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Output Feedback Stabilization of Uncertain Non-minimum Phase Nonlinear Systems

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Abstract— **An output feedback design technique is presented, by means of which it is possible to achieve semi-global practical stabilization for a class of non-minimum phase nonlinear systems, subject to parameter uncertainties. This work is an extension of the result by (Isidori, 2000). It provides a constructive controller design method for an auxiliary system, whose existence is crucial, but is assumed in (Isidori, 2000). Simulation results demonstrate satisfactory stabilization performances.**

I. INTRODUCTION

In practice, controller implementation is generally subject to limited number of measurements available for feedback, due to sensor cost and/or availability. In such cases, one relies on the design of controller based on imperfect state measurements, often termed output feedback controller design. Most existing output feedback controller designs require that the zero dynamics of the controlled plant be stable, that is, to be minimum phase. However, a number of processes exhibit non-minimum phase (i.e. inverse response) behavior. For instance, this phenomenon can be encountered in a continuous exothermic reactor, where the inlet stream flowrate is used to control the reactor temperature. In this situation, a positive step change in the inlet flowrate will cause an initial decrease in the reactor temperature [8].

For linear systems, the output feedback control of non-minimum phase systems is generally solved by factorizing the system dynamics into a minimum phase and non-minimum phase part [10]. Only the invertible minimum phase part is considered in controller design, while the non-minimum phase part, generally viewed as an obstacle to closed-loop performances, remains in the open-loop. Although attempts have been made to generate such factorization in the nonlinear case, the problem remains open. A non-minimum phase compensation structure for nonlinear systems was developed in [14], based on a synthetic output, which is minimum phase and statically equivalent to the original output. The synthetic output can either be chosen in an ISE-optimization formulation [8], or by prescribing zeros in a systematic manner [9]. However, the afore-mentioned approaches can only be applied to open-loop stable processes. In [6], a semi-global practical stabilization design tool is proposed for a general class of uncertain non-minimum phase nonlinear systems. This approach is based on the assumption of global stabilizability of an *auxiliary* system. In addition, the authors identify a class of nonlinear systems that are semi-globally stabilizable via "uniformly completely observable" (UCO) functions in the sense of [11]. In [4], an adaptive dynamic output feedback stabilization tool is proposed, for a class of nonlinear systems. The high-gain result is extended by considering the adaptation of the gain parameter as a time varying scalar function, which depends on the magnitude of the output and a quantity of the dynamic feedback compensator.

For the related problem of output tracking for non-minimum phase nonlinear systems, there are two major approaches in the literature. In the first approach proposed in [1] and [2], and further modified in [12], the stabilizing control consists of a feedforward component that generates the zero dynamics trajectory, and a feedback component that stabilizes the whole system. This approach is based on the assumption that the inverse system is kinematically hyperbolic, with slowly time-varying outputs that have small amplitude. The second approach is the nonlinear output regulator problem [5]. This approach uses center manifold theory, and gives necessary and sufficient conditions under which the closed-loop system can be driven to a center manifold contained in the output zeroing manifold. This approach yields a local result around the equilibrium point. In addition, if only the output information is available, it requires detectablility of the linearized system, which implies that the system has to be locally minimum-phase.

In this work, we examine the assumption from [6], stating that "the auxiliary system is globally stabilizable by dynamic output feedback". We provide a controller design procedure and outline the requirements for auxiliary systems with relative degree zero, a problem not addressed in [6].

This paper is organized as follows, the main results are provided in section 2, and simulation results are presented in section 3, followed by conclusions in section 4.

II. OUTPUT FEEDBACK CONTROLLER DESIGN

A. Problem Formulation and Motivation

We first give a brief review of the results in [6].

Consider a smooth nonlinear system modelled by equations of the following form

$$\begin{array}{rcl}\dot{z} &=& f_0(z, x_1, \cdots, x_{r-1}, x_r, p) \\ \dot{x}_1 &=& x_2 \\ &\vdots& \\ \dot{x}_{r-1} &=& x_r \\ \dot{x}_r &=& h_0(z, x_1, \cdots, x_{r-1}, x_r, p) + b(x_1)u \\ y &=& x_1,\end{array} \quad (1)$$

where $z \in \mathbb{R}^{n-r}$, and p is a (possibly vector-valued) unknown parameter, ranging over a compact set $\mathcal{P}$. The following is assumed for system (1).

Assumption 1: For all $p \in \mathcal{P}$

$$\begin{array}{rcl} f_0(0, \cdots, 0, p) &=& 0 \\ h_0(0, \cdots, 0, p) &=& 0 \end{array}$$

and $b(x_1) \neq 0$.

The control objective is to stabilize system (1) using a robust output feedback, given that the system is non-minimum phase. The solution of this problem is based on the existence of a dynamic output feedback controller for an auxiliary system associated with system (1). The auxiliary system is defined as follows,

$$\begin{array}{rcl} \dot{x}_a &=& f_a(x_a, u_a, p) \\ y_a &=& h_a(x_a, u_a, p) \end{array} \quad (2)$$

where

$$\begin{array}{rcl} x_a &=& \begin{pmatrix} z \\ x_1 \\ \cdots \\ x_{r-2} \\ x_{r-1} \end{pmatrix} \\ f_a(x_a, u_a, p) &=& \begin{pmatrix} f_0(z, x_1, \cdots, x_{r-1}, u_a, p) \\ x_2 \\ \cdots \\ x_{r-1} \\ u_a \end{pmatrix} \\ h_a(x_a, u_a, p) &=& h_0(z, x_1, \cdots, x_{r-1}, u_a, p). \end{array}$$

The basic hypothesis about the auxiliary system (2) is the knowledge of a robust global dynamic output feedback stabilizer of the following form ([6], Assumption 2),

$$\begin{array}{rcl} \dot{\eta} &=& L(\eta) + M y_a \\ u_a &=& N(\eta) \end{array} \quad (3)$$

in which $\eta \in \mathbb{R}^{\nu}$, $L(0) = 0, N(0) = 0$, and M is a $\nu \times 1$ constant matrix.

Under the above assumption, it is proved in [6] that system (1) can be semi-globally practically stabilized by the following dynamic output feedback law

$$\begin{array}{rcl} \dot{\xi} &=& P\xi + Qy \\ \dot{\eta} &=& L(\eta) + M\sigma_L(k[\xi_r - N(\eta)]) \\ u &=& \frac{1}{b(x_1)}\Big[\frac{\partial N}{\partial \eta}(L(\eta) + M\sigma_L(k[\xi_r - N(\eta)]) \\ && -\sigma_L(k[\xi_r - N(\eta)])\Big] \end{array} \quad (4)$$

where k is a positive number, $\sigma_L(\cdot)$ is a saturation function

$$\sigma_L(r) = \begin{cases} r, & \text{if} |r| < L \\ sgn(r)L, & \text{if} |r| \geq L, \end{cases}$$

and ξ_r is the estimation of state x_r under the following high gain observer

$$\begin{array}{rcl} \dot{\xi} &=& \begin{pmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \cdots \\ \dot{\xi}_{r-1} \\ \dot{\xi}_r \end{pmatrix} = \begin{pmatrix} \xi_2 + gc_{r-1}(y - \xi_1) \\ \xi_3 + g^2 c_{r-2}(y - \xi_1) \\ \cdots \\ \xi_r + g^{r-1}c_1(y - \xi_1) \\ g^r c_0(y - \xi_1) \end{pmatrix} \\ &=:& P\xi + Qy, \end{array}$$

in which $c_0, c_1, \cdots, c_{r-1}$ are the coefficients of some Hurwitz polynomial, g is a positive number.

There are two limitations to the above approach. First, even though it is shown in [6] that the assumption of global stabilizability of the auxiliary system is not restrictive, how to find such a dynamic output feedback is not a trivial task. Second, it is also observed that, for most applications, the auxiliary output h_a is a function of u_a, in other word, the auxiliary system (2) has relative degree zero, which makes the problem even more difficult.

It is suggested in [6] that the relative degree zero problem be solved by moving the u_a term to the control u, provided that y_a is a linear function of u_a. However, this is not always possible, as shown in the following illustrative example.

Example I Consider a continuous stirred tank reactor (CSTR), where the series/parallel van de Vusse reaction [13] is taking place:

$$\begin{array}{rcl} A & \xrightarrow{k_1} & B \xrightarrow{k_2} C \\ 2A & \xrightarrow{k_3} & D \end{array}$$

where A is the reactant, B the desired product, C and D are unwanted by-products.

The dynamics of the CSTR can be described in terms of the material balance for species A and B and an energy balance for the reactor as follows:

$$\begin{array}{rcl} \frac{dC_A}{dt} &=& -k_1(T)C_A - k_3(T)C_A^2 + (C_{A0} - C_A)u \\ \frac{dC_B}{dt} &=& k_1(T)C_A - k_2(T)C_B - C_B u \\ \frac{dT}{dt} &=& \frac{1}{\rho C_p}\Big[(-\triangle H_1)k_1(T)C_A + (-\triangle H_2)k_2(T)C_B \\ && +(-\triangle H_3)k_3(T)C_A^2 + Q\Big] + (T_0 - T)u \end{array}$$

where C_A, C_B are the concentrations of the species A and B inside the reactor, respectively; T is the temperature inside the reactor; C_{A0} is the concentration of A in the feed stream; T_0 is the feed stream temperature; $k_i(T)$ is the rate coefficient given by the Arrhenius expressions, $k_i(T) = k_{i0}\exp(-E_i/RT)$, $i = 1, 2, 3$; u is the dilution rate, given by $u = F/V$, where F is the inlet flow rate, and V is the reactor volume (assumed constant); ρ and C_p are the density and specific heat of the reaction mixture, respectively; $-\triangle H_i$, $i = 1, 2, 3$ are the heat of reactions, $-Q$ is the cooling rate per unit volume.

The control objective is to make the output $y = C_B$ track its setpoint, by manipulating the dilution rate, $u = F/V$.

Assuming that $C_B \neq 0$, the following change of variables, $z_1 = \frac{C_{A0}-C_A}{C_B}$, $z_2 = \frac{T_0 - T}{C_B}$, $x_1 = C_B$, $y = x_1$, transforms the CSTR dynamics into the normal form:

$$\begin{aligned}
\dot{z}_1 &= \tfrac{1}{x_1}\Big[(1-z_1)k_1(C_{A0}-z_1x_1) \\
&\quad +k_3(C_{A0}-z_1x_1)^2 + k_2z_1x_1\Big] \\
\dot{z}_2 &= \tfrac{-1}{\rho C_p x_1}\Big[\\
&\quad (-\triangle H_1)k_1(C_{A0}-z_1x_1) + (-\triangle H_2)k_2x_1 \\
&\quad +(-\triangle H_3)k_3(C_{A0}-z_1x_1)^2 + Q\Big] \\
&\quad -\tfrac{z_2}{x_1}[k_1(C_{A0}-z_1x_1) - k_2x_1] \\
\dot{x}_1 &= k_1(C_{A0}-z_1x_1) - k_2x_1 - x_1u \\
y &= x_1.
\end{aligned} \tag{5}$$

It can be shown later in section III-A that system (5) is locally non-minimum phase around a reference steady state.

The auxiliary system associated with (5) is as follows,

$$\begin{aligned}
\dot{z}_1 &= \tfrac{1}{u_a}\Big[(1-z_1)k_1(C_{A0}-z_1u_a) \\
&\quad +k_3(C_{A0}-z_1u_a)^2 + k_2z_1u_a\Big] \\
\dot{z}_2 &= \tfrac{-1}{\rho C_p u_a}\Big[\\
&\quad (-\triangle H_1)k_1(C_{A0}-z_1u_a) + (-\triangle H_2)k_2u_a \\
&\quad +(-\triangle H_3)k_3(C_{A0}-z_1u_a)^2 + Q\Big] \\
&\quad -\tfrac{z_2}{u_a}[k_1(C_{A0}-z_1u_a) - k_2u_a] \\
y_a &= k_1(C_{A0}-z_1u_a) - k_2u_a.
\end{aligned} \tag{6}$$

It is observed from the last equation in (6) that the term multiplying u_a is $-k_1z_1 - k_2$, which depends on the zero dynamics z_1 and z_2. Since the zero dynamics are not observable, we can not move this term to the control u to make the relative degree greater than zero as in [6].

In order to alleviate the two limitations, we provide in the next section a systematic design procedure to construct a dynamic output feedback for the relative degree zero auxiliary system (2). It is shown that the original system (1) can still be semi-globally practically stabilized.

B. Controller Design

To construct a robust dynamic output feedback for the auxiliary system (with relative degree zero), we add an integrator $\dot{u}_a = v$ on the auxiliary input u_a. The auxiliary system (2) becomes

$$\begin{aligned}
\dot{z} &= f_0(z, x_1, \cdots, x_{r-1}, u_a, p) \\
\dot{x}_1 &= x_2 \\
&\vdots \\
\dot{x}_{r-1} &= u_a \\
\dot{u}_a &= v \\
y_a &= h_0(z, x_1, \cdots, x_{r-1}, u_a, p).
\end{aligned}$$

where v is the new control. Differentiating y_a, we have

$$\begin{aligned}
\dot{z} &= f_0(z, x_1, \cdots, x_{r-1}, u_a, p) \\
\dot{x}_1 &= x_2 \\
&\vdots \\
\dot{x}_{r-1} &= u_a \\
\dot{y}_a &= \tfrac{\partial h_0}{\partial x_a} f_a(z, x_1, \cdots, x_{r-1}, u_a, p) + \tfrac{\partial h_0}{\partial u_a} v
\end{aligned} \tag{7}$$

where x_a is the state vector as in (2).

The following assumptions are made for the auxiliary system (7).

Assumption 2: The zero dynamics of (7) is stable with respect to the output y_a.

Assumption 3: $\frac{\partial h_0}{\partial x_a} f_a(z, x_1, \cdots, x_{r-1}, u_a, p)$ is globally Lipschitz in z and locally Lipschitz in $(x_2, \cdots, x_{r-1})$.

Assumption 4: $\|\frac{\partial h_0}{\partial u_a}\| > \varepsilon > 0$, and the sign of $\frac{\partial h_0}{\partial u_a}$ is known.

Remark 1: Assumption 2 is not restrictive, as shown in [6], that a memoryless feedback transformation $u \to u + hy$ can render this property with a proper choice of h.

Under these assumptions, it is guaranteed that there exists a robust dynamic output feedback for the auxiliary system (2), as shown in the following lemma.

Lemma 2.1: There exists a smooth dynamic system of the form

$$\begin{aligned}
\dot{\eta} &= L(x_1, \cdots, x_{r-1}, \eta) + My_a \\
u_a &= \eta
\end{aligned} \tag{8}$$

in which $\eta \in \mathbb{R}$, $L(0) = 0$, M is a nonzero constant. In addition, there is a positive definite and proper smooth function $V(x_a, \eta)$ whose derivative along the trajectories of the interconnected system (2) and (8) is negative definite, i.e.,

$$\frac{\partial V}{\partial x_a} f_a(x_a, \eta, p) + \frac{\partial V}{\partial \eta}[L(x_1, \cdots, x_{r-1}, \eta, p) + My_a] < 0 \tag{9}$$

for all $(x_a, \eta) \neq (0, 0)$.

Proof: Under the above assumptions, it is obvious that the following controller

$$\begin{aligned}
v &= \frac{1}{\frac{\partial h_0}{\partial u_a}}\left(-k_0y_a - \frac{\partial h_0}{\partial x_a} f_a(0, x_1, \cdots, x_{r-1}, u_a)\right) \\
&= L(x_1, \cdots, x_{r-1}, u_a) - k_0(\frac{\partial h_0}{\partial u_a})^{-1} y_a.
\end{aligned} \tag{10}$$

is able to stabilize the auxiliary system (7), provided that k_0 is a large enough positive number. By Assumption 4, there exists a positive number m such that $(\|\frac{\partial h_0}{\partial u_a}\|)^{-1} \leq m$. Denoting

$$M = -k_0 m sgn\Big(\frac{\partial h_0}{\partial u_a}\Big),$$

the following controller

$$v = L(x_1, \cdots, x_{r-1}, u_a) + My_a \tag{11}$$

is able to stabilize the auxiliary system (7) as well.

Given that $\dot{u}_a = v$ and (11) stabilizes (7), it is implied that the dynamic output feedback (8) stabilize the auxiliary system (2). In addition, by the converse Lyapunov theorem [7], there exists a Lyapunov function $V(x_a, \eta)$ such that (9) is satisfied. □

Next, consider the dynamic state feedback control

$$\begin{aligned} \dot{\eta} &= L(x_1, \cdots, x_{r-1}, \eta) + Mk[x_r - \eta] \\ u &= \frac{1}{b(x_1)}\Big[L(x_1, \cdots, x_{r-1}, \eta) + Mk(x_r - \eta) \\ &\quad -k(x_r - \eta)\Big] \end{aligned} \tag{12}$$

where k is a positive number. Changing the state variable x_r into the new variable

$$\theta = x_r - \eta,$$

the interconnection of the feedback control (12) and system (1) becomes

$$\begin{aligned} \dot{x}_a &= f_a(x_a, \theta + \eta, p) \\ \dot{\theta} &= h_a(x_a, \theta + \eta, p) - k\theta \\ \dot{\eta} &= L(x_1, \cdots, x_{r-1}, \eta) + Mk\theta. \end{aligned} \tag{13}$$

Let B_R^k denote the closed cube

$$B_R^k = \{x \in \mathbb{R}^k : |x_i| \le R, 1 \le i \le k\}. \tag{14}$$

Consider the positive definite and proper function

$$W(x_a, \eta, \theta) = V(x_a, \eta + M\theta) + \theta^2$$

and let Ω_b denote the set

$$\Omega_b = \{(x_a, \eta, \theta) : W(x_a, \eta, \theta) \le b\}. \tag{15}$$

Then Lemma 2 in [6] applies to the closed-loop system (13) in the same manner, which shows that the original system (1) is semi-globally practically stabilizable by (12).

Lemma 2.2: (Lemma 2 in [6]) For any $R > 0$ and $\epsilon > 0$, and for any $\rho > 0$ and $c > 0$ such that

$$\Omega_\rho \subset B_\epsilon^{n+1} \subset B_R^{n+1} \subset \Omega_c$$

where B_ϵ^{n+1} and B_R^{n+1} are defined as in (14), and Ω_ρ and Ω_c are defined as in (15), then there is a number k^* such that, if $k > k^*$, the derivative of the function $W(x_a, \eta, \theta)$ along the trajectories of (13) is negative at each point of the set

$$S = \{(x_a, \eta, \theta) : \rho \le W(x_a, \eta, \theta) \le c\}.$$

Proof: See [6].

The dynamic state feedback (12) uses the states $(x_2, \cdots, x_r)$, which need to be estimated. A high gain observer together with a saturation element are shown to provide a systematic design approach.

The resulting output feedback controller is as follows,

$$\begin{aligned} \dot{\xi} &= P\xi + Qy \\ \dot{\eta} &= L(y, \xi_2, \cdots, \xi_{r-1}, \eta) + M\sigma_L(k[\xi_r - \eta]) \\ u &= \frac{1}{b(y)}\Big[L(y, \xi_2 \cdots, \xi_{r-1}, \eta) + M\sigma_L(k[\xi_r - \eta]) \\ &\quad -\sigma_L(k[\xi_r - \eta])\Big]. \end{aligned} \tag{16}$$

The control law (16) takes similar form with the control law (4), which was developed in [6]. The only difference is the presence of the estimated states $(\xi_2, \cdots, \xi_{r-1})$ in (16), which does not add any difficulty in the stability proof. Therefore, Theorem 1 in [6] applies to the closed-loop system (16) and (1), which is stated below.

Theorem 2.1: Suppose Assumptions 1 to 4 hold and consider system (1). Given any arbitrary large number $R > 0$ and any arbitrary small number $\epsilon > 0$, there are numbers $k > 0$, $g > 0$, $L > 0$, $k_0 > 0$ such that, in the closed-loop system (16) and (1), any initial condition in B_R^{n+1+r} produces a trajectory which is captured by the set B_ϵ^{n+1+r}.

Proof: This proof amounts to showing that the presence of the estimated states in (16) will not affect the stability conditions imposed in the proof of the Theorem 1 in [6].

Consider the change of variable $\theta = x_r - \eta$, we have the following closed-loop system

$$\begin{aligned} \dot{x}_a &= f_a(x_a, \theta + \eta, p) \\ \dot{\theta} &= h_a(x_a, \theta + \eta, p) - \sigma_L(k[\xi_r - \eta]) \\ \dot{\eta} &= L(y, \cdots, x_{r-1}, \eta) + M\sigma_L(k[\xi_r - \eta]) \\ \dot{\xi} &= P\xi + Qy. \end{aligned} \tag{17}$$

Define the following scaled state estimation error

$$e_i = g^{r-i}(x_i - \xi_i)$$

for $i = 1, \cdots, r$, i.e.,

$$e = D_g(x - \xi)$$

in which $D_g = \text{diag}[g^{r-1}, \cdots, g, 1]$.

Then (17) can be written in the following perturbation form,

$$\begin{aligned} \dot{x}_a &= f_a(x_a, \theta + \eta, p) \\ \dot{\theta} &= h_a(x_a, \theta + \eta, p) - k\theta + \phi_1(\theta, e) \\ \dot{\eta} &= L(y, \cdots, x_{r-1}, \eta) + Mk\theta - M\phi_1(\theta, e) \\ \dot{e} &= gAe + B\phi_2(x_a, \theta, \eta, e, p) \end{aligned} \tag{18}$$

in which A is a Hurwitz matrix, $B = [0, 0, \cdots, 0, 1]^T$, with perturbations ϕ_1 and ϕ_2 as follows,

$$\begin{aligned} \phi_1(\theta, e) &= k\theta - \sigma_L(k\theta - ke_r) + \Big[L(y, \cdots, \xi_{r-1}, \eta) \\ &\quad -L(y, \cdots, x_{r-1}, \eta)\Big] \end{aligned}$$

$$\phi_2(x_a, \theta, \eta, e, p) = h_a(x_a, \theta + \eta, p) - \sigma_L(k\theta - ke_r) + \dot{\eta}.$$

It is shown in [6] that the key for the stability proof is to prove that the perturbation terms satisfy the following requirements: for all $((x_a, \eta, \theta), e) \in \Omega_{c+1} \times \mathbb{R}^r$

$$|\phi_1(\theta, e)| \le \beta_1$$
$$|\phi_2(x_a, \theta, \eta, e, p)| \le \beta_2$$
$$|\phi_1(\theta, e)| \le \gamma(\|e\|)$$

in which β_1, β_2 are fixed numbers, and $\gamma(\cdot)$ is a continuous function such that $\gamma(0) = 0$.

The only difference between the perturbation in this case with the perturbation term in [6] is the extra term $L(y, \cdots, \xi_{r-1}, \eta) - L(y, \cdots, x_{r-1}, \eta)$ in ϕ_1. However, the above requirements can be easily verified given Assumption 3.

The rest of the proof follows the proof in [6]. □

Remark 2: Note that the assumption of $\frac{\partial h_0}{\partial x_a} f_a(z, x_1, \cdots, x_{r-1}, u_a, p)$ being globally Lipschitz in z (Assumption 3) is rather restrictive. This can be relaxed to a more general locally Lipschitz assumption, which yields a semi-global stability result for the auxiliary system. Moreover, this relaxation doesn't affect the stability result in Theorem 2.1.

III. APPLICATION

A. Example I

Consider again the van de Vusse reaction system in section II-A. An example is the production of cyclopentenol (B) from cyclopentadiene (A) by acid-catalyzed electrophilic addition of water in dilute solution, where cyclopentanediol (C) and dicyclopentadiene (D) are also produced as side products [3].

The operating condition is $C_{A0} = 5$ gmol·L^{-1}, and $T_0 = 403.15$ K. In addition, the following parameters values are assumed [3]:

TABLE I
PARAMETER VALUES FOR THE VAN DE VUSSE REACTOR

$k_{10} = 1.287 \cdot 10^{12}\text{h}^{-1}$	$k_{20} = 1.287 \cdot 10^{12}\text{h}^{-1}$
$k_{30} = 9.043 \cdot 10^{9}\text{L}(\text{mol} \cdot \text{h})^{-1}$	$E_1/R = -9758.3$K
$E_2/R = -9758.3\text{K}$	$E_3/R = -8560\text{K}$
$\triangle H_1 = 4.2\text{kJ} \cdot \text{mol}^{-1}$	$\triangle H_2 = -11\text{kJ} \cdot \text{mol}^{-1}$
$\triangle H_3 = -41.85\text{kJ} \cdot \text{mol}^{-1}$	$\rho = 0.9342\text{kgL}^{-1}$
$C_p = 3.01\text{kJ}(\text{kg} \cdot \text{K})^{-1}$	$Q = -451.509\text{kJ}$

The control objective is to make the output $y = C_B$ track its setpoint, by manipulating the dilution rate, $u = F/V$. In this work, we would like the output C_B to track a setpoint change to 1.0 mol · L^{-1}, from the following reference steady-state: $C_{Bs} = 0.9$ mol · L^{-1}, $C_{As} = 1.25$ mol · L^{-1}, $T_s = 407.15$ K, which corresponds to $u_s = 19.5218$ hr^{-1}.

To check the stability of the zero dynamics of (5), we linearize the zero dynamics around the reference steady state, and get the following eigenvalues: $\lambda_1 = 122.68$, and $\lambda_2 = -11.17$. This shows that system (5) is locally non-minimum phase around the reference steady state.

Checking the stability of the zero dynamics of (6) around the reference steady state, we get the following eigenvalues: $\lambda = -21.86 \pm 8.93I$. This shows that the auxiliary system is locally minimum phase. Therefore, Assumption 2 is satisfied. For Assumption 3, only the local Lipschitz condition is satisfied. To verify Assumption 4, we check the term $\frac{\partial h_0}{\partial u_a} = -k_1 z_1 - k_2$. Since k_1, z_1 and k_2 are all positive numbers, we know that the sign of $\frac{\partial h_0}{\partial u_a}$ is always negative. In addition, $z_1 = \frac{C_{A0} - C_A}{C_B} > 0$, $k_2 = k_{20} e^{-E_2/RT} > k_{20} e^{-E_2/RT_0} = \tilde{k}_2$, since the reaction is exothermic, it follows that the temperature T is always greater than the cooling water temperature T_0. As a result, $\left(\|\frac{\partial h_0}{\partial u_a}\|\right)^{-1} < (\tilde{k}_{2h})^{-1} = m$. Assumption 4 is therefore satisfied.

In the simulation, controller (16) is implemented with the following design parameters $k_0 = 150$, $k = 150$. The initial conditions for the states are the reference steady state. The control action is restricted in the range of $5\ \text{h}^{-1} \leq u \leq 35\ \text{h}^{-1}$ [3].

Simulation results are shown in Figure 1. It can be seen that the output C_B tracks the new set point within a short period of time (about 2/3 times shorter than the result in [9]).

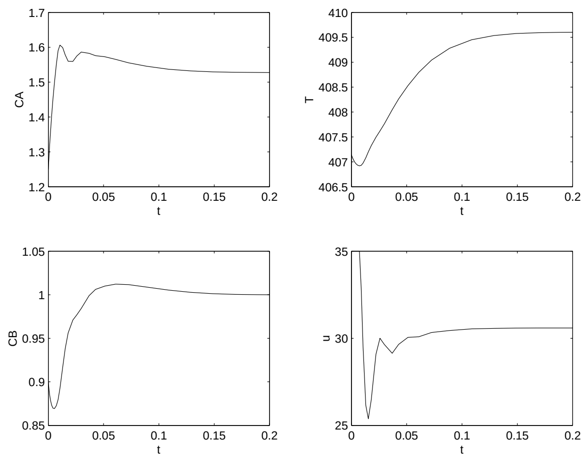

Fig. 1. State trajectory and controller performance of example I

IV. CONCLUSIONS

In this work, we proposed a robust control design that semi-globally practically stabilize a general uncertain non-minimum phase nonlinear system. Simulation results demonstrate that satisfactory controller performance is obtained. In particular, we show that the approach yields excellent performance for the control of the bench mark van de Vusse reactor.

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