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Real-time Multiple Parameter Estimation for Voltage Controlled Brushless DC Motor Actuators

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Abstract – **Brushless DC motors have been widely used as actuators in mechatronic systems. A non-dynamic motor inverse model, which neglects the electrical dynamics of the motor, is usually employed to achieve torque control of the motor. Variations of the motor parameters due to environmental factors, temperature, build variations, aging, etc. directly impart inaccuracies in the inverse motor model, and thus the dynamic performance of the system suffers. A multi-parameter estimation scheme is proposed in this paper. Improvements for the dynamic performance of the estimator are discussed. Comparison of closed loop simulations for voltage control BLDC motor in an electric power steering system confirms a lower bound of error of the estimated parameter and faster adaptation with the proposed improved estimation scheme.**

I. INTRODUCTION

Motors are the most commonly used actuators in mechatronic systems. Variations of the motor parameters such as coil resistance, coil inductance, torque constant directly impart inaccuracies in the control scheme based on the nominal values of parameters. The motor parameter variations generate output inaccuracies as a function of build, life, and temperature variations. To ensure adequate torque control and acceptable frequency domain performance, often it is desirable to compensate the control of the motor for variations in motor parameters including, but not limited to, motor resistance R and motor torque constant K_e.

Mir [1, 2] proposed on-line single parameter estimation for a brushless DC (BLDC) motor and verified the algorithm experimentally. Klienau et al [3] have proposed a current feedback error based single parameter estimation scheme for voltage controlled BLDC motors. However, when there exist errors in more than one parameter, performance of the single-parameter estimation schemes will deteriorate, and the accuracy of the control system will also suffer. Moseler [4] proposed a parameter estimation technique for a BLDC motor driven by a PWM inverter, where several parameters can be estimated by measuring the motor's input and output signals.

In this paper, a multi-parameter estimation algorithm is derived for the motor inverse model. Open loop simulation verified the effective-ness of the algorithm. Improvements for the algorithm are proposed for compensation of errors introduced by two assumptions in derivation of the estimation scheme. Closed-loop simulations presented for an electric power steering system plant demonstrate a lower bound on the error of the estimated parameter and a faster response time with the improved estimation algorithm.

II. ADAPTIVE MULTI-PARAMETER ESTIMATION SCHEME

Fig.1 shows a typical plant controlled in the closed loop. As shown in Fig. 1, an approximate inverse of the motor [5] neglecting the motor dynamics is used in the controller so that the fast motor dynamics are approximately cancelled by the controller to provide the expected torque output. Variations of the motor parameters will cause discrepancy between the motor inverse model used in the controller and the actual motor dynamics, therefore, the system in Fig. 1 has a motor-current based parameter estimation scheme.

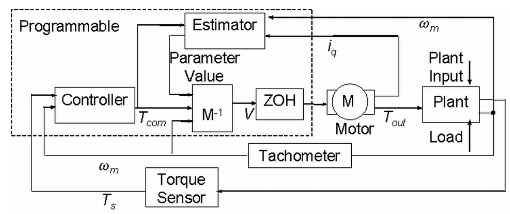

Fig. 1. The schematic diagram of a mechatronic system with motor current based parameter estimation

2.1 Non-dynamic motor inverse model

As a load control actuator of a mechatronic system, a three-phase BLDC motor can be modeled by following equation[6, 7]

$$L\frac{di_q}{dt} = -R\,i_q - \omega_e L\,i_d - K_e\,\omega_m + V\cdot\cos\delta\,,$$
$$L\frac{di_d}{dt} = -R\,i_d + \omega_e L\,i_q - V\sin\delta, \tag{1}$$
$$T_{out} = K_e i_q\,.$$

where,

i_q, i_d – q and d axis currents mapped from the three phase currents,

R – Motor coil resistance,

K_e – Torque and back EMF constant of the motor,

L – Motor coil inductance,

V – Input voltage to the motor,

ω_e – Angular velocity of electro-magnetic field,

ω_m – Angular velocity of armature, $\omega_e = \omega_m \cdot N_p/2$, N_p is the number of magnetic poles,

T_{out} – Motor torque output,

δ – Phase advance $\delta = \tan^{-1}\left(\frac{\omega_e L}{R}\right)$.

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It is desired that the torque output of the motor tracks the desired torque (T_{com}) from the controller. T_{com} defines the desired q-axis motor current by the following equation

$$i_{qcom} = \frac{T_{com}}{K_e} . \quad (2)$$

Neglecting the electrical dynamics of the motor, i.e. $\frac{di_q}{dt} \approx 0$, $\frac{di_d}{dt} \approx 0$, the voltage V to be applied to the motor in order to get the desired torque T_{com} at the output is obtained [3] from (1) as

$$V = \frac{R^2 \frac{T_{com}}{K_e} + RK_e\omega_m + \omega_e{}^2 L^2 \frac{T_{com}}{K_e}}{R\cos\delta + \omega_e L \sin\delta} . \quad (3)$$

2.2 Feedback estimation of multiple parameters

While using (3), it is assumed that the motor parameters R, L, and K_e are constant and the parameters used in the controller have the same values as those in the motor. However, aging and changing environmental factors, such as temperature, humidity, etc. will change the values of the parameters in the motor. It is necessary to take some corrective action so that the value of one or more parameters in the controller is as close as possible to the actual value in the motor.

Let's investigate the integration of the feedback current error scheme to estimate the two parameters R and K_e in the controller represented by,

$$R_c(t) = R_c(t_0) + \int_{t_0}^{t} k_1(t)\,\Delta\hat{R}(t)\,dt \quad , \quad (4)$$

$$K_{ec}(t) = K_{ec}(t_0) + \int_{t_0}^{t} k_2(t)\,\Delta\hat{K}_e(t)\,dt \quad , \quad (5)$$

where, $\Delta\hat{R}(t)$ and $\Delta\hat{K}_e(t)$ are the estimated error, $k_1(t) < 1$ and $k_2(t) < 1$ are the weighting functions. We denote the parameters and variables used for computation in the controller by suffix c. The estimated error $\Delta\hat{R}(t)$ and $\Delta\hat{K}_e(t)$ will be derived in the following paragraphs.

Applying the voltage V computed from (3) with parameter and variable values in controller to the motor model, we have

$$\frac{di_q}{dt} = \frac{R_c i_{qcom} - R\,i_q}{L} + (\omega_{ec} i_{dcom} - \omega_e i_d) + \frac{K_{ec}\omega_{mc} - K_e\omega_m}{L} , \quad (6a)$$

$$\frac{di_d}{dt} = \frac{R_c i_{dcom} - R i_d}{L} + \omega_e i_q - \omega_{ec} i_{qcom} . \quad (6b)$$

Define

$$\Delta R = R - R_c \quad (7a)$$

$$\Delta K_e = K_e - K_{ec} \quad (7b)$$

$$\Delta i_q = i_{qcom} - i_q \quad (7c)$$

$$\Delta i_d = i_{dcom} - i_d \quad (7d)$$

$$\Delta\omega_e = \omega_e - \omega_{ec} \quad (7e)$$

$$\Delta\omega_m = \omega_m - \omega_{mc} \quad (7f)$$

Assuming the system is in equilibrium: $\frac{di_q}{dt} = 0$, $\frac{di_d}{dt} = 0$, and neglecting the high order error, we get (8) from (6).

$$\Delta R_{eq} i_{qcom} + \Delta K_{eeq}\omega_m = R\Delta i_q - K_e\Delta\omega_m + \omega_e L\Delta i_d - Li_{dcom}\Delta\omega_e , \quad (8)$$

where ΔR_{eq} and ΔK_{eeq} are the errors under equilibrium condition. For most real motors, $L << R$ and $L << K_e$; because of mechanical inertia, $\Delta\omega << \Delta i_q$. Therefore, the last 3 terms in the right side of (8) can be possibly neglected if compare to the first term. In the mean time, it is reasonable to assume that these errors will not change significantly in one sampling period of 0.002 second. Thus we can find the value of ΔR_{eq} and ΔK_{eeq} by solving following equations

$$\Delta R_{eq} i_{qcom}(k) + \Delta K_{eeq}\omega_m(k) = R\Delta i_q(k) , \quad (9a)$$

$$\Delta R_{eq} i_{qcom}(k+1) + \Delta K_{eeq}\omega_m(k+1) = R\Delta i_q(k+1) . \quad (9b)$$

The estimated error $\Delta\hat{R}(t)$ and $\Delta\hat{K}_e(t)$ for the estimation scheme in (4) and (5) via feedback current error integration was proposed based on heuristics as

$$\Delta\hat{R} = \frac{[\omega_m(k+1)R\Delta i_q(k) - \omega_m(k)R\Delta i_q(k+1)]}{i_{qcom}(k)\omega_m(k+1) - i_{qcom}(k+1)\omega_m(k)} \quad (10a)$$

$$\Delta\hat{K}_e = \frac{[-i_{qcom}(k+1)R\Delta i_q(k) + i_{qcom}(k)R\Delta i_q(k+1)]}{i_{qcom}(k)\omega_m(k+1) - i_{qcom}(k+1)\omega_m(k)} \quad (10b)$$

The single parameter estimation scheme can be proven to be stable using theorems about bounded-ness of perturbation [8]. It may be possible to extend the proof to multi-parameter estimation algorithm. Here we only confirm the stability by simulation. Fig. 2 shows the performance of above estimation scheme in an open-loop simulation.

In order to evaluate the estimation scheme, simulated random required torque T_{com} and motor speed ω_m (with a lower dominant frequency than that of T_{com}) are fed into the motor inverse model, the parameter estimation model and the motor model. To avoid the error caused by singularity points in (10), $k_1(t)$ and $k_2(t)$ in (10) are set as following:

$$k_1(t) = k_2(t) = \begin{cases} 0.1 & \det(k) \ge 0.01 \\ 0 & \det(k) < 0.01 \end{cases} , \quad (11)$$

where $\det(k) = i_{qcom}(k)\omega_m(k+1) - i_{qcom}(k+1)\omega_m(k)$. Root mean square (*RMS*) value of the state variables is used to approximate the bound on each error, as shown in

$$b(\Delta) = 6 \cdot RMS(\Delta) . \quad (12)$$

R_c and K_{ec}, beginning at the initial error of ±10% and ±6%, reached the bound of $b(\Delta R) = 7.1656\times10^{-4}$ and $b(\Delta K_e) = 2.5125\times10^{-4}$ within 80 seconds.

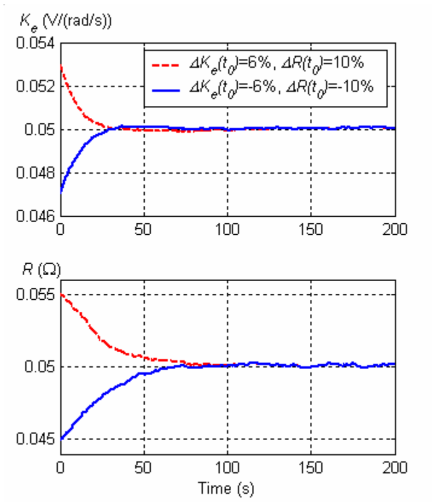


Fig. 2 Open-loop simulation of two-parameter estimation scheme.

III. IMPROVING THE DYNAMIC PERFORMANCE OF THE ADAPTIVE ALGORITHM

The basic parameter estimation scheme of (10) is derived from the non-dynamic motor inverse model of (3) and the current error feedback scheme in (9). The non-dynamic motor inverse model neglected motor electrical dynamics and introduced an error corresponding to neglected dynamics $\frac{di_q}{dt}$ and $\frac{di_d}{dt}$. If the dynamics are not neglected while solving (1) to obtain an algorithm for the applied voltage *V*, it is possible to reduce errors introduced in the earlier estimation approach. The estimation scheme of (10) neglected the effect of $\Delta\omega_m$ and $\Delta\omega_e$, which were caused by the sampling delay. This also introduced error to the basic parameter estimation scheme. If the motor speed error can be taken into the estimation scheme, the performance of the adaptive algorithm can be possibly improved.

3.1. Motor electrical dynamics

An analytical solution of (1) is not available because of its nonlinearity, as ω_m and ω_e are functions of time. However, since the motor is driving the mechanical inertia, the electrical states can change significantly in duration of the order of the electrical time constant τ, whereas the mechanical states, e.g. the motor speed ω_m, can hardly change over the same period due to slow dynamics of the mechanical system. It is possible to approximate the motor electrical dynamics by assuming ω_m is a constant during each sampling interval *T*, which is of the order of the electrical time constant τ. With this assumption, (1) becomes a finite dimensional linear time-invariant state equation during the sampling interval *T*. This equation can be solved exactly via the state transition matrix [9].

Now consider (1) in matrix form as following equation

$$\dot{y} = \frac{1}{\tau}\begin{pmatrix} -1 & -\zeta \\ \zeta & -1 \end{pmatrix}\cdot y + \frac{1}{\tau}\begin{pmatrix} \frac{\cos\delta}{R} & \frac{-K_e}{R} \\ \frac{-\sin\delta}{R} & 0 \end{pmatrix}\cdot u \qquad (13)$$

for $kT \le t \le (k+1)T$, where $y = \begin{pmatrix} i_q \\ i_d \end{pmatrix}$, $u = \begin{pmatrix} V \\ \omega_m \end{pmatrix}$, $\tau = \frac{L}{R}$, $\zeta = \frac{\omega_e L}{R}$.

Equations (13) can be solved with the state transition matrix

$$\Phi(kT,t) = e^{-\frac{t-kT}{\tau}}\begin{pmatrix} \cos[\omega_e(t-kT)] & -\sin[\omega_e(t-kT)] \\ \sin[\omega_e(t-kT)] & \cos[\omega_e(t-kT)] \end{pmatrix} \qquad (14)$$

for $kT \le t \le (k+1)T$.

With this state transition matrix, the solution of (13) is:

$$x[(k+1)T] = \Phi[kT,(k+1)T]\cdot x(kT) + \int_{kT}^{(k+1)T}\Phi(kT,\theta)Gud\theta, \qquad (15)$$

where $G = \begin{pmatrix} \frac{\cos\delta}{R} & \frac{-K_e}{R} \\ \frac{-\sin\delta}{R} & 0 \end{pmatrix}$.

G and *u* in the above equation can be treated as constants during each sampling interval because of the zero-order-hold sampling.

Given the desired torque $T_{com}(k)$, which is the expected output at $t = (k+1)T$, the value of the state variable $i_q(k+1)$ is expected to be $i_{qcom}(k)$. This value can be obtained from $i_{qcom}(k) = \frac{T_{com}(k)}{K_e}$ and the unknowns $V(k)$ can be solved using (15) as

$$V(k) = \frac{1}{I\frac{\cos\delta}{R\tau} + J\frac{\sin\delta}{R\tau}}\left\{i_{qcom}(k) + I\frac{K_e}{R\tau}\omega_m - e^{-\frac{T}{\tau}}[i_q(k)\cos(\omega_e T) - i_d(k)\sin(\omega_e T)]\right\}, \qquad (16)$$

where

$$I = \frac{(\omega_e\tau)^2}{(\omega_e\tau)^2+1}\left[\frac{1}{\omega_e}e^{-\frac{T}{\tau}}\sin(\omega_e T) + \frac{1}{\omega_e{}^2\tau}(1-e^{-\frac{T}{\tau}}\cos(\omega_e T))\right] \qquad (17a)$$

$$J=\frac{(\omega_e\tau)^2}{(\omega_e\tau)^2+1}\left[\frac{1}{\omega_e}(1-e^{-\frac{T}{\tau}}\cos(\omega_e T))-\frac{1}{\omega_e{}^2\tau}e^{-\frac{T}{\tau}}\sin(\omega_e T)\right]. \quad (17b)$$

Compared to the non-dynamic motor inverse model in (3), the state transition matrix method approximated the electrical dynamics of the motor. Therefore, when the motor speed is constant or changed at a lower frequency compare to the torque, the algorithm with the state transition matrix method can track the required torque with higher fidelity if the parameters in the controller match the actual values in the motor. If there is discrepancy between the parameter value in motor and in the controller, the output current error will more likely represent this discrepancy. Consequently, the performance of the estimation scheme and the controller will be improved.

Fig. 3 shows comparison of the performance of estimation scheme with the non-dynamic motor inverse model and the dynamic motor inverse model in the open-loop simulation. A random T_{com} and constant ω_m were fed into the motor inverse algorithm so as to compare the performances of the non-dynamic algorithm and the state transition matrix algorithm. The gain $k_1(t)$ and $k_2(t)$ are set to the same value for both model, as in (11).

In the model with the approximate dynamic motor inverse model (case 2), the steady state error bounds (after 150s) are $b(\Delta R)=4.492\times10^{-5}, b(\Delta K_e)=3.365\times10^{-6}$, which are about 17.5% and 29.4% of corresponding values in the model of basic scheme (case 1). The overshoot of K_{ec} in case 2 is 47% of that in case 1. The rise time of R_c in case 2 is about 10 seconds shorter than in case 1.

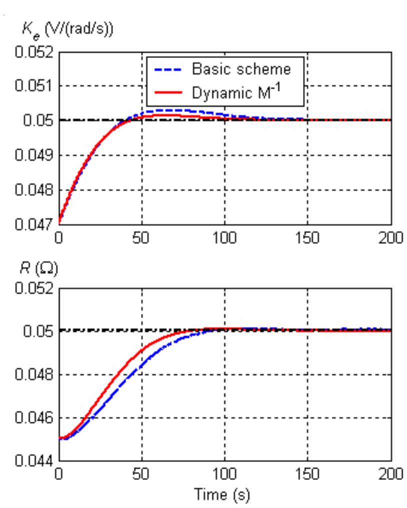


Fig. 3 Performance comparison of the two-parameter estimation scheme between case 1 and case 2

3.2. Motor speed sampling delay

While calculating the voltage using (3) or (16), the desired q-axis current during the next sampling period is known but the motor velocity during the next sampling period is not. The error $\Delta\omega_m$ is caused by the use of the latest available sampled motor velocity instead, and, it is unavoidable. The parameter estimation scheme of (9) neglected this error caused by the sampling delay. It is possible to reduce the bound of error during estimation if $\Delta\omega_m$ is taken into account.

The motor velocity is not constant during the next sampling period but is varies from $\omega_m(k)$ to $\omega_m(k+1)$. It is reasonable to approximate the motor velocity error by $\Delta\omega_m(k)=\frac{1}{2}[\omega_m(k+1)-\omega_m(k)]$ during the next sampling period. Therefore, the estimation scheme becomes

$$\Delta R_{eq}i_{qcom}(k)+\Delta K_{eeq}\omega_m(k)= R\Delta i_q(k)-K_e\Delta\omega_m(k)-Li_{dcom}(k)\Delta\omega_e(k), \quad (18a)$$

$$\Delta R_{eq}i_{qcom}(k+1)+\Delta K_{eeq}\omega_m(k+1)= R\Delta i_q(k+1)-K_e\Delta\omega_m(k+1)-Li_{dcom}(k+1)\Delta\omega_e(k+1). \quad (18b)$$

$$\Delta\hat{R}=\frac{[\omega_m(k+1)\Delta S(k)-\omega_m(k)\Delta S(k+1)]}{i_{qcom}(k)\omega_m(k+1)-i_{qcom}(k+1)\omega_m(k)}, \quad (19a)$$

$$\Delta\hat{K}_e=\frac{[-i_{qcom}(k+1)\Delta S(k)+i_{qcom}(k)\Delta S(k+1)]}{i_{qcom}(k)\omega_m(k+1)-i_{qcom}(k+1)\omega_m(k)}. \quad (19b)$$

where $\Delta S(k)=R\Delta i_q(k)-K_e\Delta\omega_m(k)-Li_{dcom}(k)\Delta\omega_e(k)$.

Fig. 4 shows the performance comparison of the estimation scheme with and without the approximate $\Delta\omega_m$ compensation. Inputs for the open-loop system simulation are a random T_{com} and a random ω_m.

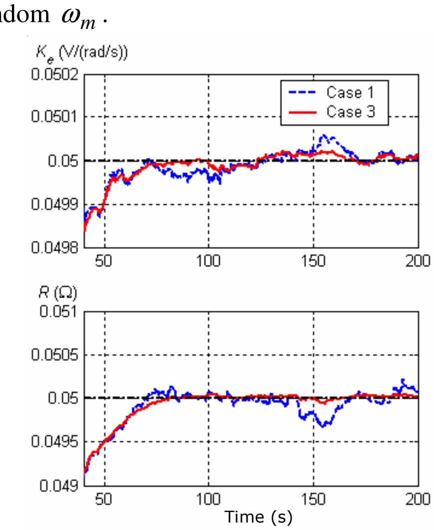


Fig 4. Performance comparison of estimation scheme with and without $\Delta\omega_m$ compensation

As shown in Fig. 4, the scheme with $\Delta\omega_m$ compensation (case 3) has lower steady state error bounds (after 150s): $b(\Delta R) = 1.638\times10^{-4}$, $b(\Delta K_e) = 6.39\times10^{-5}$, which are about 19.6% and 42.8% of corresponding values in the basic estimation model (case 1).

Since the two ways of performance improvement discussed before are working on motor inverse model and parameter estimation algorithm respectively, it is possible to combine them together and tighten the bound of error, thereafter further improve the precision of the system.

IV. CLOSED-LOOP SYSTEM SIMULATIONS

In the above open loop simulations, uncorrelated ω_m and T_{com} were used to compare the performance of the adaptive algorithm. However, in the application of the closed-loop control of a system, ω_m and T_{com} cannot be completely uncorrelated because that ω_m and T_{com} are both excited by the plant input in Fig.1. In order to demonstrate the performance of the derived algorithm in a closed loop setting, its application to an electric power steering system is considered.

As shown in Fig. 5, the closed-loop automotive electric power steering system consists of a controller, a BLDC motor, torque sensor, a steering hand wheel, and the rack and pinion. The torque of the motor is transferred to the steering column via a worm and worm gear assembly. Torque on the steering column is measured by the torque sensor (T_s). The motor speed is measured by a tachometer. These signals along with motor position are collected by the controller, which generates T_{com} accordingly and calculates the voltage to be applied to the motor. The steering system plant is modeled by a two-mass mechanical system with viscous friction [10]. The controller is designed to stabilize the system.

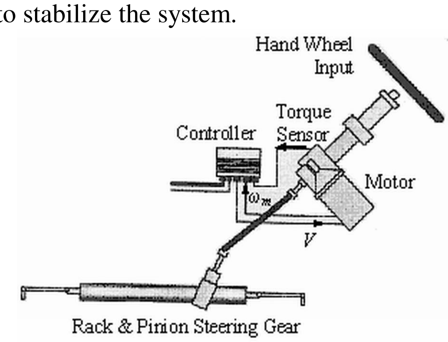
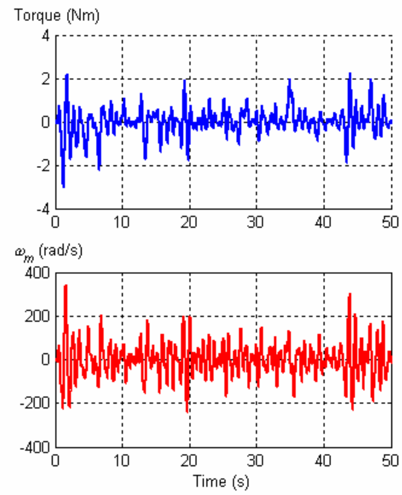
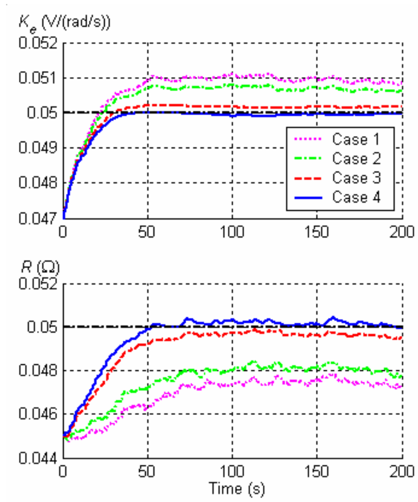


Fig. 5. Schematic diagram of the automotive electric power steering system [10].

In the closed-loop model, the nominal parameter values are as following: $L = 0.0001\text{H}$, $R = 0.05\Omega$ and $Ke = 0.05\text{V/(rad/s)}$. A random hand-wheel torque is applied to the plant model. Consequently, correlated ω_m and T_s are measured by the tachometer and the torque sensor, and are sent to the controller as inputs. A low pass filter with a cut-off frequency $f_c \approx 0.8Hz$ is added to the input of estimation scheme to suppress the measurement noise and act as an anti-aliasing filter. Fig. 6(a) shows the T_{com} generated from the controller and ω_m measured from the plant. Fig. 6(b) compares the bounds of error (ΔR, ΔK_e and Δi_q) in the four different cases of estimation scheme in the closed-loop simulation.



(a). Input: T_{com} and ω_m



(b). parameter estimation performance

Fig. 6. Closed loop simulation :

If $\Delta\omega_m$ is neglected from the parameter estimation and frequency of ω_m is high, high error will be introduced. This error should be theoretically symmetric about zero for uncorrelated random signals T_{com} and ω_m. However, T_{com} and ω_m are excited by the same driver torque input, resulting in similarity of phases of these two signals, as shown in Fig. 6(a). This may result in offset error for the estimated parameters, as in Fig. 6(b). Case 3 and case 4 both implemented the approximate $\Delta\omega_m$ compensator, which resulted in a lower bound of error. Compared to case 3, case 4 implemented the approximate dynamic motor inverse model, which helped to tighten the bound of error in parameter implementation. Table 1 compares the bound of errors with the four cases of parameter estimation scheme in the closed loop simulation.

TABLE. 1 COMPARISON OF THE BOUND OF ERRORS IN CLOSED LOOP SIMULATION

Case	Motor Inverse Model	Estimation scheme	Bound of ΔR ($\times 10^{-3}$)	Percentage of $b(\Delta R)$ compared to Case #1	Bound of ΔK_e ($\times 10^{-4}$)	Percentage of $b(\Delta K_e)$ compared to Case #1
1	Non-Dynamic equation (3)	Basic scheme equation (10)	17.0	100%	55.77	100%
2	Dynamic equation (16)	Basic scheme equation (10)	13.06	76.8%	40.18	72.04%
3	Non-dynamic equation (3)	$\Delta\omega_m$ compensator equation (19)	3.35	19.7%	9.42	16.9%
4	Dynamic equation (16)	$\Delta\omega_m$ compensator equation (19)	1.77	10.43%	3.76	6.75%

V. CONCLUSION

An adaptive multi-parameter estimation scheme for a BLDC motor is derived. Open loop simulation shows the effectiveness of the estimation scheme. Two suggested improvements are proposed for the scheme, *viz*., approximation of the motor electrical dynamics in the motor inverse model and compensator of the motor speed sampling delay error. The state transition matrix method and compensator of sampling delay of motor speed were implemented to realize the suggested performance improvements, respectively. In the application to an electric power steering system, where T_{com} and ω_m are both excited by the same driver input, the motor speed error compensation had a big performance payoff for the estimator as shown in the relative performance comparison of Table. 1, while the state-transition matrix based motor inverse model had a marginal payoff in comparison. For the best performance, it is desirable to implement both these suggestions as in case 4 of Table. 1. The motor speed error correction is computationally inexpensive and the state transition matrix is relatively computationally expensive. If the computational resources available in the controller are low, then only the motor speed error compensation should be implemented as in case 3. If we are solely considering the performance of the estimator, implementing case 2 alone does not make sense given the expense and the availability of case 3.

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