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Satellite Structure Attitude Control with Parameter Robust Risk-Sensitive Control Synthesis

Chang-Hee Won, *Member, IEEE*

***Abstract*— We present the Parameter Robust Risk-Sensitive control theory to deal with the parameter uncertainties of a system. In this synthesis the parameter uncertainties are decomposed into three parts, and are reflected in the cost function and the noise models. This new method improves the tolerance of a controller to parameter uncertainties in a system. We apply the newly developed method to satellite structure attitude control application. The linearized version of the satellite structure attitude model is derived for the attitude hold mode. To determine the performance of the Parameter Robust Risk-Sensitive control method, the steady-state mean square histories of the state and input variables are calculated by finding the covariance matrices. Moreover, the performance robustness of Parameter Robust Risk-Sensitive control are compared with the classical Linear Quadratic Gaussian control. Finally, as an application of this Parameter Robust Risk-Sensitive control, we consider commercially operating remote sensing satellite. The numerical simulations show that the Parameter Robust Risk-Sensitive control method has better performance and stability characteristics than the classical Linear Quadratic Gaussian control method, especially when the parameter uncertainties are large.**

I. Introduction

THE linear quadratic Gaussian (LQG) control method has found applications in the spacecraft control as early as 1960s [1]. In the paper by Wie and *et al*., an approach to control the attitude of the space station has been proposed [2]. There, they found the linearized equations of motion and attitude kinematics, and used the linear quadratic regulator (LQR) and the pole placement techniques to control the attitude of the space station. In 1992, Parlos and Sunkel derived the fully coupled equations of motion and linearized them around an equilibrium point for a spacecraft with control moment gyros [3]. They used a full state feedback LQR (H_2) controller with gain scheduled adaptation. For a recent survey of attitude representations see [4]. In 1996, Ballois and Duc noted that one LQR controller is required for one position of the solar array, and the controller change is needed for other positions. Thus they proposed an H_∞ controller to meet the performance and robustness objectives [5]. But the H_∞ controller had its share of disadvantages: the order of the controllers was high and the controller was overly conservative. As recently as 1997, Paynter and Bishop investigated the use of nonlinear feedback linearization for attitude control and momentum management of an evolutionary spacecraft [6].

Even though the LQG method has been widely used in spacecraft maneuver applications, the Risk-Sensitive (RS) control, which is a generalization of LQG control, is relatively unused in space applications. In RS control, the cost function is given as the expected value of the exponential of the usual LQG cost function, and RS cost function reduces to the classical LQG cost function when the risk-sensitivity parameter assumes a particular value (i.e., the risk sensitivity parameter is zero, $\sigma = 0$). This RS idea, which is related to the dynamic games and H_∞ control methods, seems to have originated from Jacobson in 1973 [7]. The linear finite horizon RS problem of the full-state-feedback RS control case has been introduced and solved by Jacobson. In 1985 Bensoussan and van Schuppen solved the partially observable RS control problem [8]. In 1991 Whittle introduced the risk-sensitive maximum principle in [9], and coined the term Risk-Sensitive control. Furthermore, Runolfsson solved the infinite horizon RS case for the risk-aversive case (the risk sensitivity parameter is positive, $\sigma > 0$) [10]. The performance and stability characteristics of RS controlled structures have been studied in [11]..

Even though LQG control has good nominal performance and stability characteristics, it can be sensitive to modeling error [12]. Doyle and Stein proposed LQG/LTR method to deal with this problem [13]. More recently, in 1995 Lin developed an algorithm for the multi-input multi-output robust control system design by the LEQG/LTR method [14]. Another technique to deal with parameter uncertainties came from Tahk and Speyer in 1987 [15]. Tahk and Speyer developed a parameter robust linear-quadratic-Gaussian (PRLQG) method. They modeled the parameter variation using an internal feedback loop. Then the parameter variation is represented using the input/output decomposition. Furthermore, Lin and Mingori introduced LQG synthesis with reduced parameter sensitivity in 1992 [16].

LQG, PRLQG and RS control theory have been developed and studied by various researchers, but parameter robust risk-sensitive (PRRS) control theory has only been addressed by the author [17]. Thus, here we present the abridged version of the PRRS theory. This PRRS synthesis can be considered as an extension of the Lin and Mingori's PRLQG synthesis. Then we propose to use the presented PRRS method in satellite attitude control.

In the next section we present a linearized satellite attitude model for the attitude hold mode. The PRRS control design method is developed after modeling of the satellite attitude. Then the average behavior and stability robustness of PRRS control are considered. Numerical simulations are performed with the real parameters of the KOMPSAT, Korea Multipurpose Satellite. Finally, conclusions are given in the last section.

II. Linear Satellite Attitude Model

We use the nonlinear satellite attitude model of with thrusters, magnetic torquers, and reaction wheels and find the linear model. The reaction wheel configuration is shown in Fig. 1. We incorporate the gravitational torque into the system equation, but consider magnetic and aerodynamic torques as external disturbances. The largest torque affecting the satellite is the gravitational torque. Magnetic and aerodynamic torques are an order of magnitude smaller, and the solar radiation pressure (diffuse reflection and specular reflections) is two orders of magnitude smaller. We are also ignoring the effects due to solar luna gravitation and charged particles.

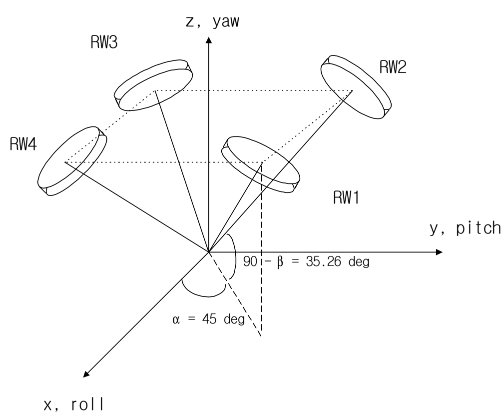

Fig. 1. Reaction wheel cluster configuration in BFC

The nonlinear model is

$$I_g \underline{\dot{\omega}} = -\underline{\omega}\times\left(I_t\underline{\omega}+L' I_w \underline{\Omega}\right) - L'\underline{\tau}_w + \underline{\tau}_{thruster} + 3n^2 \underline{c}_3^{\times} I_t \underline{c}_3 + \underline{w}$$

where $\underline{w} = \underline{\tau}_{aero} + \underline{\tau}_{magnetc} + \underline{\tau}_{srp}$,

I_t : Total moment of inertia for the satellite body (3x3),

I_w : Moment of inertial matrix for the wheels (4x4),

$I_g = I_t - L' I_w L$: Total moment of inertia minus the moment of inertia of the wheels (3x3),

L : Wheel attitude matrix (4x3),

$\underline{\omega}$: Angular velocity in body fixed coordinate (3x1),

$\underline{\Omega}$: Wheel speed vector,

$\underline{\tau}_w$: Absolute torque due to the reaction wheels,

$\underline{\tau}_{thruster}$: Torque due to the thrusters,

$\underline{\tau}_{gravity}$: Torque due to the Earth's gravity gradient,

$\underline{\tau}_{aero}$: Torque due to the aerodynamic atmospheric drag,

$\underline{\tau}_{magnetic}$: Torque due to the magnetic field,

$\underline{\tau}_{srp}$: Torque due to the solar radiation pressure,

$\underline{h}_w$: Angular momentum of the wheel cluster,

and the attitude matrix L is given by

$$L = \begin{bmatrix} \cos\alpha\sin\beta & \sin\alpha\sin\beta & \cos\beta \\ -\sin\alpha\sin\beta & \cos\alpha\sin\beta & \cos\beta \\ -\cos\alpha\sin\beta & -\sin\alpha\sin\beta & \cos\beta \\ \sin\alpha\sin\beta & -\cos\alpha\sin\beta & \cos\beta \end{bmatrix}$$

where $\alpha = 45^o$ and $\alpha = 54.74^o$.

If we assume small attitude variations from LVLH coordinates, the above equation can be linearized. To have zero initial conditions we let $\delta\omega_y = \omega_y + n$ where n is the orbital rate, and find the general linear equation for any torque equilibrium attitude (TEA) of the following states,

$$\underline{x} = [\phi,\theta,\psi,\omega_x,\delta\omega_y,\omega_z,\Omega_1,\Omega_2,\Omega_3,\Omega_4]' \equiv [\underline{\phi_e},\underline{\omega},\underline{\Omega}]' .$$

The linear equation can be found for any TEA. Moreover it is possible to find the estimated average TEA value for a given configuration over an orbit, after equilibrium conditions have been reached. For the simplicity sake, here we assume the case of the attitude hold mode where the TEA values are fixed as a null vector: $\underline{x}_0 = [0,0,0,0,0,0,0,0,0,0]'$. To uncouple the pitch from yaw/roll and simplify the calculations, we assume that the products of inertia of the satellite body are small (i.e., $It(3,2) = It(2,3) = It(1,2) = It(2,1) = It(3,1) = It(1,3) = 0$).

Assuming 2-3-1 (Pitch-yaw-roll) body-axes sequence attitude kinematics and Pitch(θ)-yaw(ψ)-Roll(ϕ) are small in magnitude, the nonlinear attitude kinematics can be linearized around the TEA to obtain

$$\frac{d}{dt}\begin{bmatrix} \phi \\ \theta \\ \psi \\ \omega_x \\ \delta\omega_y \\ \omega_z \\ \Omega_1 \\ \Omega_2 \\ \Omega_3 \\ \Omega_4 \end{bmatrix} = A \begin{bmatrix} \phi \\ \theta \\ \psi \\ \omega_x \\ \delta\omega_y \\ \omega_z \\ \Omega_1 \\ \Omega_2 \\ \Omega_3 \\ \Omega_4 \end{bmatrix} + B \begin{bmatrix} \underline{\tau}_w \\ \underline{\tau}_{thruster} \end{bmatrix} + E \ \underline{w} \qquad (1)$$

where

$$A=\begin{bmatrix} 0 & 0 & n & 1 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 1 & 0 & 0_{3\times4} \\ -n & 0 & 0 & 0 & 0 & 1 & \\ 3I_g^{-1}n^2N_3 & & & I_g^{-1}nN_1 & & & I_g^{-1}nN_2 \\ -L3I_g^{-1}n^2N_3 & & & -LI_g^{-1}nN_1 & & & -LI_g^{-1}nN_2 \end{bmatrix}$$

$$B=\begin{bmatrix} 0_{3\times4} & 0_{3\times3} \\ -I_g^{-1}L' & I_g^{-1} \\ I_w^{-1}+LI_g^{-1}L' & -LI_g^{-1} \end{bmatrix}$$

$$E=\begin{bmatrix} 0_{3\times3} \\ I_g^{-1} \\ -LI_g^{-1} \end{bmatrix}, \text{ where}$$

$$N_1=\begin{bmatrix} 0 & 0 & I_t(3,3)-I_t(2,2) \\ 0 & 0 & 0 \\ I_t(2,2)-I_t(1,1) & 0 & 0 \end{bmatrix}$$

$$N_2=\begin{bmatrix} I_w(1,1)L_t(3,1) & I_w(2,2)L_t(3,2) & I_w(3,3)L_t(3,3) & I_w(4,4)L_t(3,4) \\ 0 & 0 & 0 & 0 \\ -I_w(1,1)L_t(1,1) & -I_w(2,2)L_t(1,2) & -I_w(3,3)L_t(1,3) & -I_w(4,4)L_t(1,4) \end{bmatrix}$$

$$\text{and } N_3=\begin{bmatrix} I_t(3,3)-I_t(2,2) & 0 & 0 \\ 0 & I_t(3,3)-I_t(1,1) & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

III. Parameter Robust RS Control Synthesis

To deal with the parameter uncertainties, the parameter robust Risk-Sensitive (PRRS) control synthesis is developed in this section. Consider a linear time-invariant system with parameter uncertainties:

$$\frac{dx(t)}{dt} = (A + \Delta A)x + (B + \Delta B)u + E\,w \qquad (2)$$

$$y = (C + \Delta C)x + v,$$

where x, u and y denote the state vector, the input vector, and the output vector respectively. The w and v represent Gaussian white noise with zero mean and covariances of $E\{\,w(t)\,w'(\tau)\} = W\delta(t-\tau)$ and $E\{\,v(t)\,v'(\tau)\} = V\delta(t-\tau)$. Furthermore $A, B,$ and C represent the nominal system matrix: and $\Delta A, \Delta B$, and ΔC are appropriate perturbation matrices. The PRRS performance index is given as

$$J(x,t_0) = \sigma E\left\{\exp\left(\frac{\sigma}{2}\right)\hat{J}(T)\right\}, \qquad (3)$$

where σ is a real number called the *PRRS parameter*, and

$$\hat{J}(T) = x'(T)\,Q_T\,x(T) + \int_0^T \left[x'(t)\,Q\,x(t) + u'(t)\,R\,u(t)\right] dt,$$

in which superscript $(')$ denotes vector transposition. The word, risk-sensitive, is introduced because this PRRS parameter increases or decreases the cost (risk) depending on the sign of the PRRS parameter [9]. As is customary in such costs, the matrices Q and R are both symmetric, with Q being positive semidefinite and R being positive definite. Also assume that *(A,B)* and *(A,E)* are both controllable pairs, and that *(Q,A)* is an observable pair.

We repeat the results of Lin and show the Risk-Sensitive control solution of the nominal system. Using the results of Lin [15], we apply the separation theorem and obtain the following state estimation equation for the nominal system.

$$\dot{\hat{x}} = A\,\hat{x} + B\,u + K_f(y - C\,\hat{x}),$$

where K_f is the Kalman filter gain given by $K_f = PC'V^{-1}$. The error covariance matrix, $P = E\{\,(x-\hat{x})(x-\hat{x})'\}$ is given by

$$\dot{P} = AP + PA' + EWE' - PC'V^{-1}CP, \qquad (4)$$

where the initial condition is $P(0) = P_0$. The controller to this output feedback RS problem is given as follows.

$$u^*(t) = -R^{-1}B'S\,\hat{x}$$

where S is a positive-definite symmetric matrix which satisfies the Riccati equation,

$$-\dot{S} = Q + SA + A'S - S(BR^{-1}B' - \sigma K_f V K_f')S,$$

with the final condition $S(T) = Q_T$. The dynamic controller is described by the following set of equations.

$$\dot{\hat{x}} = A_c\,\hat{x} + B_c\,y \qquad (5)$$

$$u = C_c\hat{x}$$

where

$$A_c = A - B\,K_c - K_f\,C,\; B_c = L,\; C_c = -K_c$$

$$K_c = R^{-1}B'S.$$

In frequency domain the controller is given as

$$K(s) = K_c(sI - Ac)^{-1}K_f.$$

Now, we are ready to deal with the parameter uncertainties of the PRRS system. The perturbations are decomposed into three parts; input, output, and feedback matrices. We have

$$\Delta A = M_a\,L_a(\varepsilon)\,N_a,\; \Delta B = M_b\,L_b(\varepsilon)N_b,$$
$$\Delta C = M_c\,L_c(\varepsilon)N_c \qquad (6)$$

where $L_a(\varepsilon)$, $L_b(\varepsilon)$, and $L_c(\varepsilon)$ are diagonal matrices which are zero when ε is zero, and $M_a, N_a, M_b, N_b, M_c, N_c$ are non-unique matrices with full rank. This is called the input/output decomposition method, and for further details refer to [16]. Note that when ε is zero then we have the nominal PRRS (i.e., RS) case without any parameter uncertainties.

We introduce three fictitious white noise inputs, w_a, w_b, and w_c with unit intensity and uncorrelated with each other. Also introduce three outputs z_a, z_b, and z_c, where they are related to the fictitious inputs by the following equations

$$w_a = L_a(\varepsilon)\, z_a,\ w_b = L_b(\varepsilon)\, z_b,\ w_c = L_c(\varepsilon)\ z_c, \quad z_a = N_a\, x,\ z_b = N_b u,\ z_c = N_c\ x \quad . \qquad (7)$$

The internal loops are related to the magnitude of z_a, z_b, z_c and to the effects of w_a, w_b, w_c on u and y. Heuristically we may think of reducing the effects of parameter variations by keeping auxiliary outputs (z_a, z_b, z_c) small while auxiliary disturbance inputs (w_a, w_b, w_c) attenuated. This leads to a new PRRS problem:

$$\dot{x} = (A + \mu_a\, M_a\, L_a(\varepsilon)\, N_a)\, x + (B + \mu_b\, M_b\, L_b(\varepsilon)\, N_b)\, u + \mu_e\, E\, w$$
$$y = (C + \mu_c M_c L_c(\varepsilon))x + \mu\ \ v$$

In order to deal with the parameter uncertainties, we use equation (7). The parameter uncertainties are reformulated using the auxiliary disturbance inputs (w_a, w_b, w_c) and the process disturbance w. Moreover, we assume that (w_a, w_b, w_c) and w are all mutually uncorrelated of each other. Then let

$$\eta_a(t) = \mu_a\, M_a\, w_a + \mu_b M_b\, w_b + \mu_e\, E w,$$

to obtain the new set of equations:

$$x = Ax + Bu + \eta_a$$
$$y = Cx + \xi_a$$

where

$$E\{\eta_a(t)\, \eta_a'(\tau)\} = [\mu_e^2\, EWE' + \mu_a^2\, M_a\, M_a' + \mu_b^2 M_b\, M_b']\, \delta(t-\tau)$$

and

$$E\{\xi_a(t)\xi_a'(\tau)\} = [\mu^2\, V + \mu_c^2\, M_c\, M_c']\, \delta(t-\tau).$$

To simplify the derivation without losing the generality, we let $Q_T = 0$ and $z = \sqrt{Q}x$. Then the cost given by equation (3), can be rewritten as

$$J(x,t_0) = \sigma\ E\left\{\exp\left(\frac{\sigma}{2}\int_0^T z'z + u'Ru\, dt\right)\right\}.$$

Now we introduce augmented $z_{new} = [z, z_a, z_b, z_c]'$ and new design parameters to obtain

$$J(x,t_0) = \sigma\ \mathrm{E}\left\{\exp\left(\frac{\sigma}{2}\int_0^T \rho_z^2 x'Qx + \rho_a^2 x'N_a'N_a\, x + \rho_b^2\, u'\, N_b'N_b\, u + \rho_c^2\, x'N_c'N_c\, x + \rho^2\ u'Ru\, dt\right)\right\}$$

Ten new design parameters ρ's and μ's are introduced. The parameters ρ_z, ρ, μ_e, and μ are related to the nominal performance and $\rho_a, \rho_b, \rho_c, \mu_a, \mu_b$ and μ_c are related to the robustness enhancement. In this design the uncertainties in the parameter variations are taken into account through the weighting matrices. Thus, this design can enhance robustness and performance by varying the new design parameters.

IV. Average Behavior of PRRS Control

It is important to know the average behavior of a stochastic system. Thus, in this section we derive the average behavior of the PRRS control system. From equation (2) and (5), we form an augmented closed loop system;

$$\frac{d}{dt}\begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} = A_{fd}\begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} + \begin{bmatrix} E & 0 \\ 0 & K_f \end{bmatrix}\begin{bmatrix} w(t) \\ v(t) \end{bmatrix},$$

where $A_{fd} = \begin{bmatrix} A + \Delta A & -(B + \Delta B)K_c \\ K_f(C + \Delta C) & A_c \end{bmatrix}$. Then we obtain

$$\begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} = \int_0^t \exp\ (A_{fd}\ (t-s))\begin{bmatrix} E & 0 \\ 0 & K_f \end{bmatrix}\begin{bmatrix} w(s) \\ v(s) \end{bmatrix} ds,$$

for $x(0) = x_0, \hat{x}(0) = 0$. Using the above equation we obtain

$$\Sigma(t) = E\left\{\begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix}\begin{bmatrix} x'(t) & \hat{x}'(t) \end{bmatrix}\right\} \equiv \begin{bmatrix} \Sigma_{xx} & \Sigma_{x\hat{x}} \\ \Sigma_{\hat{x}x} & \hat{\Sigma}_{\hat{x}\hat{x}} \end{bmatrix}$$
$$= \int_0^t \left[\exp(A_{fd}(t-s))\begin{bmatrix} EWE' & 0 \\ 0 & K_f V K_f' \end{bmatrix}\exp(A_{fd}'(t-s))\right] ds.$$

Now we take time derivative and use the Leibniz formula to get

$$\frac{d\Sigma(t)}{dt} = \begin{bmatrix} EWE' & 0 \\ 0 & K_f V K_f' \end{bmatrix} + A_{fd}\ \Sigma(t) + \Sigma(t) A_{fd}'.$$

Let error $e = x - \hat{x}$, then we have

$$E\{x\, x'\} = E\{(e + \hat{x})(e + \hat{x})'\} = E\{ee' + \hat{x}\, e' + e\, \hat{x}' + \hat{x}\, \hat{x}'\}$$

and assuming that e is uncorrelated with $\hat{x}$ we get

$$E\{xx'\} = E\{ee' + \hat{x}\,\hat{x}'\} = P(t) + \hat{\Sigma}_{\hat{x}\hat{x}}(t),$$

where $P(t)$ is obtained from equation (4). And because $u = -R^{-1} B'S\hat{x}$, we finally obtain the following formula for average control.

$$E\{u\, u'\} = R^{-1}\, B'S\ \hat{\Sigma}_{\hat{x}\hat{x}} S\, B\, R^{-1}.$$

To find the steady-state mean square histories of the state variables, we have $V_x \equiv \lim_{t\to\infty} E\{x'(t)x(t)\} = \lim_{t\to\infty} Tr(E\{x(t)x'(t)\})$ and the steady-state mean square histories of the input variables,

$V_u \equiv \lim_{t\to\infty} E\{u'(t)u(t)\} = \lim_{t\to\infty} Tr(E\{u(t)u'(t)\})$. These values are used to determine the robust performance of a controller.

V. Simulation Results

This section contains various numerical simulations for the controllers described in the previous section, The actual parameters of the low earth orbiting remote sensing satellite, KOMPSAT, has been used in this simulation. The sensors and actuators in the KOMPSAT.

We shall assume that roll, pitch, and yaw angles are available using conical earth sensor, fine sun sensor, and gyro. Furthermore the wheel speeds are also available from the reaction wheel tachometer. Following parameters are used for the KOMPSAT model. The orbital rate is n =0.0010636 rad/s. Total moment of inertia for s/c body (3x3) assuming small products of inertia is given by

$$I_t = \begin{bmatrix} 2.967\times10^2 & 0 & 0 \\ 0 & 1.304\times10^2 & 0 \\ 0 & 0 & 2.111\times10^2 \end{bmatrix} \text{kg m}^2.$$

Moment of inertia matrix for the reaction wheels (4x4) is $I_w = 0.0077 \quad I_4$ kg m^2, where, I_4 represents a four by four identity matrix. The two angles associated with the wheel attitude matrix are $\alpha = 45\times\pi/180$ rad, and $\beta = 54.74\times\pi/180$ rad. KOMPSAT attitude is modeled by the following differential equation,

$$\frac{dx(t)}{dt} = (A + \Delta A)x + (B + \Delta B)u + E\,w$$
$$y = I_{10}x + v$$

where $A, B,$ and E are obtained using equation (1) with the above parameters. For the simulation purpose, we assumed the parameter uncertainties in the A matrix. The uncertainties in the A matrix may come from the satellite configuration change due to the solar panel and also due to the mass properties change during the mission lifetime. We perform input/output decomposition and let $M_a = A, L_a(\varepsilon) = \varepsilon,$ and $N_a = I_{10}$ in equation (6) to obtain $\Delta A = A\varepsilon$, and $\Delta B = 0$. Furthermore we use the following design parameters:

$Q = 10\times I_{10}, R = 1\times10^{-3}\times I_7, W = 1\times10^{-2}\times I_3,$
$V = 1\times10^{-3}\times I_{10},\ \rho_b^2 = \rho_c^2 = \mu_a^2 = \mu_b^2 = \mu_c^2 = 0$, and
$\rho_z^2 = \rho^2 = \mu_e^2 = \mu^2 = 1.$

Figure 2 shows the plot of the steady-state mean square histories of the state variables, V_x , versus parameter variation ε for the LQG $(\rho_a^2 = 0, \sigma = 0)$, PRLQG $(\rho_a^2 = 10, \sigma = 0)$, RS $(\rho_a^2 = 0, \sigma = 100)$, and PRRS $(\rho_a^2 = 10, \sigma = 100)$ cases. Note that PRRS control has the smallest steady-state mean square histories of the state variables for the same parameters. For example, PRRS control has 13.5% smaller steady-state mean square histories of the state variables than LQG control steady-state mean square histories of the state variables when ε =4.1. Note that ρ_a^2 and μ_a^2 can be varied to adjust robustness.

Figure 3 shows the plot of the steady-state mean square histories of the input values, V_u , versus parameter variation ε for LQG $(\rho_a^2 = 0, \sigma = 0)$, PRLQG $(\rho_a^2 = 100, \sigma = 0)$, RS $(\rho_a^2 = 0, \sigma = 100)$, and PRRS $(\rho_a^2 = 100, \sigma = 100)$ cases. PRRS control has the largest steady-state mean square histories of the input values for ε between –0.9 and 5.9. In all the cases, the steady-state mean square histories of the input values are largest when $\varepsilon = 5.9$ and smallest when $\varepsilon = -0.9$. This implies that more control effort is required as the uncertainties become larger. We also note that the PRRS case requires the most control effort and PRLQG case the least.

Figures 4 shows the average roll, pitch, and yaw values as the parameter uncertainty, ε , varies. In all three cases, for ε between –0.9 and 5.9, the PRRS case has the smallest values and then the RS case, and then the LQG case. Moreover, the LQG and PRLQG average angles increase much faster than the RS and PRRS cases. Thus, we note that PRRS is more robust when the uncertainties are large. It is interesting to note that PRLQG case shows a little bit larger average roll, pitch, and yaw values compared to LQG case for small ε , and then becomes smaller when ε is between 4 and 5.

Even though it is not shown, comparing PRRS and LQG control efforts, we note that PRRS control effort is larger. RS and PRLQG falls in between LQG and PRRS curves.

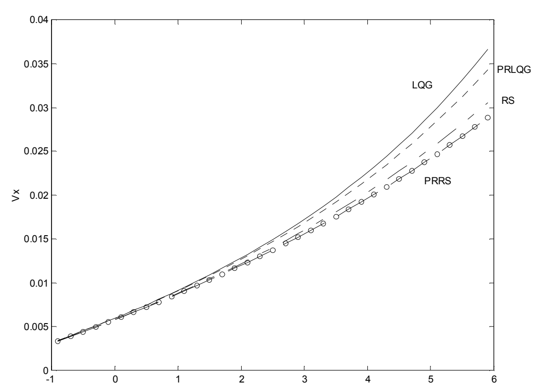

Fig. 2. Steady-State Mean Square Histories of the State Variables versus Parameter Uncertainty

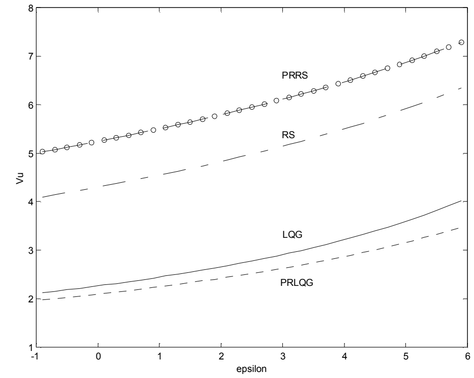

Fig. 3. Steady-State Mean Square Histories of the Input Variables versus Parameter Uncertainty

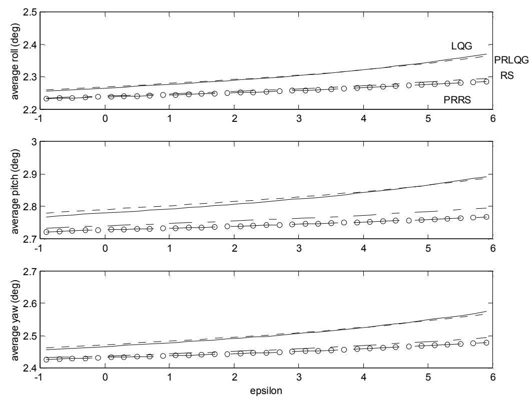

Fig. 4. Average Roll, Pitch, and Yaw Angle versus Parameter Uncertainty

VI. CONCLUSION

The RS method is introduced in satellite attitude control. The parameter robust Risk-Sensitive control synthesis method is newly derived to deal with the structural parameter uncertainties in a satellite attitude model. The steady-state mean square values are used to determine its performances, and the maximum real parts of the closed loop eigenvalues are used to determine the stability characteristics. It is shown that the performance of PRRS control out performs classical LQG, RS, and PRLQG controllers, especially when the uncertainties become larger.

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