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Contouring Control Of Stewart Platform Based Machine Tools

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Abstract— **In this paper, two novel robust adaptive Cartesian space control algorithms are proposed for friction compensation in the six degrees of freedom high performance Stewart Platform based machine tools. The first controller utilizes an adaptive friction compensation scheme based on a postulated linear-in-the-parameters friction model. The proposed friction compensation algorithm explicitly accounts for time varying normal forces as well as dependence of the friction coefficient on velocity. The Stribeck friction characteristic and varying spherical joint static friction are treated as bounded disturbances, and compensated by a sliding mode robust controller. In the second controller, a new form of Takagi-Sugeno Multi-Input Multi-Output fuzzy system is developed to adaptively learn unknown friction behavior and compensate for it. This approach assumes that no a priori knowledge about frictional effects in the strut joints is available. The simulation results indicate that large contouring errors caused by friction at the velocity reversals when conventional control algorithms are used, are reduced greatly by the adaptive controllers.**

I. INTRODUCTION

In late nineties, machine tool manufacturers have introduced 6 degree-of-freedom machine tools with structures based on parallel linkage mechanisms and have promoted their use for the machining of sculptured surfaces [15]. The specific type of parallel-link machine of interest here is the Stewart Platform mechanism [10]. One arrangement of the Stewart Platform is shown in Figure 1. Considerable research attention in the past 10 years has been focused on the kinematics and design of the mechanism [15]. Analysis suggests that the stiffness of S.P. machine tools is very sensitive to its location in the workspace, and is also strongly influenced by the machine geometric configuration [13]. This has resulted in very conservative use of high speed capabilities of the S.P. Very few researchers have worked on the multi-axial motion control of such mechanisms as well as the development of specialized control strategies which benefit from the manipulator's parallel structure and offers better performance characteristics. Nguyen et al (1993) proposed an adaptive joint space controller for the S.P.; however, the results were unsatisfactory according to machine tool standards. Harib and Srinivasan (1998) presented a disturbance observer based cross-coupling controller in Cartesian space. They demonstrated through simulation studies the effectiveness of the controller, achieving a contour error of 5 microns for a circular contour of radius 0.5 m traversed at feed rates of 12 m/min assuming frictionless joints. In this paper, attention is focused on the effects of frictional forces and torques on machine accuracy, as well as on inclusion of compensation techniques for such phenomena through nonlinear control algorithms. Two Cartesian space robust adaptive decoupling and linearizing controllers are developed which consist of an adaptive/robust controller that is able to simultaneously adapt on-line for the adverse effects resulting from the nonlinear system dynamics, and an adaptive model based friction compensation or adaptive nonparametric fuzzy friction compensator that is able to handle uncertainties associated with frictional effects.

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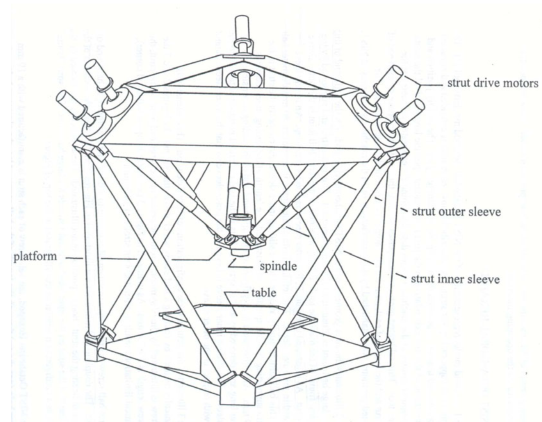

Fig. 1. The NIST Octahedral Machine Toll

II. PARAMETRIZATION OF THE HEXAPOD'S DYNAMIC EQUATIONS OF MOTION

The dynamic model of the Hexapod machining center, which was derived using the Lagrangian and Newton-Euler formulation by Harib (1997), Harib and Srinivasan (2003),

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provides the equations of motion in a compact analytical form containing the inertia matrix, the matrix containing Coriolis/centripetal terms, and the gravity vector. The rigid body and actuator dynamic equations, written in Cartesian space, are given as [5]

$$\mathbf{M(q)\ddot{q} + N(q,\dot{q}) + G(q) = Y(q,\dot{q},\ddot{q})\ \Theta = \tau}$$
$$\left(\mathbf{M_a J^{-1}}\right)\ddot{\mathbf{q}} + \left(\mathbf{V_a J^{-1}} + \mathbf{M_a}\frac{\mathbf{dJ^{-1}}}{\mathbf{dt}}\right)\dot{\mathbf{q}} + \mathbf{K_a J^T \tau} = \boldsymbol{\tau}_{\mathrm{m}} \qquad (1)$$

where $\boldsymbol{\tau}$ is the Cartesian space force/torque vector, $\mathbf{M(q)}$ is the machine inertia matrix, $\mathbf{N(q,\dot{q})}$ is the Cartesian space vector of Coriolis and centrifugal forces and torques, and $\mathbf{G(q)}$ is the Cartesian space vector of gravity forces and torques. The Cartesian space coordinate vector $\mathbf{q}$, whose elements are the six variables chosen to describe the position and orientation of the platform, is defined as $\mathbf{q}=\begin{pmatrix}X & Y & Z & \phi & \theta & \psi\end{pmatrix}^T$. The platform orientation is given as a set of Euler angles $\begin{pmatrix}\phi & \theta & \psi\end{pmatrix}$ which uniquely determines the orientation of a rigid body after the certain sequence of rotations. The term $\mathbf{Y(\dot{q},\dot{q},\ddot{q})} \in \Re^{nxp}$ is the regressor matrix of the parametrized linear-in-parameters rigid body dynamics model defined shortly. $\mathbf{M_a}$ is the actuator inertia diagonal matrix, $\mathbf{V_a}$ is the actuator viscous damping coefficient diagonal matrix, and $\mathbf{K_a}$ is the actuator gain diagonal matrix. $\boldsymbol{\tau}_{\mathrm{m}}$ is the vector of motor torques. $\mathbf{J}$ is the Jacobian matrix which inverse is defined in [5]

$$\mathbf{J}^{-1} = \mathbf{J}_1{}^{-1}\mathbf{J}_2{}^{-1} \qquad (2)$$

It was shown by Garagic (2002) that the term $\mathbf{Y(q,\dot{q},\ddot{q})\Theta}$, equation (1), required for the adaptive law can be derived using the Newton-Euler formulation of the rigid body dynamics, which ultimately results in a computationally efficient control algorithm.

Substituting the first of equations (1) into the second one results in the combined linear-in-the-parameters model of the rigid body and mechanical actuator dynamics in Cartesian coordinate space with respect to the set of unknown parameter vector $\boldsymbol{\Theta}_I$, rewritten as

$$\boldsymbol{\tau}_m = \left[\left(\mathbf{J}^{-1}\ddot{\mathbf{q}} + \frac{d\mathbf{J}^{-1}}{dt}\dot{\mathbf{q}}\right)_{6x1} \quad \left(\mathbf{J}^{-1}\dot{\mathbf{q}}\right)_{6x1} \quad \left(\mathbf{K_a J}^T\mathbf{Y}\right)_{6x7}\right]\boldsymbol{\Theta}_{\mathbf{I}_{9x1}} \qquad (3)$$
$$= \mathbf{Y_I(q,\dot{q},\ddot{q})_{6x9}}\boldsymbol{\Theta}_{\mathbf{I}_{9x1}}$$

where

$$\boldsymbol{\Theta}_{\mathbf{I}_{9x1}} = \begin{bmatrix}M_a & V_a & m_p & m_1 & m_2 & I_m + I_s & I_{xx} & I_{yy} & I_{zz}\end{bmatrix}^T$$

and $\mathbf{Y(q,\dot{q},\ddot{q})}_{(6x7)}$ is the regressor matrix of the parametrized linear-in-parameters model of the rigid body dynamics defined in [1].

Because the Stewart Platform contains all of the distinct features of an entire class of parallel mechanisms, the representation of the dynamic model given by equation (3) is relevant for the general field of parallel-link kinematic structures. This form of the dynamic model is useful for system identification and development of adaptive control algorithms.

III. JOINTS FRICTION MODEL

In the proposed work, attention is focused on the effects of frictional forces and torques on machine accuracy, as well as on inclusion of compensation techniques for such phenomena through nonlinear control algorithms. Frictional effects at both powered and unpowered joints of the parallel manipulator, shown in Figure 1, are significant [9], since even straight line motion of the cutter relative to the workpiece in a Stewart Platform involves multiple axes and direction reversals. Joint friction causes bending of the struts resulting in an error in their effective lengths. The elastic deformation of the strut is dependent on the direction of motion causing angular reversal error [9]. Friction losses in the linear actuators are due to sliding contact between the inner and outer sleeves of the struts, and sliding motion between screw and nut threads. Due to the fact that strut velocity reversals occur depending on the type of the desired trajectory being executed, low velocity friction at the prismatic joints will have a great impact on the tracking error. The load dependent friction at the prismatic joints of the full-scale machine is more significant due to the larger normal forces at the joints. Therefore, the friction model for the powered joints must account for the time varying normal reaction forces, as well as functional dependence of the coefficient of friction on the strut extension rate. On the other hand, frictional analysis of three-DOF spherical joints requires availability of information on relative motion and reaction forces at the joints. The prismatic joint friction forces are modeled as functions of coefficients of friction that vary with the strut extension rate, as well as the magnitudes of the time varying normal forces acting at the points of sliding contact between the inner tube and outer sleeve of the strut. The friction model is split into frictional effects which involve linear dependence on unknown parameters (viz. Coulomb and viscous friction), and those which involve nonlinear dependence on parameters (viz. Stribeck friction). The friction in the spherical joints is a function of relative motion and reaction forces acting on the strut at the base and platform joint. The spherical joint is viewed as a revolute joint having a pure rotation about an instantaneous screw axis. The linear-in-parameters load dependent friction model for the frictional effects in the spherical joints to be used in the adaptive model based friction compensation scheme was derived as [1]

$$\mathbf{K_a J_I^T C_F = K_a J_I^T Y}_{sf}\mathbf{(q,\dot{q},\ddot{q})}\boldsymbol{\Theta}_{sf} \qquad (4)$$

The overall linear-in-parameters dynamic model of the Stewart Platform mechanism in Cartesian coordinate space is given by

$$\boldsymbol{\tau}_m = \left[\left(\mathbf{J}^{-1}\ddot{\mathbf{q}} + \frac{d\mathbf{J}^{-1}}{dt}\dot{\mathbf{q}} \right) \quad \left(\mathbf{J}^{-1}\dot{\mathbf{q}} \right) \quad \left(\mathbf{K_a}\mathbf{J}^T\mathbf{C_{par}} \right) \right] \boldsymbol{\Theta}_I +$$
$$+\mathbf{K_a} diag\left(\left\| \vec{F}_i^T \right\| \text{sgn}(\dot{l}_i) \right) \boldsymbol{\Theta}_{FC} + \mathbf{K_a} diag\left(\left\| \vec{F}_i^T \right\| \left| \dot{l}_i \right| \text{sgn}(\dot{l}_i) \right) \boldsymbol{\Theta}_{FV} \quad (5)$$
$$+\mathbf{K_a}\mathbf{F}_{str} + \mathbf{K_a}\mathbf{J_1}^T\mathbf{Y}_{sf}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\boldsymbol{\Theta}_{sf}$$

$\mathbf{F}_{str}$ is a (6x1) vector representing Stribeck friction at the prismatic joints

$$F_{stri} = \left((\mu_{sl_i} - \mu_{c_i l}) \exp\left(-\left(\frac{\dot{l}_i}{v_{srl_i}} \right)^2 \right) \left\| \vec{F}_i^T(q,\dot{q},\ddot{q}) \right\| \text{sgn}(\dot{l}_i) \right)$$

$\text{i} = 1,2,..6$

$\vec{F}_i^T(q,\dot{q},\ddot{q})$ is the normal component of the reaction force acting at the point of sliding contact between the drive components. $\mu_{sl_i}, \mu_{cl_i}, \mu_{v_i l}$ are the static, Coulomb and viscous friction coefficients of the ith strut respectively, and v_{srl} is the rate of the Stribeck effect, assumed here to be a constant [1].

IV. ROBUST ADAPTIVE FRICTION COMPENSATION

Stewart Platforms are actuated through six prismatic joints. These six linear motion axes constitute the joint space coordinates. The motion of the six prismatic joints results in motion of the end effector described by three DOF linear motion and three DOF angular motion. These six variables constitute Cartesian space coordinates. The motion control problem formulated in Cartesian space naturally separates position and orientation coordinates. The main problem with Cartesian space control for Stewart Platform machine tools, however, is in obtaining Cartesian space coordinates in real time from joint space measurements (i.e. the lengths of six struts), or solving the forward kinematics problem [15] .

In this paper we use an iterative approach based on Newton-Raphson's method [7], to solve for forward kinematics problem of the Stewart Platform based mechanism. As shown in [7] this iterative method works well in tracking control problems where it is employed to compute the actual position and orientation of the payload platform with respect to the base platform using the actuator lengths. This occurs because the current guess is based on the previous position and orientation of the payload platform, which is close to the correct solution provided that the desired path is tracked closely. The use of a control scheme combining adaptive and robust control is explored in this section.

A. Cartesian Space Direct Robust Adaptive Controller with Model Based Adaptive Friction Compensation

The Cartesian space following error and its derivative are defined:

$$\tilde{\mathbf{q}} = \mathbf{q_d} - \mathbf{q} \cong \mathbf{J}(\mathbf{q_d})(\mathbf{l_d} - \mathbf{l}) \;,\; \dot{\tilde{\mathbf{q}}} = \dot{\mathbf{q}}_\mathbf{d} - \dot{\mathbf{q}} \quad (6)$$

where $\mathbf{q_d}, \dot{\mathbf{q}}_\mathbf{d} \in \Re^{nx1}$ represent the desired Cartesian space position and velocity vectors and $\mathbf{l_d}, \mathbf{l} \in \Re^{nx1}$ represent the desired and actual strut length (joint space) variables. Equation (6) gives an approximate estimate of the Cartesian space error vector based on the joint space error vector obtained after two iterations of a numerical solution of the forward kinematic problem based on the Newton-Raphson method, as described in [4]. We assume that $\mathbf{q}$ is close enough to the desired Cartesian space position, and $\mathbf{l}$ is close enough to the corresponding desired joint space position, which will be guaranteed by effective closed loop control. $\mathbf{J}$ is the Jacobian matrix defined by equation (2). Then we define vector $\dot{\mathbf{q}}_\mathbf{r} \in \Re^{nx1}$ by

$$\dot{\mathbf{q}}_\mathbf{r} = \dot{\mathbf{q}}_\mathbf{d} + \boldsymbol{\Lambda}\tilde{\mathbf{q}} \quad (7)$$

where $\boldsymbol{\Lambda} = diag(\lambda_1 ....\lambda_{n=6})$ is a positive definite matrix. These terms will enable us express the nonlinear compensation and decoupling terms as functions of the desired velocity and acceleration, corrected by the current estimates of Cartesian position and velocity, $\mathbf{q}, \dot{\mathbf{q}}$.

To achieve robust tracking control, a sliding surface is defined as

$$\boldsymbol{\sigma} = \dot{\mathbf{q}}_\mathbf{r} - \dot{\mathbf{q}} = \dot{\mathbf{q}}_\mathbf{d} + \boldsymbol{\Lambda}\tilde{\mathbf{q}} - \dot{\mathbf{q}} = \dot{\tilde{\mathbf{q}}} + \boldsymbol{\Lambda}\tilde{\mathbf{q}} \quad (8)$$

where $\boldsymbol{\sigma} \in \Re^{nx1}$ is a single n-dimensional vector sliding manifold. The control law that combines the computed torque/inverse dynamics approach is defined as

$$\mathbf{u_1} = \mathbf{Y_I}(\mathbf{q}, \dot{\mathbf{q}}_\mathbf{r}, \ddot{\mathbf{q}}_\mathbf{r})\hat{\boldsymbol{\Theta}}_\mathbf{I} + \mathbf{Y_F}(\mathbf{q}, \dot{\mathbf{q}}_\mathbf{r}, \ddot{\mathbf{q}}_\mathbf{r}, \dot{\mathbf{l}})\hat{\boldsymbol{\Theta}}_\mathbf{F} + \mathbf{K_D}\boldsymbol{\sigma} \quad (9)$$

where $\hat{\boldsymbol{\Theta}}_\mathbf{I} \in \Re^{\mathbf{px1}}, \hat{\boldsymbol{\Theta}}_\mathbf{F} \in \Re^{\mathbf{pfx1}}$ are estimates of the machine mass/inertial parameters and friction parameters respectively, and obtained using adaptive laws defined shortly. Note that the terms $\mathbf{Y_I}(\mathbf{q}, \dot{\mathbf{q}}_\mathbf{r}, \ddot{\mathbf{q}}_\mathbf{r}) \in \Re^{nxp}$ and $\mathbf{Y_F}(\mathbf{q}, \dot{\mathbf{q}}_\mathbf{r}, \ddot{\mathbf{q}}_\mathbf{r}, \dot{\mathbf{l}}) \in \Re^{nxpf}$, given by equations (3) and (5), are derived using the Newton-Euler formulation of the rigid body dynamics [2], which ultimately results in a computationally efficient control algorithm. It is also important to note that these terms do not depend on the actual Cartesian space acceleration, but only on its desired value.

The term $\mathbf{Y_F}(\mathbf{q}, \dot{\mathbf{q}}_\mathbf{r}, \ddot{\mathbf{q}}_\mathbf{r}, \dot{\mathbf{l}}) \in \Re^{nxpf}$ is the regressor matrix of the parametrized linear-in-parameters friction model and can be split into the regressor matrix $\mathbf{Y_{FC}}(\mathbf{q}, \dot{\mathbf{q}}_\mathbf{r}, \ddot{\mathbf{q}}_\mathbf{r}, \dot{\mathbf{l}}) \in \Re^{nxpf}$ given in equation (5), related to the unknown Coulomb friction parameters and the regressor matrix $\mathbf{Y_{FV}}(\mathbf{q}, \dot{\mathbf{q}}_\mathbf{r}, \ddot{\mathbf{q}}_\mathbf{r}, \dot{\mathbf{l}}) \in \Re^{nxpf}$ given in equation (5), related to the unknown viscous friction parameters of the prismatic joints. The Stribeck friction, $\mathbf{F_{str}} \in \Re^{nx1}$, in the prismatic joints will be viewed as a bounded disturbance.

It is important to stress that the computation of reaction forces in the regressor matrices $\mathbf{Y}_{FC}(\mathbf{q},\dot{\mathbf{q}}_\mathbf{r},\ddot{\mathbf{q}}_\mathbf{r},\dot{\mathbf{l}})$ and $\mathbf{Y}_{FV}(\mathbf{q},\dot{\mathbf{q}}_\mathbf{r},\ddot{\mathbf{q}}_\mathbf{r},\dot{\mathbf{l}})$ will not be implicit in acceleration and therefore will not require an iterative root finding solution. Since the calculation of reaction forces will require knowledge of the parameter vector $\mathbf{\Theta}_I$, the previous estimate of the parameter vector $\mathbf{\Theta}_I$ will be utilized, resulting in a one integration step delay.

On the other hand, the linear-in-parameters spherical joint friction model is more computationally involved as shown in [1]. Since we know that the parameter vector of unknown spherical joints friction coefficients lies in a known bounded open convex set $\Omega_{\Theta_{sf}}$, $\Theta_{sf} \in \Omega_{\Theta_{sf}}$. It can be shown that the regressor matrix $\mathbf{K_a J_1^T Y}_{sf}(\mathbf{q},\dot{\mathbf{q}}_\mathbf{r},\ddot{\mathbf{q}}_\mathbf{r})$ in equation (4) is bounded by a known scalar function

$$\begin{aligned}\left\|\mathbf{K_a J_1^T Y}_{sf}(\mathbf{q},\dot{\mathbf{q}}_\mathbf{r},\ddot{\mathbf{q}}_\mathbf{r})\mathbf{\Theta}_{sf}\right\| &\le f_{sf}(\mathbf{q},\dot{\mathbf{q}}_\mathbf{r},\ddot{\mathbf{q}}_\mathbf{r}) \\ &= \left\|\mathbf{K_a J_1^T Y}_{sf}(\mathbf{q},\dot{\mathbf{q}}_\mathbf{r},\ddot{\mathbf{q}}_\mathbf{r})\right\|\mathbf{\Theta}_{sf_{\max}}\end{aligned} \quad (10)$$

since $\Omega_{\Theta_{sf}}$ is a bounded set and therefore there exists a $\mathbf{\Theta}_{sf_{\max}}$. The robust adaptive law that combines the parameter projection algorithm with the switching σ-modification is developed here. For parameter projection, we need to know a convex region in the parameter space of $\Theta_I, \Theta_{Fc}, \Theta_{Fv}$, which contains the true parameter $\Theta_I^*, \Theta_{Fc}^*, \Theta_{Fv}^*$ [1]. The robust adaptive law is

$$\dot{\hat{\mathbf{\Theta}}}_I = \mathbf{\Gamma}_I^{-1}\left(\mathbf{Y}_I(\mathbf{q},\dot{\mathbf{q}}_r,\ddot{\mathbf{q}}_r)^T\boldsymbol{\sigma} - \boldsymbol{\sigma}_s\hat{\mathbf{\Theta}}_I\right) + \mathbf{Pr}_I \quad (11)$$

where $\mathbf{\Gamma}_I = diag(\gamma_1 \quad . \quad . \quad . \quad \gamma_6) > 0$ is strictly positive definite (s.p.d.) matrix, and "Pr" is a projection function defined in [1]. The robustification of the adaptive law is accomplished by using the switching-σ term, $\boldsymbol{\sigma}_S$ [12]. The parameter projection algorithm will ensure that for

$$\hat{\Theta}_I = \begin{bmatrix} M_a & V_a & m_p & m_1 & m_2 & (I_m + I_s) & I_{xx} & I_{yy} & I_{zz}\end{bmatrix}^T$$

, we have

$$\hat{\Theta}_{I_1}^{\min} \le \hat{\Theta}_1 \le \hat{\Theta}_{I_1}^{\max}, \hat{\Theta}_{I2}^{\min} \le \hat{\Theta}_2 \le \hat{\Theta}_{I2}^{\max}, \ldots, \hat{\Theta}_{I_9}^{\min} \le \hat{\Theta}_9 \le \hat{\Theta}_{I_9}^{\max}$$

for some constants $\hat{\Theta}_{I_j}^{\max}, \hat{\Theta}_{I_j}^{\min} > 0, j = 1,2,\ldots n_{\Theta_I}$.

Similarly, the robust adaptive friction laws are defined as [1]

$$\dot{\hat{\mathbf{\Theta}}}_{Fc} = \mathbf{\Gamma}_{Fc}^{-1}\left(\mathbf{Y}_{Fc}(\mathbf{q},\dot{\mathbf{q}}_r,\ddot{\mathbf{q}}_r)^T\boldsymbol{\sigma} - \boldsymbol{\sigma}_s\hat{\mathbf{\Theta}}_{Fc}\right) + \mathbf{Pr}_{Fc} \quad (12)$$

$$\dot{\hat{\mathbf{\Theta}}}_{Fv} = \mathbf{\Gamma}_{Fv}^{-1}\left(\mathbf{Y}_{Fv}(\mathbf{q},\dot{\mathbf{q}}_r,\ddot{\mathbf{q}}_r)^T\boldsymbol{\sigma} - \boldsymbol{\sigma}_{sFv}\hat{\mathbf{\Theta}}_{Fv}\right) + \mathbf{Pr}_{Fv} \quad (13)$$

where $\mathbf{\Gamma}_{Fc} = diag(\nu_1 \quad . \quad . \quad . \quad \nu_6) > 0 > 0$, $\mathbf{\Gamma}_{Fv} = diag(\eta_1 \quad . \quad . \quad . \quad \eta_6) > 0$ are s.p.d. matrices. "Pr" is a projection algorithm, and $\boldsymbol{\sigma}_{SFc}$, $\boldsymbol{\sigma}_{SFv}$ are the switching-σ terms derived in [1]. The error between the ideal controller , $\mathbf{u_i}$,and its approximation, equation (9), is represented by $d(\mathbf{q},\dot{\mathbf{q}}_\mathbf{r},\ddot{\mathbf{q}}_\mathbf{r})$. This term results from unmodeled dynamic effects such as Stribeck friction and spherical joints friction as well as the modeling error in representing the rigid body and actuator dynamics by $\mathbf{M}^*(\mathbf{q})\ddot{\mathbf{q}}_\mathbf{r} + \mathbf{Q}(\mathbf{q},\dot{\mathbf{q}})^*\dot{\mathbf{q}}_\mathbf{r} + \mathbf{G}(\mathbf{q})^* = \mathbf{Y_I}(\dot{\mathbf{q}},\dot{\mathbf{q}}_\mathbf{r},\ddot{\mathbf{q}}_\mathbf{r})\mathbf{\Theta_I}$. We can assume that the modeling error is bounded by a known scalar function $y_m(q,\dot{q}_r,\ddot{q}_r)$ due to the fact that Θ_I is bounded and belongs to the bounded set $\Theta_I \in \Omega_{\Theta_I}$. Since the adaptive law, equation (11), guarantees the boundedness of the parameter estimate, $\hat{\Theta}_I \in \Omega_{\Theta_I}$, there must exist a $\tilde{\Theta}_I^{\max}$ such that $\left\|\left(\hat{\mathbf{\Theta}}_\mathbf{I} - \mathbf{\Theta}_\mathbf{I}\right)\right\| \le \tilde{\Theta}_I^{\max}$ which implies

$$\left\|\mathbf{Y_I}(\dot{\mathbf{q}},\dot{\mathbf{q}}_\mathbf{r},\ddot{\mathbf{q}}_\mathbf{r})\left(\hat{\mathbf{\Theta}}_\mathbf{I} - \mathbf{\Theta}_\mathbf{I}\right)\right\| \le y_m(q,\dot{q}_r,\ddot{q}_r) \quad (14)$$

Note that y_m is only required to be a bounding function and that a simpler y_m than the one given by equation (14) can be chosen to reduce the computation time required for real time implementation.

To account for the modeling and ensure that the system output follows the desired trajectory, a "smoothed" sliding mode control is added to the control law given in equation (9) as

$$\mathbf{u} = \mathbf{Y_I}(\mathbf{q},\dot{\mathbf{q}}_\mathbf{r},\ddot{\mathbf{q}}_\mathbf{r})\hat{\mathbf{\Theta}}_\mathbf{I} + \mathbf{Y_F}(\mathbf{q},\dot{\mathbf{q}}_\mathbf{r},\ddot{\mathbf{q}}_\mathbf{r})\hat{\mathbf{\Theta}}_\mathbf{F} + \mathbf{K_D}\boldsymbol{\sigma} + \mathbf{u}_{sl} \quad (15)$$

$$\mathbf{u}_{sl} = \begin{cases} \tau_0 \dfrac{\boldsymbol{\sigma}}{\|\boldsymbol{\sigma}\|_2} & for\,\|\boldsymbol{\sigma}\| \ge \varepsilon \\ \tau_0 \dfrac{\boldsymbol{\sigma}}{\varepsilon} & for\,\|\boldsymbol{\sigma}\| < \varepsilon \end{cases} \quad (16)$$

where τ_0 is finite and must satisfy

$$\begin{aligned}\tau_0 > &\left\|\mathbf{Y_I}(\mathbf{q},\dot{\mathbf{q}}_\mathbf{r},\ddot{\mathbf{q}}_\mathbf{r})\left(\hat{\mathbf{\Theta}}_\mathbf{I} - \mathbf{\Theta}_\mathbf{I}\right)\right\| + \left\|K_a J_1^T \mathbf{Y}_{sf}(\mathbf{q},\dot{\mathbf{q}}_\mathbf{r},\ddot{\mathbf{q}}_\mathbf{r})\mathbf{\Theta}_{sf}\right\| \\ &+ \left\|\mathbf{K_a F}_{str}(\mathbf{q},\dot{\mathbf{q}}_r,\ddot{\mathbf{q}}_r,\dot{\mathbf{l}})\right\| = d(\mathbf{q},\dot{\mathbf{q}}_\mathbf{r},\ddot{\mathbf{q}}_\mathbf{r},\dot{\mathbf{l}})\end{aligned} \quad (17)$$

The stability analysis of the closed loop system is based on Lyapunov stability theory. The candidate Lyapunov function is used to describe error in tracking and error between the desired controller and the current controller, and to account for the uncertainties associated with unknown manipulator dynamics and friction. Before we proceed with the choice of the Lyapunov function, it is necessary to define the closed loop error dynamics as

$$\begin{aligned}\mathbf{M}^*\dot{\boldsymbol{\sigma}} = &-\mathbf{Y}(\mathbf{q},\dot{\mathbf{q}}_\mathbf{r},\ddot{\mathbf{q}}_\mathbf{r})\tilde{\mathbf{\Theta}}_\mathbf{I} - \mathbf{Y}_{Fc}(\mathbf{q},\dot{\mathbf{q}}_\mathbf{r},\ddot{\mathbf{q}}_\mathbf{r},\dot{\mathbf{l}})\tilde{\mathbf{\Theta}}_{Fc} - \\ &\mathbf{Y}_{Fv}(\mathbf{q},\dot{\mathbf{q}}_\mathbf{r},\ddot{\mathbf{q}}_\mathbf{r},\dot{\mathbf{l}})\tilde{\mathbf{\Theta}}_{Fv} - \mathbf{K_D}\boldsymbol{\sigma} - \mathbf{Q}_{All}\boldsymbol{\sigma} - \\ &\mathbf{u}_{sl} + \mathbf{d}(\mathbf{q},\dot{\mathbf{q}}_\mathbf{r},\ddot{\mathbf{q}}_r,\dot{\mathbf{l}})\end{aligned} \quad (18)$$

where the mass matrix $\mathbf{M}^* = \left(\mathbf{M_a J^{-1}} + \mathbf{K_a J^T M(q)}\right)$, and the term $\mathbf{d}(\mathbf{q},\dot{\mathbf{q}}_\mathbf{r},\ddot{\mathbf{q}}_\mathbf{r},\dot{\mathbf{l}})$ is associated with disturbance due to unmodeled manipulator dynamics. An important property of the manipulator model given by equation (3) is that the time derivative of the mass matrix, $\dot{\mathbf{M}}^*(\mathbf{q})$, and the overall Coriolis/centripetal matrix, $\mathbf{Q}^*$, are skew symmetric [8], [14]. The candidate Lyapunov function is given by

$$\mathbf{V} = \frac{1}{2}\boldsymbol{\sigma}^{\mathbf{T}}\mathbf{M}^*\boldsymbol{\sigma} + \frac{1}{2}\tilde{\boldsymbol{\Theta}}_I^{\mathbf{T}}\boldsymbol{\Gamma}_I\tilde{\boldsymbol{\Theta}}_I + \frac{1}{2}\tilde{\boldsymbol{\Theta}}_{Fc}^{\mathbf{T}}\boldsymbol{\Gamma}_{Fc}\tilde{\boldsymbol{\Theta}}_{Fc} + \frac{1}{2}\tilde{\boldsymbol{\Theta}}_{Fv}^{\mathbf{T}}\boldsymbol{\Gamma}_{Fv}\tilde{\boldsymbol{\Theta}}_{Fv} \quad (19)$$

In the stability analysis there will be two cases to consider:

Case 1. $\|\boldsymbol{\sigma}\| \geq \varepsilon$. In this case, according to the definition of the sliding mode control gain in equation (16), $\mathbf{u}_{sl}$, it can be shown that the time derivative of (19) is [1]

$$\begin{aligned}\dot{\mathbf{V}} &\leq -\boldsymbol{\sigma}^{\mathbf{T}}\mathbf{K}_{\mathbf{D}}\boldsymbol{\sigma} - \boldsymbol{\sigma}^{\mathbf{T}}\mathbf{u}_{sl} + \|\mathbf{d}(q,\dot{q}_r,\ddot{q}_r)\|\cdot\|\boldsymbol{\sigma}\| \\ &\leq -\lambda_{\min}(\mathbf{K}_{\mathbf{D}})\|\boldsymbol{\sigma}\|^2 - \|\boldsymbol{\sigma}\|^2\frac{\tau_0}{\|\boldsymbol{\sigma}\|} + \tau_0\|\boldsymbol{\sigma}\| \leq -\lambda_{\min}(\mathbf{K}_{\mathbf{D}})\|\boldsymbol{\sigma}\|^2\end{aligned} \quad (20)$$

where $\mathbf{K}_{\mathbf{D}} > 0$ is a s.p.d. matrix. Using the property of the matrix norm we have $\lambda_{\min}(\mathbf{K}_{\mathbf{D}})\|\boldsymbol{\sigma}\|^2 \leq \boldsymbol{\sigma}^{\mathbf{T}}\mathbf{K}_{\mathbf{D}}\boldsymbol{\sigma}$, with $\lambda_{\min}(\mathbf{K}_{\mathbf{D}})$ being the smallest singular value of the gain matrix $\mathbf{K}_{\mathbf{D}}$. It follows, from equation (20), that **V** is decreasing with time along the system's trajectory. By Barbalat's lemma (Ioannou and Sun, 1996), this implies that σ converges asymptotically to zero which implies convergence to zero of $\tilde{q}$ and $\dot{\tilde{q}}$, $\dot{q}_r \rightarrow \dot{q}_d, \ddot{q}_r \rightarrow \ddot{q}_d$ and the boundedness of $\hat{\Theta}, \dot{\hat{\Theta}}$. In addition, the control law given by equation (38) guarantees fast iterative numerical solution of the forward kinematics problem based on the Newton-Raphson method, because it guarantees that $q \rightarrow q_d$ with high accuracy (Garagic, 2002). Therefore, $\dot{V}$ is negative definite in terms of $\|\boldsymbol{\sigma}\|$. Hence, V is decreasing in this region and $\|\boldsymbol{\sigma}\|$ decreases towards ε.

Case 2. $\|\boldsymbol{\sigma}\| < \varepsilon$. In this case, according to the definition of the sliding mode control gain in equation (16), $\mathbf{u}_{sl}$, we have

$$\begin{aligned}\dot{\mathbf{V}} &\leq -\boldsymbol{\sigma}^{\mathbf{T}}\mathbf{K}_{\mathbf{D}}\boldsymbol{\sigma} - \boldsymbol{\sigma}^{\mathbf{T}}\mathbf{u}_{sl} + \|\mathbf{d}(q,\dot{q}_r,\ddot{q}_r)\|\cdot\|\boldsymbol{\sigma}\| \\ &\leq -\lambda_{\min}(\mathbf{K}_{\mathbf{D}})\|\boldsymbol{\sigma}\|^2 + \|\boldsymbol{\sigma}\|\tau_0\left(1 - \frac{\|\boldsymbol{\sigma}\|}{\varepsilon}\right)\end{aligned} \quad (21)$$

The last term is generally positive in this region ($\|\boldsymbol{\sigma}\| < \varepsilon$), so that nothing can be said about whether V is increasing or decreasing. If V is increasing in this region, then $\|\boldsymbol{\sigma}\|$ is increasing towards ε.

B. Robust Adaptive Controller with Takagi-Sugeno Fuzzy Adaptive Friction Compensation

The physical nature of friction is such that it often changes with time and may depend in an unknown way on environmental factors (i.e. temperature changes, lubricant condition etc). Therefore, as an alternative to model-based friction compensation, we developed a multi-input multi-output fuzzy systems approach. The special form of the Takagi-Sugeno fuzzy system [11] will be utilized to adaptively learn friction behavior and compensate for it [2] The Takagi-Sugeno fuzzy systems used for friction compensation for the prismatic and spherical joints of the Stewart Platform mechanism will be partitioned into n=6 subsystems. This is a reasonable assumption since joint frictional effects in the machine joint space can be viewed as decoupled due to the machine configuration (parallel structure with six identical drives). Therefore, the j[th] (j=1,2...6) Takagi – Sugeno fuzzy systems with center average defuzzification consists of Takagi-Sugeno fuzzy rules as follows:

$$\begin{aligned}&R_i^j: \text{ If } x_1\ F_1^i, \text{ If } x_2\ F_2^i, \ldots\ldots, \text{ If } x_n\ F_n^i \\ &\text{then } g_i^j(\underline{x}) = a^j_{i,0} + a^j_{i,1}\cdot z^j_1(\underline{x}) + \ldots + a^j_{i,m-1}\cdot z^j_{m-1}(\underline{x})\end{aligned} \quad (22)$$

where $j = 1,2....6$ is the number of partitioned fuzzy subsystems, $i = 1,2,....R$ is the number of fuzzy rules for the j[th] fuzzy subsystem. F_j^i is the linguistic value of the membership function (i.e. slow, medium or fast). The singleton fuzzification of the input vector $\underline{x} = [x_1..........x_n]^T$ is assumed. The output consequence function $g^j_i(\underline{x})$ is defined as a linear combination of a set of functions $z_k^j(\underline{x}) \in \Re, k = 1,2,...m-1$, so that the output of the jth fuzzy system is inferred as follows:

$$\tilde{f}^j_{ts}(\underline{x}) = \frac{\sum_{i=1}^{R} g^j_i(\underline{x})\cdot\mu^j_i(\underline{x})}{\sum_{i=1}^{R}\mu^j_i(\underline{x})} = \mathbf{z}_j^T \mathrm{A}_{fr_j}\boldsymbol{\varsigma}_j \quad (23)$$

where R is the number of fuzzy rules for the j[th] fuzzy subsystem, and μ^j_i is the value of the membership function (i.e. Gaussian type) for the premise of the i[th] rule given the input $\underline{x}$ for the j[th] fuzzy subsystem. It is assumed that the fuzzy system in equation (23) is constructed in such a way that $\sum_{i=1}^{R}\mu^j_i(\underline{x}) \neq 0, \forall \underline{x} \in \Re^n$. Define

$$z_j = \begin{bmatrix} 1 & z_1^j(\underline{x}) & .... & z_{m-1}^j(\underline{x}) \end{bmatrix}^T \in \Re^m$$

$$\varsigma_j^T = \frac{\begin{bmatrix}\mu^j_1(\underline{x}) & . & . & . & \mu^j_R(\underline{x})\end{bmatrix}}{\sum_{i=1}^{R}\mu_i^j(\underline{x})} \quad (24)$$

$$A_{fr_j(mxR)} = \begin{bmatrix} a^j_{1,0} & a^j_{2,0} & .... & a^j_{R,0} \\ . & . & .... & . \\ a^j_{1,m-1} & a^j_{2,m-1} & .... & a^j_{R,m-1}\end{bmatrix}$$

The overall partitioned Takagi-Sugeno fuzzy systems can be written in compact matrix form utilizing the properties of the direct matrix sum, ⊕,

$$\tilde{F}_{ts}(\underline{x})_{Nx1} = \left(z_1^T \oplus ... \oplus z_N^T\right)\left(A_{fr_1} \oplus ... \oplus A_{fr_N}\right)\left[\varsigma_1 ... \varsigma_N\right]^T = \mathbf{Z}^* \hat{\mathbf{A}}_{\mathbf{FR}} \mathbf{\Xi}^* \quad (25)$$

The unknown nonlinear function **F(x)** represents frictional effects in the prismatic and spherical joints of the Stewart Platform mechanism and can be defined using equations (4) and (5) as

$$\mathbf{F}(\underline{x}) = \mathbf{K_a J_l^{\mathit{T}} C_F}(\mathbf{q}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r) + \mathbf{Y_F}(\mathbf{q}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r, \dot{\mathbf{l}})\hat{\mathbf{\Theta}}_\mathbf{F} + \mathbf{K_a F}_{str}(\mathbf{q}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r, \dot{\mathbf{l}}) \quad (26)$$

According to the universal approximation property of Takagi-Sugeno fuzzy systems, there is a T-S fuzzy system such that

$$\mathbf{F}(\underline{x}) = \mathbf{Z}^* \mathbf{A}^*_{Fr} \mathbf{\Xi}^* + \mathbf{d}_{fr} \quad (27)$$

where d_{fr}, the approximation error that arises when $\mathbf{F}(\underline{x})$ is represented with a fuzzy system, and $\mathbf{A}^*_{Fr}(t)$, represent the optimum, and unknown, fuzzy system parameters that minimize the approximation errors. The approximation error is bounded on a compact set by $\|\mathbf{d}_{fr}\| < D_{fr}$, with D_{fr} a known bound. We shall require an assumption that the ideal fuzzy parameters given in $\mathbf{A}^*_{Fr}$ are bounded on any compact subset of $\Re^n$, so that $\|\mathbf{A}^*_{Fr}\|_F \le \mathbf{A}_{\max}$, with $\mathbf{A}_{\max}$ known and $\|\cdot\|_F$ being the Frobenius norm. The following fuzzy system $\hat{F}(\underline{x})$ will be used to approximate $F(\underline{x})$:

$$\hat{F}(\underline{x}) = \mathbf{Z}^* \hat{\mathbf{A}} \mathbf{\Xi}^* \quad (28)$$

$\hat{A}_{fr_j}, j = 1,2...6$ of the actual values of the T-S fuzzy subsystems given by the adaptive algorithm to be specified yet. The choice of inputs to the Takagi-Sugeno fuzzy subsystems is discussed in details in [1]. The operational ranges of these inputs for the specified input trajectories are known heuristically. This information is used to assign the Gaussian membership functions with the linguistic values $\mathrm{F}_\mathrm{l}^\mathrm{i}, \mathrm{l} = 1,2,...\mathrm{q}$ (e.g. slow, medium or fast strut extension rate) to the operational space of the input variable [1]. In order to account for load dependent friction, we select the vector $z_j \in \Re^m$ in equation (24) as

$$z_j = \begin{bmatrix} 1 & \ddot{X}_r & \ddot{Y}_r & \ddot{Z}_r & \ddot{\phi}_r & \ddot{\theta}_r & \dot{l}_j & \mathrm{sgn}(\dot{l}_j) \end{bmatrix}^T \quad \forall j \quad (29)$$

Now, we select the control law that combines the computed inverse dynamics term given by equation (9) excluding model based friction compensation, a robust sliding mode controller yet to be defined, u_{sl}, and an adaptive friction compensator based on T-S fuzzy systems as follows:

$$\mathbf{u} = \mathbf{Y_I}\hat{\mathbf{\Theta}}_\mathbf{I} + \mathbf{K_D}\boldsymbol{\sigma} + \mathbf{u_{sl}} + \mathbf{Z}^*\hat{\mathbf{A}}_\mathbf{FR}\mathbf{\Xi}^* \quad (30)$$

with $\mathbf{K_D} > 0$, a positive definite matrix. The adaptive law to tune $\hat{\mathbf{\Theta}}_\mathbf{I}$ is given by equation (11), while the parameters, $\hat{\mathbf{A}}_\mathbf{FR}$, of the T-S fuzzy system (27) are updated using the following update law

$$\dot{\hat{\mathbf{A}}}_{\mathbf{fr_j}} = \mathbf{\Gamma_{fr_j}}^{-1}\mathbf{z}_j\boldsymbol{\varsigma}^\mathbf{T}_{\mathbf{fr}_j} \quad \forall j = 1,2,...6 \quad (31)$$

with $\mathbf{\Gamma_{fr_j}}^{-1}$, j = 1,2,..6 being any constant positive definite design matrices. The sliding mode control term, $\mathbf{u}_{sl}$, is defined as

$$\mathbf{u}_{sl} = \tau_0 \,\mathrm{sgn}(\boldsymbol{\sigma}),\ \tau_0 \ge D_{fr} + \|Y_I(q, \dot{q}_r, \ddot{q}_r)\|\Theta_I^{\max} > 0 \quad (32)$$

where τ_0 is only required to be a bounding function and a simpler τ_0 than the one given by equation (32) can be chosen to reduce the computation time required for real time implementation. Also, a smoothed version of the sliding control law in equation (32) can be used in accordance with equation (16).

The stability of this controller is proved in the same way as the previous one. We define the candidate Lyapunov function as

$$\mathbf{V} = \frac{1}{2}\boldsymbol{\sigma}^\mathbf{T}\mathbf{M}\boldsymbol{\sigma} + \frac{1}{2}\tilde{\mathbf{\Theta}}_I^T\mathbf{\Gamma}_I\tilde{\mathbf{\Theta}}_I + \frac{1}{2}\sum_j^{N=6} tr\left(\mathbf{\Phi}^T_{fr_j}\mathbf{\Gamma}_{fr_j}\mathbf{\Phi}_{fr_j}\right) \quad (33)$$

where $\Phi_{fr\,j} = \hat{A}_{fr}(t) - A_{fr\,j}^*$, j = 1,2,..6 is defined in equation (31). The derivative of equation (33) yields

$$\dot{\mathbf{V}} \le -\boldsymbol{\sigma}^\mathbf{T}\mathbf{K_D}\boldsymbol{\sigma} + \sum_j^{6}\mathbf{tr}(\mathbf{\Phi}^\mathbf{T}_{\mathbf{fr_j}}(\mathbf{\Gamma}_{\mathbf{fr_j}}\dot{\mathbf{\Phi}}_{\mathbf{fr_j}} - \mathbf{z}_j\boldsymbol{\varsigma}^\mathbf{T}_\mathbf{j}\boldsymbol{\sigma})) - \boldsymbol{\sigma}^\mathbf{T}\mathbf{u_R} + \boldsymbol{\sigma}^\mathbf{T}(\mathbf{d_I} + \mathbf{d_F}) - \boldsymbol{\sigma}^\mathbf{T}\mathbf{u_{sl}} \quad (34)$$

The update law for $\mathbf{\Phi}_{fr_j}$ is given by

$$\dot{\mathbf{\Phi}}_{fr_j} = \mathbf{\Gamma}_{fr_j}^{-1}\mathbf{z}_j\boldsymbol{\varsigma}^\mathbf{T}_j\boldsymbol{\sigma} \quad (35)$$

where Γ_{fr_j} is a positive definite adaptation gain matrix. To assure that $\hat{\mathbf{A}}_\mathbf{FR}$ stays bounded, we will use an *update law with projection [2]*, which will guarantee that $\hat{A}_{fr_j} \in \Omega_{fr_j}$, a compact parameter set. The bounds for the parameter matrix $\hat{A}_{fr_j}$ should be in range of the static friction level for both directions of motion. Using the projection algorithm, we also ensure that $\dot{V} \le -\boldsymbol{\sigma}^\mathbf{T}\mathbf{K_D}\boldsymbol{\sigma} \le 0$ it follows that V is decreasing with time along the system's trajectory.

V. CONTOURING PERFORMANCE EVALUATION

A computer simulation study is performed to evaluate the effectiveness of the controllers described in the preceding sections. The Cartesian space direct robust adaptive controller (CDAC) with model-based adaptive friction compensation, as well as the Cartesian space direct robust adaptive controller with Takagi-Sugeno fuzzy adaptive friction compensation (FCDAC), are compared with the Cartesian space computed torque controller (CCTPID) [3]. For Stewart Platform based machine tools, which involve active control of all six DOF of motion, the controller performance is characterized in terms of position and orientation contour errors [4].

The first case studied involves controller performance evaluation for horizontal circular trajectory: the radius is 0.2 meter and the contour starts at (-0.1, -0.1732, 0) meter and ends at (0.2, -0.1732, 0) meter while the orientation coordinate starts at (0, -90, 0) degrees and end at (0, 90, 0) degrees. Also, a maximum feedrate of 0.05 m/s for the positional displacement along the trajectory is used, with acceleration/deceleration limits of ±2 m/sec^2 at the beginning and end of the trapezoidal trajectory.

While the first proposed controller is model dependent, the second adaptive controller — the Cartesian space robust direct adaptive controller with fuzzy adaptive friction compensation (FCDAC) — is capable of compensating for frictional effects and assumes no a priori knowledge of frictional effects in the machine joints is available. Adequate location of the centers of the membership functions as well as their spread will also influence performance of the fuzzy identifier and should be investigated before the number of rules is increased, since their adjustment will not result in increase in the computational complexity of the algorithm.

Figure 2 shows the contour and orientation errors ε and γ for the CCTPID, CDAC and CFDAC controllers. As can be seen from Figure 2, the contouring error and orientation error with the CCTPID controller contain a prominent friction induced error – the glitch. Twelve glitches are seen on the contouring error due to 12 reversals that occur (two per leg) for every revolution. However, as the geometry of the Hexapod machining center incorporates three parallel leg pairs, the size of six of the glitches is in the range of 10 μm, while the size of the other six glitches is in the range of 3 μm. By contrast, the CDAC and FCDAC controllers effectively compensate for frictional effects in the machine joints and, as a result, the contouring error and orientation error are free of the glitches. The contouring error for the CDAC and FCDAC controllers is kept under 20 μm and the maximum orientation error is 0.08 minutes. The proposed adaptive controllers are capable of compensating for errors induced by friction when the axes change direction, thus eliminating the resulting glitches, in addition to compensating for unknown inertial, Coriolis, and gravity effects.

Controller performance evaluation for cornering contour: the cornering contour consists of two straight-line segments that make a 90-degree angle. The first segment starts at point (0, 0, 0) and ends at point (0.05, 0.05, 0.01) meter, relative to the center of the workspace, while the second segment ends at approximately (0, 0.098, 0.02) meter. Here also, a constant feedrate of 0.2 m/sec is used, with a trapezoidal velocity profile at the beginning of the first segment and end of the second segment. At the corner, the commanded velocity directions are changed without acceleration/deceleration limits. The orientation is kept constant in this test, at (0, 90, 0) degrees. Because of the abrupt change in direction at the corner, a large contour error will result when the CCTPID controller is used, as shown in Figure 3. The corresponding contour and orientation errors for the CDAC controller are shown in Figure 4. At the corner, the contour error is reduced from 350 μm to 15 μm when using the CDAC and FCDAC controllers.

VI. Conclusion

The use of a Cartesian space control scheme combining adaptive and robust control for a 6 degree of freedom Stewart Platform machine tool are developed in this paper. Measurements in this space is derived in real time from joint space measurements by an approximate solution of the forward kinematic relationships.

The control scheme can account for frictional effects which may be unmodeled, as well as being able to deal with uncertainties caused by unknown manipulator parameters and nonlinear effects. The first controller utilizes an adaptive friction compensation scheme based on a postulated linear-in-the-parameters friction model. The proposed friction compensation algorithm explicitly accounts for time varying normal forces as well as dependence of the friction coefficient on velocity. The Stribeck friction characteristic and varying spherical joint static friction are treated as bounded disturbances, and compensated by a sliding mode robust controller. In the second controller, a special form of Takagi-Sugeno multi-input multi-output fuzzy system is utilized to adaptively learn unknown friction behavior and compensate for it. This approach assumes that no a priori knowledge about frictional effects in the strut joints is available. The performance of the two proposed robust adaptive controllers with friction compensation is evaluated on the simulation for a number of representative Cartesian space trajectories, and compared with the response of a Cartesian space computed torque PID controller (CCTPID). The proposed controllers clearly outperform the CCTPID controller. The large tracking errors caused by friction at the velocity reversals are reduced greatly by the adaptive controllers. In addition, the Cartesian space robust adaptive controllers presented in this paper can be extended to applications where the problem of controlling interaction between the machine tool tip and the environment is of concern.

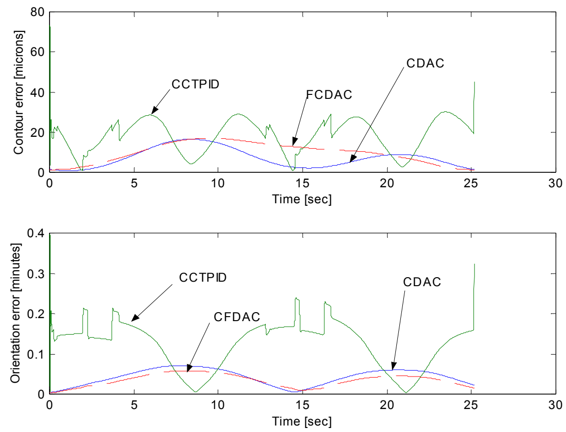

Fig.2. Contour and orientation errors for horizontal circular trajectory: Radius=0.2 m , feedrate = 0.05 m/sec.

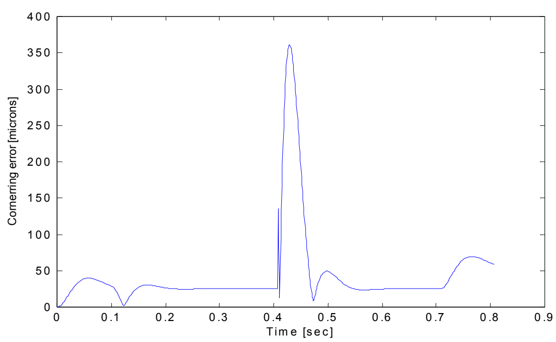

Fig. 3. Contour error for cornering contour with 0.2 m/sec feed rate: CCTPID control ($\omega_0 = 100$ rad/sec, $\omega_0 = 100$ rad/sec).

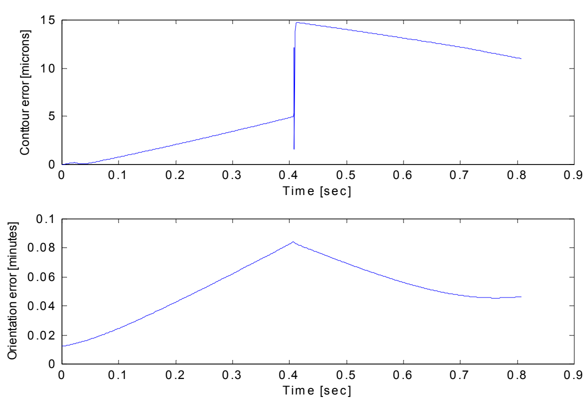

Fig. 4. Contour error for cornering contour with 0.2 m/sec feed rate: CDAC control.

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