relations shown in the N^2 diagram for state s_i evaluate *true*. Equivalently, the state s_i is said to be *active*. The system will change to state s_j if and only if an event occurs such that the conditions associated with transition t_{ij} evaluate *true*, assuming t_{ij} exists; in this situation, transition t_{ij} is said to be *active*. Nonexistent transitions are represented in the N^2 diagram by empty squares in the grid.

Although useful for illustrative purposes, Fig. 3 assumes an equivalent and more compact and convenient format when associated with state and transition tables, illustrated for the MRD example in Fig. 4 and Tables I and II. Note $\wedge$ and $\vee$ represent the logical *and* and *or* operations.

TABLE I

STATE TABLE FOR BOUC-WEN HYSTERESIS MODEL

State	Description
s_1	$(\dot{x} \geq 0) \wedge (z \geq 0)$
s_2	$(\dot{x} \geq 0) \wedge (z < 0)$
s_3	$(\dot{x} \leq 0) \wedge (z \leq 0)$
s_4	$(\dot{x} \leq 0) \wedge (z > 0)$

TABLE II

TRANSITION TABLE FOR BOUC-WEN HYSTERESIS MODEL

Transition	Description
t_{13}	$(\dot{x} < 0) \wedge (z = 0)$
t_{14}	$(\dot{x} \leq 0) \wedge (z > 0)$
t_{21}	$(\dot{x} \geq 0) \wedge (z = 0)$
t_{23}	$(\dot{x} < 0) \wedge (z < 0)$
t_{31}	$(\dot{x} > 0) \wedge (z = 0)$
t_{32}	$(\dot{x} > 0) \wedge (z < 0)$
t_{41}	$(\dot{x} > 0) \wedge (z > 0)$
t_{43}	$(\dot{x} \leq 0) \wedge (z = 0)$

C. Operating Assumptions and Constraints

In the sequel, a single-threaded serial process will be assumed. Extension of the model to parallel processes is reasonably straightforward and is omitted for brevity. The following modeling assumptions are designed to facilitate control under fault conditions by eliminating ambiguity with respect to active states and transitions and also to admit a specialized state space representation [7].

Assumption 1 Each state in an N^2 diagram is unique.

Assumption 2 At most one state in an N^2 diagram is active at any given time.

Assumption 3 At most one transition in an N^2 diagram is active at any given time.

Assumption 4 Ideally, the time interval over which a transition is active has measure zero.

Assumption 4 implies that plant activity, including faults, cannot occur when a transition is active. This greatly simplifies modeling and design.

If the system is in a given state s_i, and the logical relations associated with it evaluate *false*, then the system must change state. The new state is determined by examining the transitions t_{ij}, $1 \leq j \leq n$, $i \neq j$. If, for example, transition t_{ik} evaluates *true*, then the system has changed to state s_k.

D. A State Space Representation of N^2 Diagrams

A means of realizing a state space representation of an N^2 diagram is as follows. Let $B_L = \{0, 1\}$ where 1 and 0 denote logical *true* and *false*, and let B_L^n denote the n-dimensional Euclidean product $B_L \times B_L \times \cdots \times B_L$. Let $x_i \in B_L, i = 1, \ldots, n$ be elements of a state vector $x \in B_L^n$, where $x = [x_1 x_2 \ldots x_n]^T$. Associate element x_i in the state vector x with state s_i (grid position (i, i) in the N^2 diagram), and define $x_i = 1$ to mean that state s_i is active. Given $x_i = 1$, then Assumption 1 requires $x_j = 0$ for all $j \neq i$.

Define an event k, $k = 0, 1, 2, \ldots$, to be a change of the plant's operating conditions at time $t_k \in R$, $t_k \geq 0$ such that the plant changes its state, and require $t_j < t_k$ if $j < k$. Assumption 4 requires such changes to be modeled as being instantaneous. Changes requiring nonnegligible intervals of time are modeled by creating a special state that is active during the change. Let $x(k)$ denote the value of the state vector x for the time interval $[t_k, t_{k+1})$. For example, if the plant is in state s_i during the time interval $[t_{k-1}, t_k)$, then $x_i(k-1) = 1$ and $x_j(k-1) = 0$ for $1 \leq j \leq n$, $j \neq i$. Upon event k, suppose the plant moves into state s_q, $1 \leq q \leq n$, $q \neq i$ (*i.e.*, transition t_{iq} is true at time t_k). Then for the time interval $[t_k, t_{k+1})$, the plant is in state s_q, and this is reflected in the model by the fact that $x_q(k) = 1$ and $x_j(k) = 0$ for $1 \leq j \leq n$, $j \neq n$.

The model described herein is of a form similar to that of a time-varying discrete time linear system,

$$x(k+1) = A(k)x(k) + B(k)u(k), \tag{3}$$

except that in this realization, k represents the occurrence of the kth *event* and is not necessarily meant to imply that a fixed amount of time has elapsed. In this representation, the state vector x is defined above, $u \in B_L^m$ is a vector of input signals, $A \in B_L^{n \times n}$ is a matrix mapping $A : B_L^n \to B_L^n$ and $B \in B_L^{n \times m}$ is a matrix mapping $B : B_L^m \to B_L^n$.

The construction of the state matrix A is conceptually straightforward: for a given point in time, refer to the logical relations associated with an N^2 diagram (*e.g.*, Tables I and II). If the logical relations associated with state s_i in the state table are true, then $A_{ii} = 1$ (true), otherwise $A_{ii} = 0$ (false). If the logical relations associated with transition t_{ij} in the transition table are true, then $A_{ji} = 1$, otherwise $A_{ji} = 0$ (observe the transposition).

Let $b \in B$ and let the value of b remain fixed over the time interval $T_k = [t_k, t_{k+1})$. For a given event k, if an element on the main diagonal $A_{ii}(k) = b$, then $A_{ii}(k) = b$ during the entire time interval T_k. The off-diagonal elements of A correspond to transition activity, and these elements are false except at the points in time corresponding to events, t_k, $k = 0, 1, \ldots$, and at these points in time exactly one of the off-diagonal elements of A will be true.

Construction of B proceeds in a similar manner and corresponds to the applied control input u, and the elements of B determine under what operating conditions the input u is permitted to affect the state x.

As an example, consider the hysteresis in the MRD system described above in state s_1 (*e.g.*, $\dot{x} \geq 0, z \geq 0$) so that $x(0) = [1\,0\,0\,0]^T$. If $\dot{x}$ and z change sign, then the model changes state to s_3 via transition A3 in the N^2 diagram. Because this plant is autonomous, the all elements of the state input matrix B are false, and it can be neglected without loss in this case. The state space model representation of this event is

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (4)$$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (5)$$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}. \quad (6)$$

A caveat is appropriate at this point; the representation above uses the integers 1 and 0 to represent the Boolean *true* and *false*. The implied matrix multiplication is defined in a manner consistent with Boolean operations. In the example, for $i = 1, 2, 3, 4$,

$$\begin{aligned} x_i(k+1) \quad = \quad & (A_{i1}(k) \wedge x_1(k)) \vee (A_{i2}(k) \wedge x_2(k)) \vee \\ & (A_{i3}(k) \wedge x_3(k)) \vee (A_{i4}(k) \wedge x_4(k)). \quad (7) \end{aligned}$$

From the description above, clearly the elements in matrices A and B are functions of more than just the event index k. For the example provided, values of the elements of A are given by functions of the state vector $x_p = [\dot{x}\ z]^T$ of the physical plant. For more general cases, the values of the elements of A and B can be given as functions of time. An alternative representation, omitted from this discussion, argues that the control input enters via the functions that determine the values of the elements of A. In this latter case, Eq. 3 has the form

$$x(k+1) = A(k, x_p, t, u)x(k). \quad (8)$$

III. Hierarchical N^2 Diagrams

Hierarchical N^2 diagrams have been developed for processor-based control of complex precision servomechanisms under nominal and fault conditions. Applications include wind and seismic damping systems with certain semi-passive actuators, wherein the plant dynamics can change significantly with direction, distance and rate of travel; dynamic reconfiguration, redundancy management or graceful degradation in systems utilizing networks of sensors and actuators; and autonomous or semi-autonomous initialization, testing or calibration of control system components that are not readily accessible.

To illustrate the hierarchical concept for N^2 diagrams, consider the hypothetical bubble chart state diagrams shown in Figs. 5 and 6 for a general supervisory servo controller. In these diagrams, although the transitions paths are shown, their associated logical conditions are omitted without loss in this discussion. The N^2 diagrams corresponding to these are shown in Figs. 7 and 8.

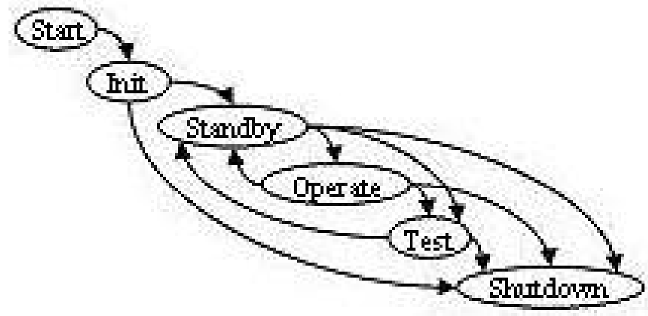

Fig. 5. Top-level bubble chart state diagram for general supervisory servo controller.

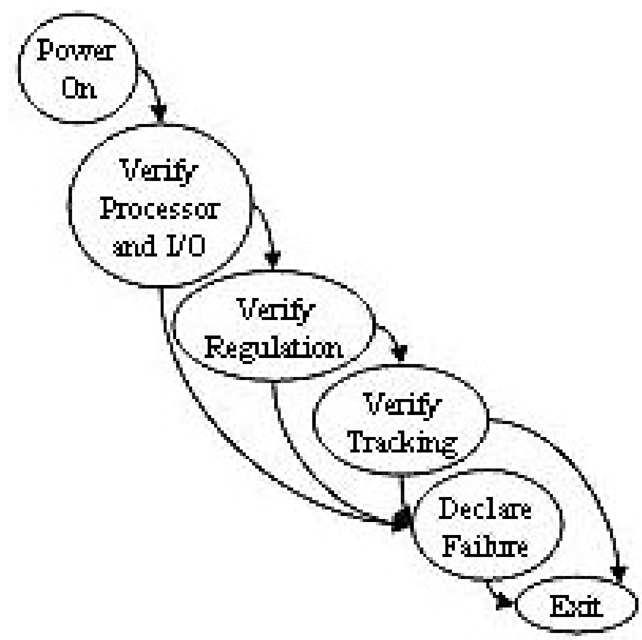

Fig. 6. Bubble subchart state diagram for initialization state of servo controller.

	1	2	3	4	5	6
A	Start	t_{12}				
B		Init	t_{23}			t_{26}
C			Standby	t_{34}	t_{35}	t_{36}
D			t_{43}	Operate	t_{45}	t_{46}
E			t_{53}		Test	t_{56}
F						Shutdn

Fig. 7. N^2 diagram for top-level supervisory controller.

	1	2	3	4	5	6
A	s_1	t_{12}				
B		s_2	t_{23}		t_{25}	
C			s_3	t_{34}	t_{35}	
D				s_4	t_{45}	t_{46}
E					s_5	t_{56}
F						s_6

Fig. 8. N^2 diagram for the Initialization state of the top-level supervisory controller in Fig. 7.

Referring to the top-level N^2 diagram in Fig. 7, the supervisory controller activity begins upon entrance to the Start state in position A1. Upon completion of any necessary activity in this state, the model then moves to the initialization state in grid location B2 once the conditions for transition t_{12} to become active are satisfied. The initialization state, however, is actually a superstate, or a parent state, in that it has six substates, shown in Figs. 6 and 8. Upon entering the initialization state in grid location B2 in the top-level N^2 diagram in Fig. 7, controller activity immediately passes to the Power On state in grid location A1 of the N^2 diagram shown in Fig. 8. When activity in the Initialization N^2 diagram is completed, which is equivalent to the model reaching the Exit state in grid location F6 in Fig. 8, controller activity immediately returns to the parent diagram in Fig. 7. In many ways, this is similar to a software subroutine being called by a parent routine. Upon returning to the parent N^2 diagram, the controller evaluates the results of the activity performed in the initialization state, and either transition t_{23} or t_{26} is activated, moving the model into Standby or Shutdown, respectively.

The advantages of the hierarchical structure outlined above are, in many ways, similar to those put forth for writing subroutines, following the rules of so-called structured programming. The controller can be structured into smaller pieces, and with each piece being of a tractable size (*e.g.,* able to fit on a single sheet of paper is often a good rule) and complexity. More importantly, each of these pieces can be verified independently with regard to qualitative characteristics discussed in the next section, such as performance and reachability.

IV. A Basis for Qualitative Results

At this point, a constructive approach for relating a specialized state space realization to the classical N^2 diagram and associated state charts has been demonstrated. The appeal of the state space representation is that classical controls concepts like stability, reachability and detectability are readily formulated in a rigorous manner. The latter two concepts are of particular interest in discrete event control designs for operational mode and fault management. They provide a means of assessing whether a system can get into and out of operational modes as required, and also whether one can determine or observe what conditions triggered a particular response from the controller.

The motivation for the above approach is driven by the need to analytically assess (as opposed to experimentally test) complex discrete event systems. For numerous systems, exhaustive testing of a supervisory controller is prohibitively expensive from a cost and schedule point of view. Even more importantly, testing of failure mechanisms is often too dangerous or damaging, and so the designer falls back on simulation and analysis, because exhaustive testing is not practical or desirable. Simulation, however, is often just as complicated as the design itself, and constructing an exhaustive simulation for all possible configurations of states and transition firing patterns is sufficiently costly and complex that one continues to search for an alternative means of assessing a discrete event controls design. The mathematical state space model formulation presented in this paper is intended to suggest such an alternative by providing a basis for analytical determination of many standard classical controller properties such as stability, reachability, detectability, disturbance rejection and so forth. The model proposed herein, however, is sufficiently rich and interesting that it requires mathematical adaptation to obtain the analogues of many classical controls results. Some of the adaptations being pursued are presented below.

A. Features of Using Boolean Operators

The state space model defined herein utilizes Boolean operators $\wedge$ (logical *and*) and $\vee$ (logical *or*) instead of multiplication and addition, and the field of real numbers R is replaced with $B_L = 0,\ 1$, where 0 denotes logical *false* and 1 denotes logical *true*. An immediate feature that arises is that no inverse exists for the $\vee$ operator. This is somewhat analogous to losing the mathematical ability to subtract. Given that feedback control has a long history of using subtraction to generate error signals at feedback junctions, the question arises of how to generate an error signal for the Boolean representation being discussed. One potential solution is to use the logical exclusive or operation (*xor*), which has the truth table shown in Table III. Observe that when the command and feedback signals differ, the error signal has a value of 1, and is 0 otherwise. A discrete event controller could be designed to act whenever the error signal is true. Clearly, this can be extended to the case where the command, feedback and error signals are vectors as opposed to scalars.

TABLE III
Truth Table for Logical XOR Function

Command	Feedback	Error
0	0	0
0	1	1
1	0	1
1	1	0

B. Reachability

A major question asked of discrete event control designs is whether there exist states that cannot be entered under any conditions, and correspondingly, whether there are states that once entered, cannot be left. These questions are addressed by the concept of *reachability*. Using the state space model form presented in this paper, an exhaustive simulation can be constructed to determine the answer. A more elegant mathematical solution is sought, however, building upon the similarity of the discrete event state equation in Eq. 3 to the discrete time, time-varying state equation. The reachability results for the discrete time case, although widespread (*e.g.,* [8], must be modified to work with the Boolean framework of this model.

V. CONCLUSION

The specialized state space realization provides a foundation for mathematically analyzing qualitative properties of the discrete event system. Of specific interest are the classical ideas of stability, reachability and observability similar to those of linear discrete time time-varying systems, and stability and performance criteria. The generation of analytical qualitative results is a key potential feature of the paradigm because rigorous validation of the discrete event controller design is then possible. In particular, the concept of reachability could be developed into a means of analyzing whether a plant can get into and out of operational modes as required without the necessity of conducting exhaustive (and potentially damaging in the case of faults and failures) simulations or tests.

VI. ACKNOWLEDGMENT

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